

Ex 04

$$f: [0, 2\pi] \times [0, 2\pi] \longrightarrow \mathbb{R}^3, \quad 0 < r < R$$

$$(u, \vartheta) \longmapsto \begin{pmatrix} (R+r\cos u)\cos\vartheta \\ (R+r\cos u)\sin\vartheta \\ r\sin u \end{pmatrix}$$

1) Régularité?

$$f_u(u, \vartheta) = \begin{pmatrix} -r\sin u \cos\vartheta \\ -r\sin u \sin\vartheta \\ r\cos u \end{pmatrix}, \quad f_\vartheta(u, \vartheta) = \begin{pmatrix} -(R+r\cos u) \\ (R+r\cos u) \\ 0 \end{pmatrix}$$

$$(f_u \wedge f_\vartheta)(u, \vartheta) = \begin{pmatrix} -r(R+r\cos u)\cos u \cos\vartheta \\ -r(R+r\cos u)\sin u \cos\vartheta \\ -R(\sin u)(R+r\cos u) \end{pmatrix}$$

$$\|f_u \wedge f_\vartheta\|^2 = r(R+r\cos u) \neq 0 \text{ pour } r \neq 0 \\ \text{et } r < R$$

donc la surface est régulière