Timed Automata: Automates Temporisés

Cours TOV

M1: GLSD

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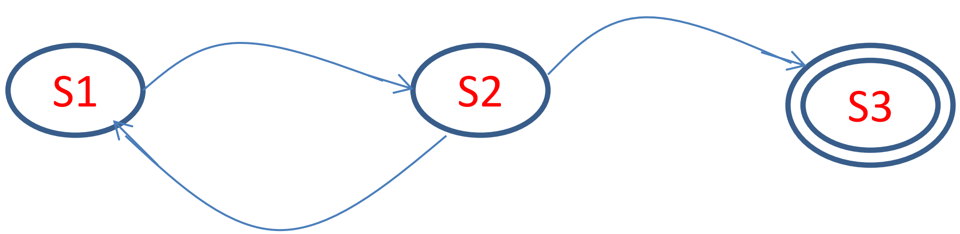
Outlines

1. Introduction : motivation (why?): actions, clocks, co-actions, reset, guards, invariants;
2. Definition: informal, formal
3. Semantics of Timed Automata: the use of LTSs (Labelled transition systems)
4. Networks of TA and their semantics

Timed Automata why? (1)

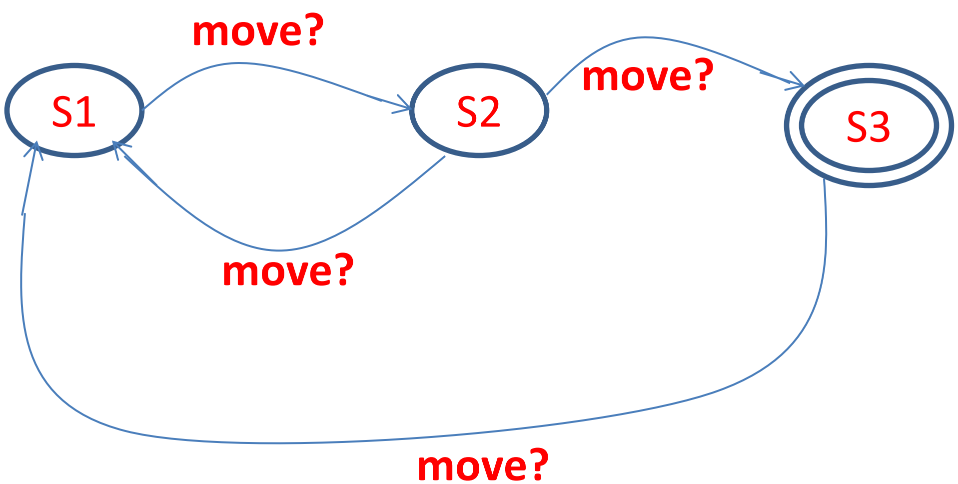
- To give more **<u>expressiveness</u>** for models;
- Example:

  Modelling a train which serves three stations S1, S2, S3. The train can move from S1 to S2, from S2 to S3, from S2 to S1, and from S3 to S1:



Timed automata
why? (2): actions and co-actions

- Then: in order to move from station to station, the train **<u>needs to receive a command</u>**: “move”. The model must be as following.



- **move?** Is **<u>an action</u>** and **move!** Is called its **<u>co-action</u>** (and vice versa)

Timed automata why? (3)

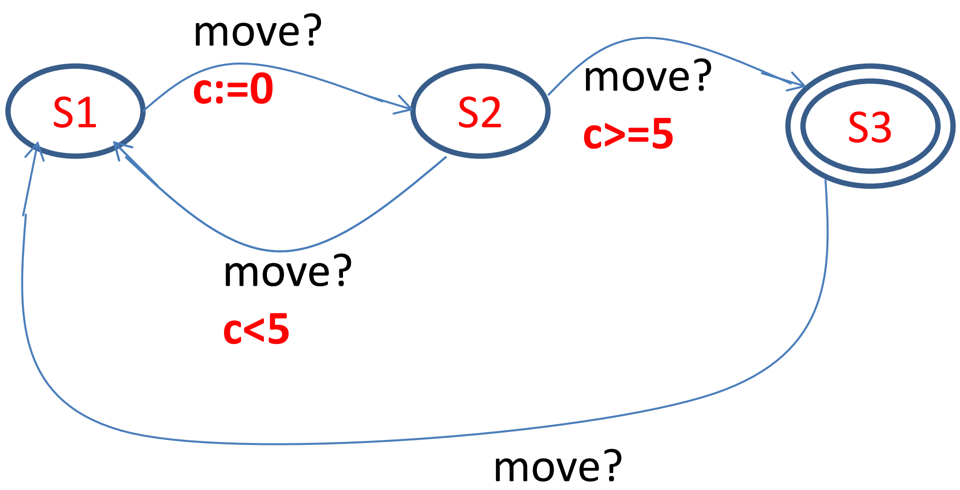
- Next: in order to move from station to station the train uses a **clock c**.

- If the train is in the s1 and it receives the command "move", it **resets c** and moves to s2.

- In s2, if the train receives the command **before 5 seconds** then it will return to S1 else it will go to S3.

- In s3, the train **waits** the command to **return** to S1.

Timed automata why? (4): guards and resets



- C:=0 is called a **reset action**;
- C<5, c>=5 are called **guards**. They are the conditions to be fulfilled to transit the edge;

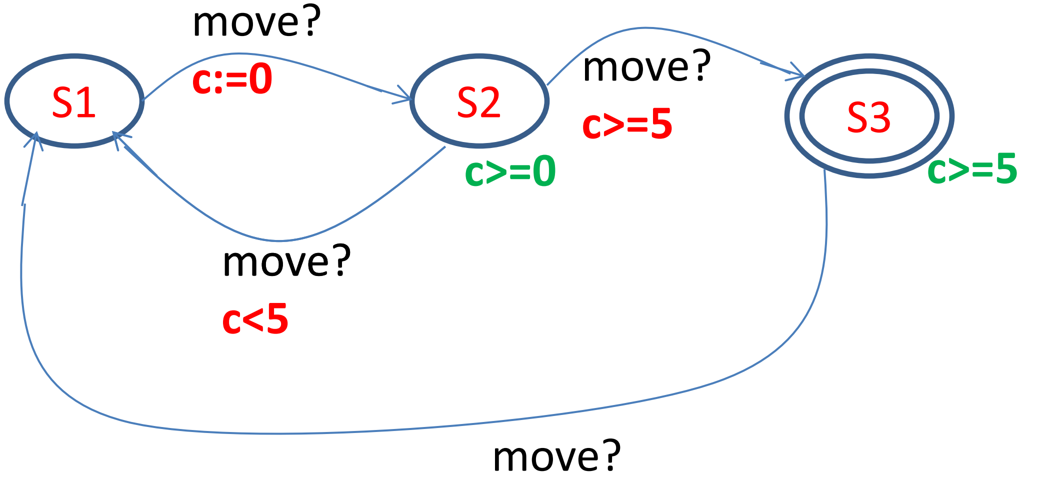
Timed automata why? (6): Invariants

- Finally:

-On the stations S2, we have always c>=0,

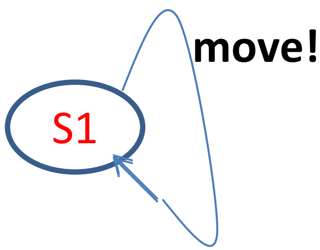
-On the station S3, we have always c>=5.

These two logic expressions are called **invariants**.



Timed automata why?(7): synchronisation

- The previous automaton can be, now, synchronized with the following one:



Timed automata
Definition

- A timed automaton (informally):
  - a **finite-state** machine;
  - extended with **clock variables (real values);**
  - All the clocks **progress synchronously**.

Timed automata:
Formal Definition (1)

A timed automaton is a tuple:

TA=(***L***, l_0, ***C***, ***A***, ***E***, ***Inv***), suh that:

- ***L*** is a set of locations,
- $l_0 \in L$ is the initial location,
- ***C*** is the set of clocks,

Timed automata:

Formal Definition (2)

TA=($\boldsymbol{L, l_0, C, A, E, Inv}$),

- ***A*** is a set of **actions**, **co-actions** and the internal **τ-action**,
- $\boldsymbol{E \subseteq L \times A \times B(C) \times 2^C \times L}$ is a set of edges between **locations** with an **action**, a **guard** and a **set of clocks to be reset**,
- $\boldsymbol{Inv : L \rightarrow B(C)}$ assigns **invariants** to locations.

Timed automata:
Formal Definition (3)

Remark:

B(C) is the set of **conjunctions** over simple conditions of the form $c \; \alpha \; n$ or $c_1 - c_2 \; \alpha \; n$, where:

- $c, c_1, c_2 \in C$ (i.e. c, c_1 and c_2 are clocks)
- $n \in N$ (n is a natural number)
- $\alpha \in \{<, \leq, =, \geq, >\}$

Timed automata:

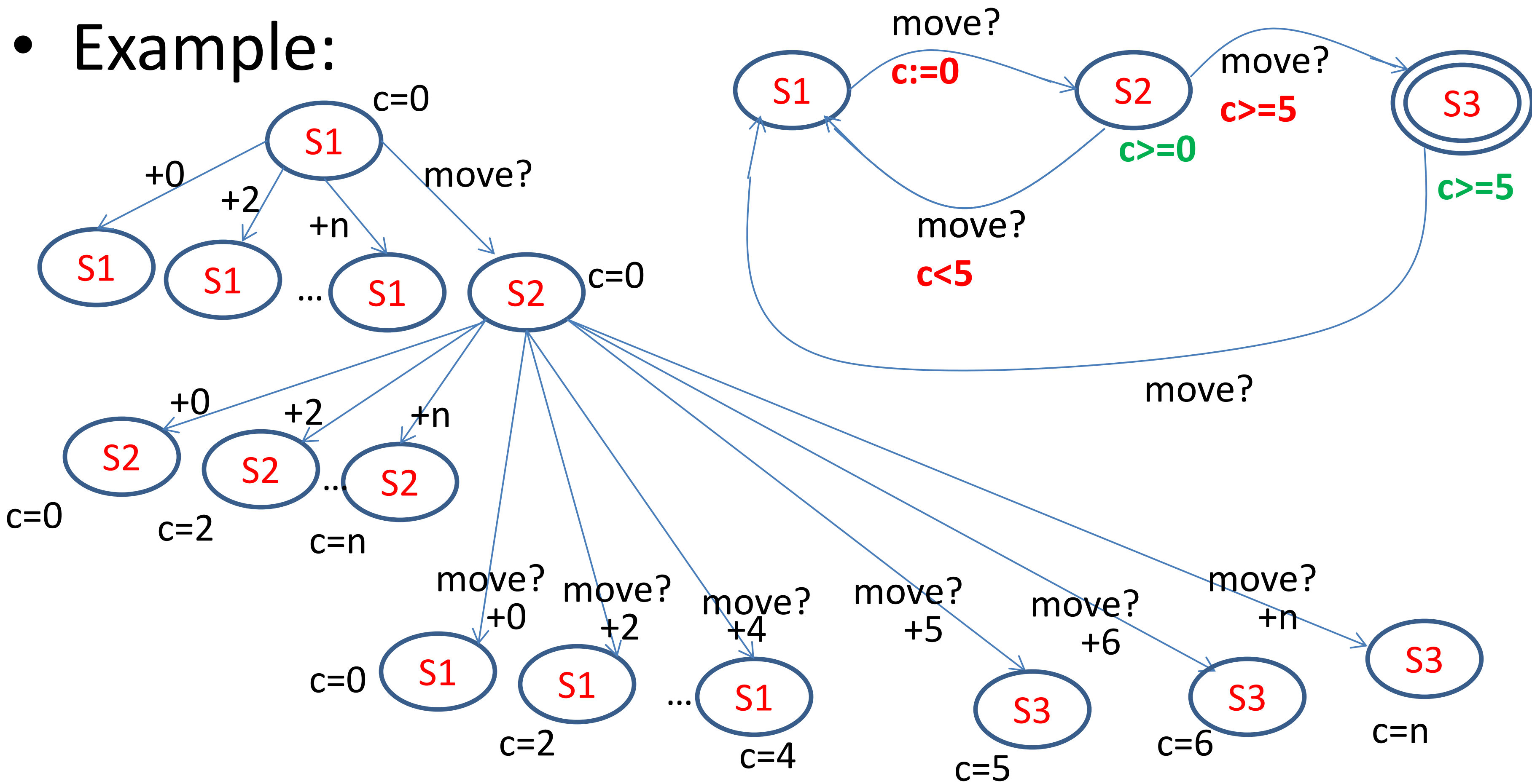
Semantics: How the automaton is executed? (1)

- The semantics of a timed automaton is given through an **LTS** (**Labelled Transitions System**),
- An **LTS** is an **infinite automaton** and it represents the execution of the Timed Automaton,
- Each state of the LTS represents a "**state of the TA**" with a "**valuation of the set of clocks**" (i. e: values of the clocks at this state),
- The "**initial state**" of the LTS represents the "**initial location**" of the TA with the "**initialisation"** of the set of clocks,
- Each edge in the LTS will be **labelled**

Timed Automata:

Semantics: How the automaton is executed? (2)

- Example:



Timed automata:

Semantics: How the automaton is executed? (3)

- The execution of the automaton means to update the values of the clocks ➔ clocks will have values by a function ***u***

- The function ***u*** (the valuation of clocks) is defined as: $u : C \rightarrow R_{\geq 0}$

- We have:

  $u_0(c)=\mathbf{0}$ for each clock c in C

  R^C is the set of all clock valuations

Timed automata:

Formal Semantics

If TA=($\boldsymbol{L, l_0, C, A, E, Inv}$) is a timed automaton,

then its LTS is the triple: ($\boldsymbol{S, s_0, \rightarrow}$), where:

- $\boldsymbol{S \subseteq L \times R^C}$ (each state in the LTS is a location in the AT with the valuation of clocks at this location)
- $\boldsymbol{s_0 = (l_0, u_0)}$ (the initial state in the the LTS is the intial location in te AT with the initialisation of all the clocks)
- $\boldsymbol{\rightarrow \subseteq S \times (R_{\geq 0} \cup A) \times S}$ (a triple composed of a source state, a destination state, and a label of the edge. The label is a couple: values of clocks and an action)

Timed automata:
Formal Semantics

The transition relation → is defined for each clock ***x*** as:

(1) A delay transition: The clock ***x*** changes in the same location:

$$(l, u(x)) \rightarrow^{d} (l, u(x)+d)$$

if ∀d': $0 \leq d' \leq d \Rightarrow (u(x)+d') \in Inv(l)$,

Timed automata:
Formal Semantics

(2) **An action transition**: The location changes from ***l*** to ***l' :***

$$(l,\ u(x)) \rightarrow^{a} (l',\ u'(x))$$

if there exists an edge $e=(l,\ a,\ g,\ r,\ l') \in E$

(***l*** for source location, ***a*** for action, ***g*** for guard, ***r*** for clocks to be reset, and ***l'*** for destination location)

such that:

- $u(x) \in g$**:** means ***u(x)*** satisfies the guard ***g***
- $u'(x) \in Inv(l')$**:** means ***u' (x)*** satisifes the invariant in ***l'***
- $u'(x)=u_0(x)$**:** resets the clock ***x*** to be reset (***x*** in ***r***)

Network of Timed Automata

- The real systems require the use of **several** synchronised automata;

- These automata **share** a set of actions and clocks;

- How the **semantics** of a network of automata will be defined?

Network of Timed Automata

« composition »

Let A1=($\boldsymbol{L_1, l_{01}, C_1, A_1, E_1, Inv_1}$) and A2= $\boldsymbol{(L_2, l_{02}, C_2, A_2, E_2, Inv_2}$).

The composition of A1 and A2 is the TA :

A3=($\boldsymbol{L_1 x L_2, (l_{01}, l_{02}), C_1} \cup \boldsymbol{C_2, A_1} \cup \boldsymbol{A_2, E, Inv}$),

Network of Timed Automata

« composition »

such that:

- ***E***: for each two edges: ***e1=(l1, a1, g1, r1, l'1) , e2=(l2, a2, g2, r2, l'2),*** we have two cases:

- Progression in one automaton

e=((l1,l2), a1, g1, r1, (l'1,l2))

or

e=((l1,l2), a2, g2, r2, (l1,l'2))

- Synchronisation

e=((l1,l2), a1a2, g1 $\wedge$ ***g2, r1*** $\cup$ ***r2, (l'1,l'2))***

- For all ***(l, l')*** in $L_1 x L_2$: ***Inv((l,l'))=Inv(l)***$\wedge$***Inv(l')***

Networks of Timed Automata

« Semantics: Formally»

- Let $A_i = (L_i, l^0_i, C, A, E_i, I_i)$ be a network of ***n*** timed automata. Let $l_0 = (l_{01}, \ldots, l_{0n})$ be the initial locations vector.

- The semantics is defined as a transition system $(S, s_0, \rightarrow)$, where:

  - $S = (L_1 \times \ldots \times L_n) \times R^C$ is the set of states,
  - $s_0 = (l_0, u_0)$ is the initial state,
  - $\rightarrow \subseteq S \times S$ is the transition relation defined by:

Networks of Timed Automata

« Semantics: informally»

Informally, Three cases are possible

1) A **delay transition**: where the network does not change location. Only **a progression in the clocks**. This progression must satisfy the invariants of the location;

2) A **silent transition**: **one location** is changed in the vector of locations. The silent transition must reset the necessary clocks;

3) A **synchronisation transition**: **two locations** will be updated in the vector. This change must respect the invariant of the new location.

Networks of Timed Automata

« Semantics: Formally»

-**delay transition**: $(l\,,u) \rightarrow^{d} (l, u+d)$

$\text{if } \forall d' : 0 \leq d' \leq d \Rightarrow u + d' \in Inv(l).$

- **Silent Action transition**: $(l\,,u) \rightarrow^{\tau} (l[l'_i \,/\, l_i], u')$

if there exists $l_i \rightarrow^{\tau} l'_i$ *such that* $u \in g$, $u' = [r \rightarrow 0]u$ *and* $u' \in Inv(l[l'_i / l_i])$.

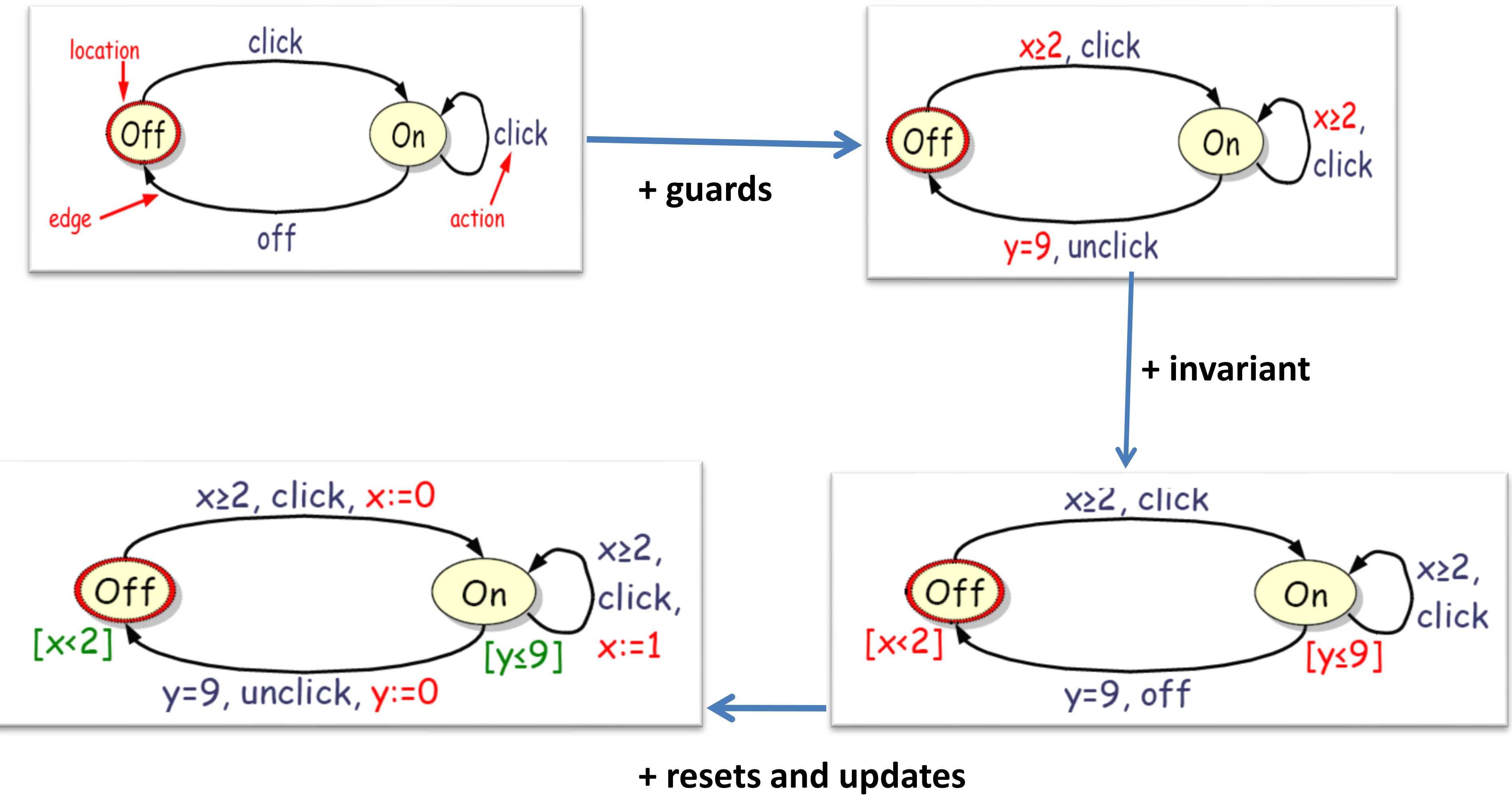
- **Synchronisation Action transition** : $(l\,,u) \rightarrow^{a?a!} (l[l'_j / l_j, l'_i / l_i], u')$

if there exist $l_i \rightarrow^{a?} l'_i$ and $l_j \rightarrow^{a!} l'_j$

such that :

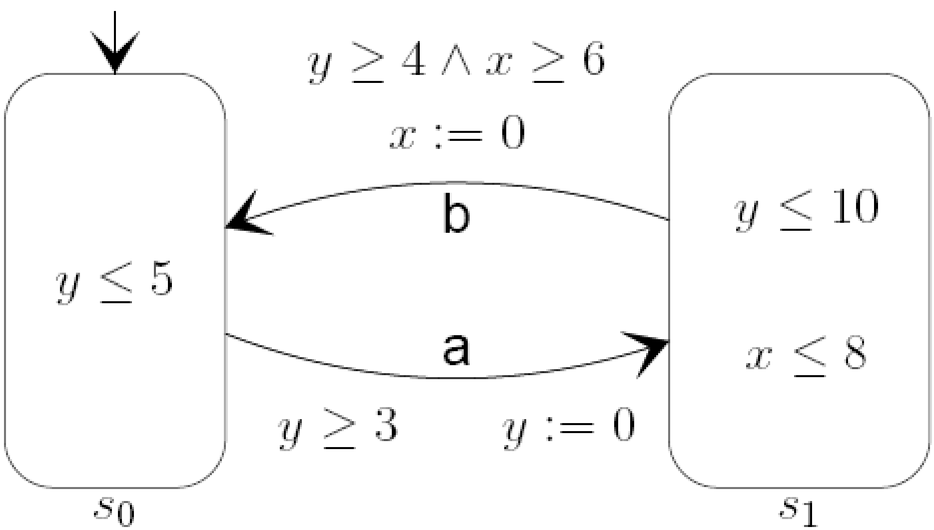
$u \in (g_i \wedge g_j)$, $u' = [r_i \cup r_j \rightarrow 0]u$ and $u' \in Inv(l\,[l'_j / l_j, l'_i / l_i])$.

Example 1

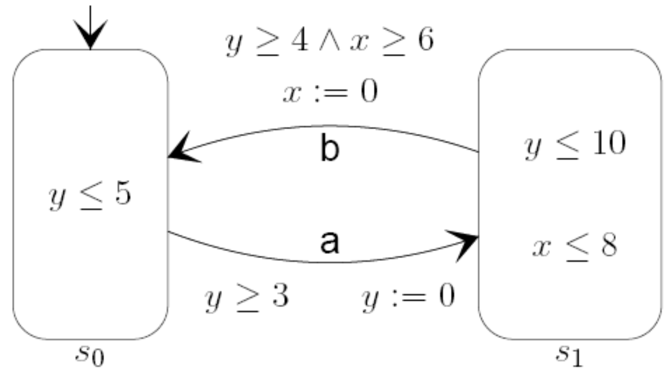


location
click
Off
On
click
edge
off
action
+ guards
x≥2, click
Off
On
x≥2,
click
y=9, unclick
+ invariant
x≥2, click
Off
On
x≥2,
click
[x<2]
[y≤9]
y=9, off
x≥2, click, x:=0
Off
On
x≥2,
click,
x:=1
[x<2]
[y≤9]
y=9, unclick, y:=0
+ resets and updates

Example 2



Example 2: execution



(l,u)=(S_i, x, y)

$$(s0,0,0) \longrightarrow^{3} (s0,3,3) \longrightarrow^{a} (s1,3,0) \longrightarrow^{4} (s1,7,4) \longrightarrow^{b} (s0,0,4) \longrightarrow^{1} (s0,1,5)$$

$$\longrightarrow^{a} (s1,1,0)\dots$$

Fin

Questions