2) L'Aire de la surface ?

$$Air(S) = \int_0^{2\pi}\int_0^{2\pi} \| f_u \wedge f_v \| \, du\, dv$$

$$= \int_0^{2\pi}\int_0^{2\pi} r(R + r\cos u) \, du\, dv$$

$$= rR \int_0^{2\pi}\int_0^{2\pi} du\, dv + r^2 \int_0^{2\pi}\int_0^{2\pi} \cos u \, du\, dv$$

$$= 4\pi^2 rR + 2\pi r^2 \left[\sin u\right]_0^{2\pi}$$

$$= \boxed{4\pi^2 rR}$$

3) $C1 = \{ f(0,v) / v \in [0,2\pi] \}$

$$= ((R+r)\cos v, (R+r)\sin v, 0)$$

$C1$ cercle de centre $(0,0,0)$ et de rayon $(r+R)$ dans le plan (xy)

$C2 = \{ f(u,0) / u \in [0,2\pi] \}$

$$= ((R + r\cos u), 0, r\sin u)$$

Cercle de centre $(R,0,0)$ et de rayon r dans (xz)

4) On a calculer $f_u \wedge f_v \neq \begin{pmatrix}0\\0\\0\end{pmatrix}$ donc $f_u \perp f_v$

5) Courbure de Gauss $K(u,v)$?

$$G(u,v) = \begin{pmatrix} \langle f_u, f_u\rangle & \langle f_u, f_v\rangle \\ \langle f_v, f_u\rangle & \langle f_v, f_v\rangle \end{pmatrix}$$

$$= \begin{pmatrix} r^2 & 0 \\ 0 & r^2(R+r\cos u)^2 \end{pmatrix}$$

$$\boxed{\det G(u,v) = r^4 (R + r\cos u)^2}$$

$$N = \frac{f_u \wedge f_v}{\| f_u \wedge f_v \|} = \begin{pmatrix} -\cos u \cos v \\ -\cos u \sin v \\ \cancel{\tfrac{R}{r}} \sin u \end{pmatrix}$$

$$\frac{\partial^2 f}{\partial^2 u} = \begin{pmatrix} -r\cos u \cos v \\ -r \cos u \sin v \\ -r \sin u \end{pmatrix}, \quad \frac{\partial^2 f}{\partial^2 v} = \begin{pmatrix} -(R+r\cos u)\cos v \\ -(R+r\cos u)\sin v \\ 0 \end{pmatrix}$$

$$\frac{\partial^2 f}{\partial u \partial v} = \begin{pmatrix} r \sin u \sin v \\ -r \sin u \cos v \\ 0 \end{pmatrix}, \quad \frac{\partial^2 f}{\partial v \partial u} = \begin{pmatrix} +r \sin u \sin v \\ -r \sin u \cos v \\ 0 \end{pmatrix}$$

$$H(u,v) = \begin{pmatrix} \left\langle \frac{\partial^2 f}{\partial^2 u}, N \right\rangle & \left\langle \frac{\partial f}{\partial u \partial v}, N \right\rangle \\ \left\langle \frac{\partial^2 f}{\partial u \partial v}, N \right\rangle & \left\langle \frac{\partial^2 f}{\partial^2 v}, N \right\rangle \end{pmatrix}$$

$$= \begin{pmatrix} r^2 \cos^2 u + R \sin^2 v & 0 \\ 0 & (R + r\cos u)\cos u \end{pmatrix}$$

$$K_{Gauss}(u,v) = \frac{\det H(u,v)}{\det G(u,v)}$$

$$= \frac{(r^2 \cos^2 u + R \sin^2 v)\cos u}{r^4 (R + r\cos u)}$$

Exo 5

$$G(v,u) = \begin{pmatrix} \frac{1}{u^2} & 0 \\ 0 & \frac{1}{u^2} \end{pmatrix}$$

$$c(t) = (0,t) \quad , \quad \gamma(t) = f(c(t))$$

1) $L(\gamma) = ?$

$$L(\gamma) = \int_a^b \sqrt{\langle G(c(t))\dot{c}(t), \dot{c}(t) \rangle}\, dt$$

$$G(c(t)) = \begin{pmatrix} \frac{1}{t^2} & 0 \\ 0 & \frac{1}{t^2} \end{pmatrix} \quad , \quad \dot{c}(t) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$G(c(t)) \cdot \dot{c}(t) = \begin{pmatrix} 0 \\ \frac{1}{t^2} \end{pmatrix} ,$$

$$L(\gamma) = \int_a^b \sqrt{\left\langle \begin{pmatrix} 0 \\ \frac{1}{t^2} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\rangle}\, dt = \int_a^b \frac{1}{t}\, dt = \ln b - \ln$$

2) $\lim\limits_{a \to 0} L(\gamma) = +\infty$