Engineering Mechanics

Brief Introduction and Overview

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Engineering Mechanics
Brief Introduction and Overview

- Statics
- Dynamics
- Mechanics of Materials (Deformable Solids)

Prerequisites: Calculus I, University Physics I

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Objectives :

- To provide a brief overview of engineering mechanics.
- To introduce the basic sub-disciplines of mechanics.
- To explain the scopes and relations of three common engineering mechanics courses: statics, dynamics and mechanics of materials.

Question 1: What is Mechanics?

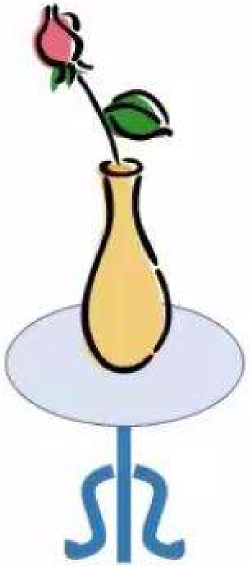
What is *Mechanics*?

1. A branch of **Physics**.

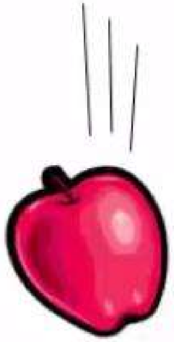
2. **Engineering mechanics** has a focus on the **applications**.

3. Calculates, describes and predicts the effects of **forces** on a **system**.

Question 2: What are some examples of what force can do?



Keep an object STATIC



A gravitational Force

Make things MOVE

Force can DEFORM an object

Make things ROTATE

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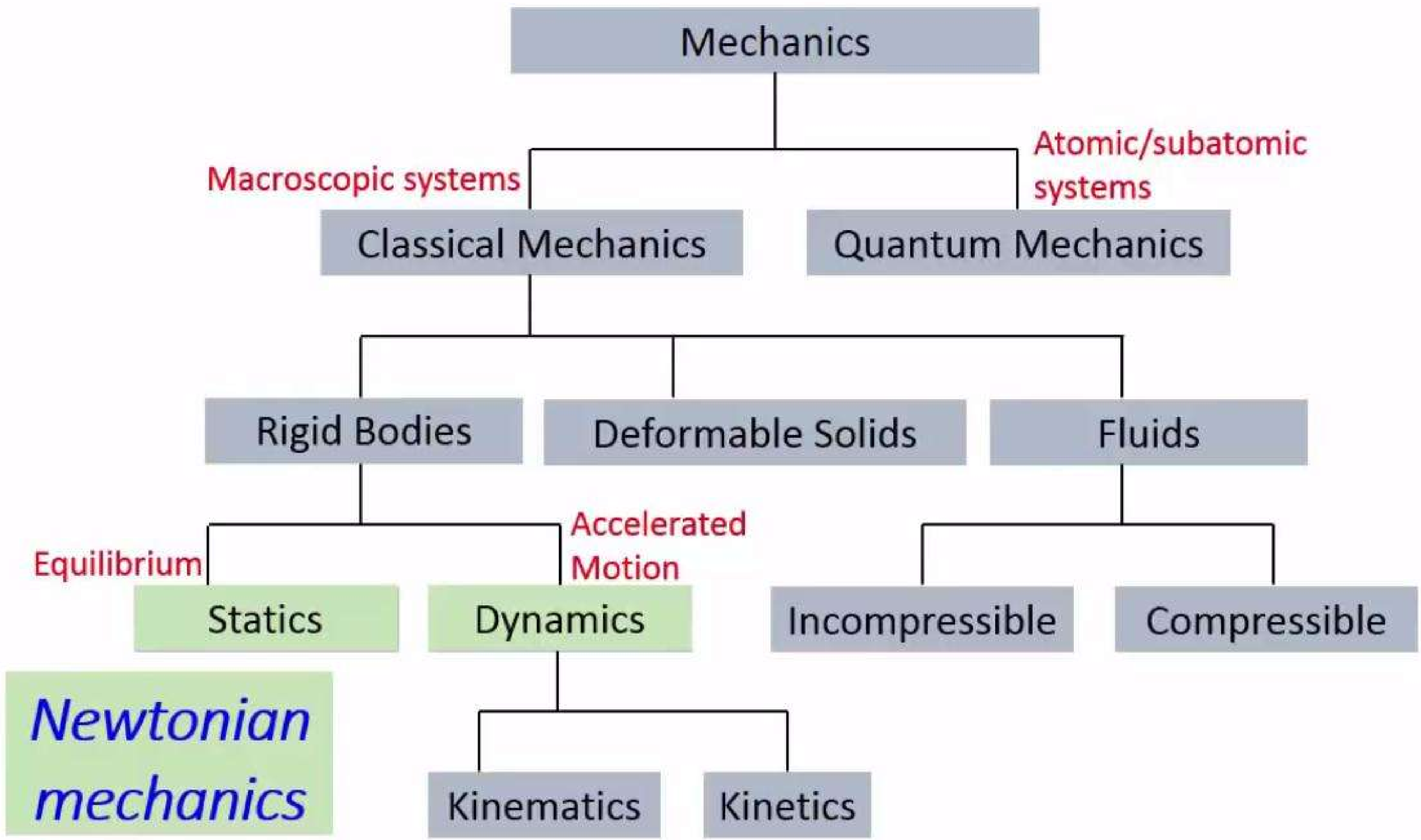

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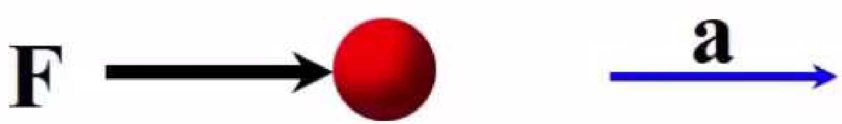
Newton's second law

$$\mathbf{F} = m\mathbf{a}$$

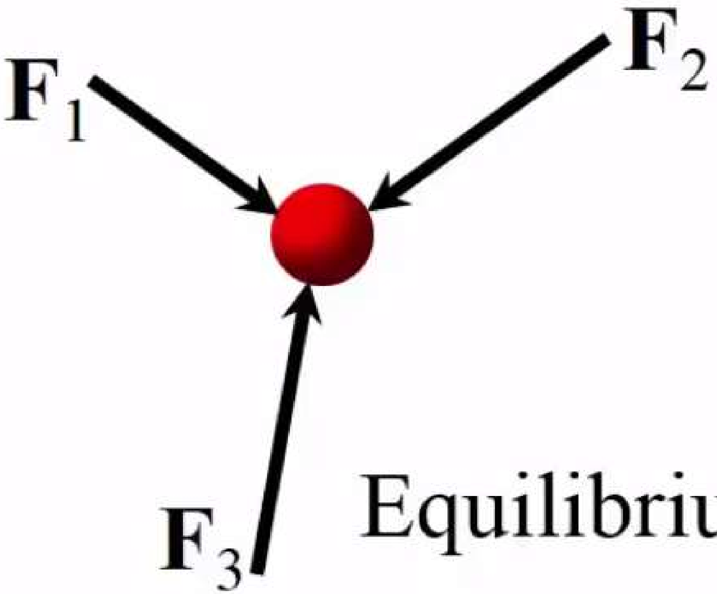


Accelerated motion

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

The acceleration of the movement of an object is ***proportional to the resultant force***, and is also in the ***same direction*** of the resultant force.

Newton's first law



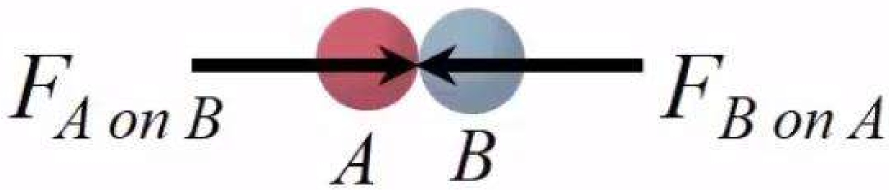
$$\mathbf{F}_R = \mathbf{0}$$

$$\mathbf{a} = \frac{\mathbf{F}_R}{m} = \mathbf{0}$$

Equilibrium

An object will remain its original state of motion (***rest*** or moving at ***constant velocity*** in a straight line) if there is no unbalanced force acting on it.

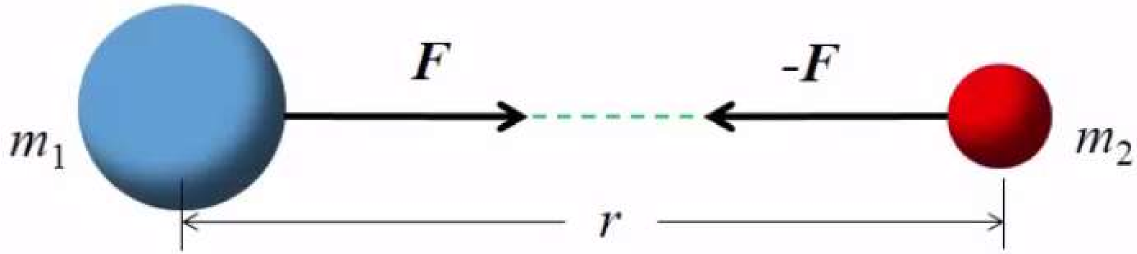
Newton's third law



Action and reaction

The forces of action and reaction between two objects are of the ***equal***, ***collinear*** and ***opposite***.

Newton's law of gravitation

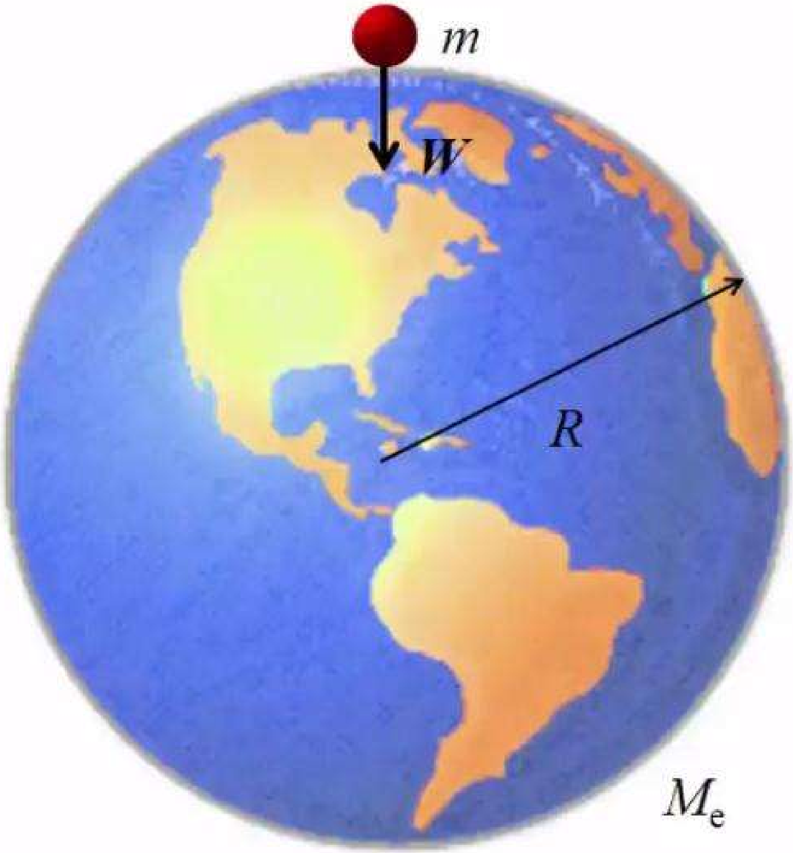


$$F = G\frac{m_1 m_2}{r^2}$$

G: universal constant of gravitation,
66.73×10^{-12} $m^3/(kg\cdot s^2)$

The gravitational attraction forces between any two objects are equal and opposite.

Newton's law of gravitation



$$W = G\frac{mM_e}{R^2}$$

R: radius of the earth

M_e: mass of the earth

Let $g = G\frac{M_e}{R^2}$

$$\therefore W = mg$$

g: constant of gravitation of the earth, 9.81 m/s^2 or 32.2 ft/s^2.

Solid System

Status

Rigid Body

(Particle)

Equilibrium

$$\mathbf{F}_R = \mathbf{0}$$

Deformable Solid

Accelerated Motion

$$\mathbf{F}_R = m\mathbf{a}$$

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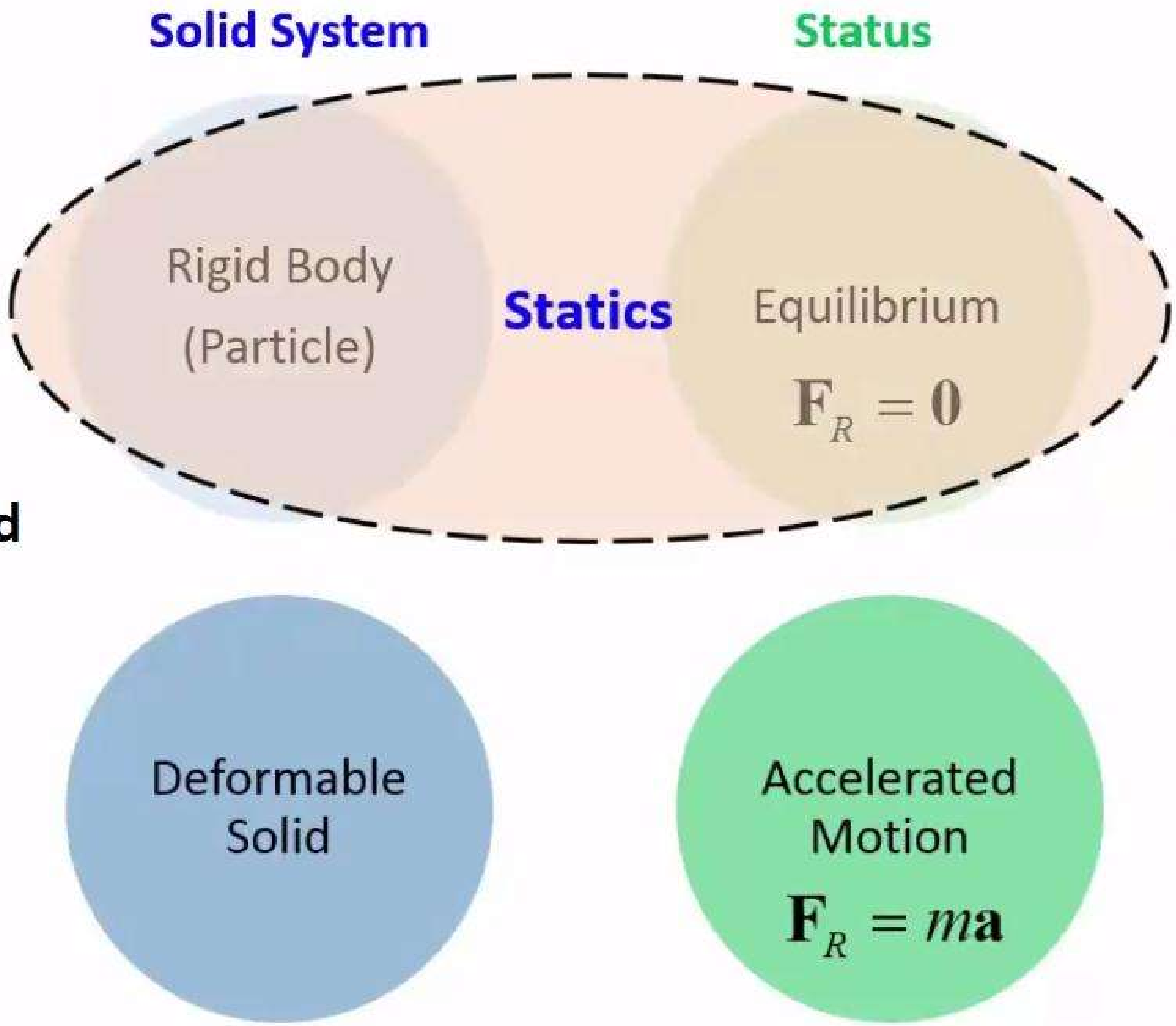
Engineering Mechanics: Statics



Statics studies rigid body Equilibrium

It is the easiest class of the three.

At the same time, it is the most important since it builds the theoretical foundation for the other two courses

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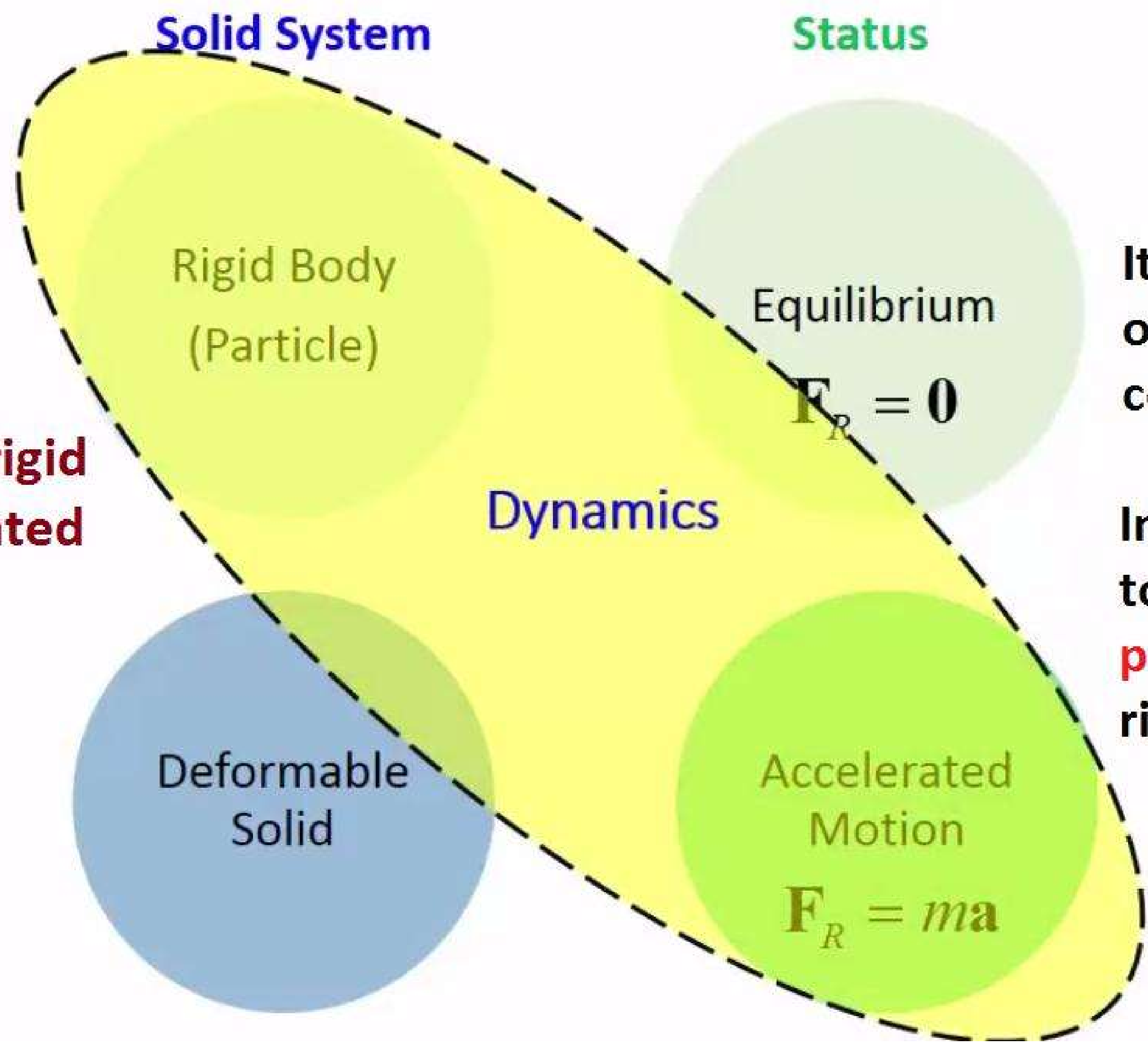
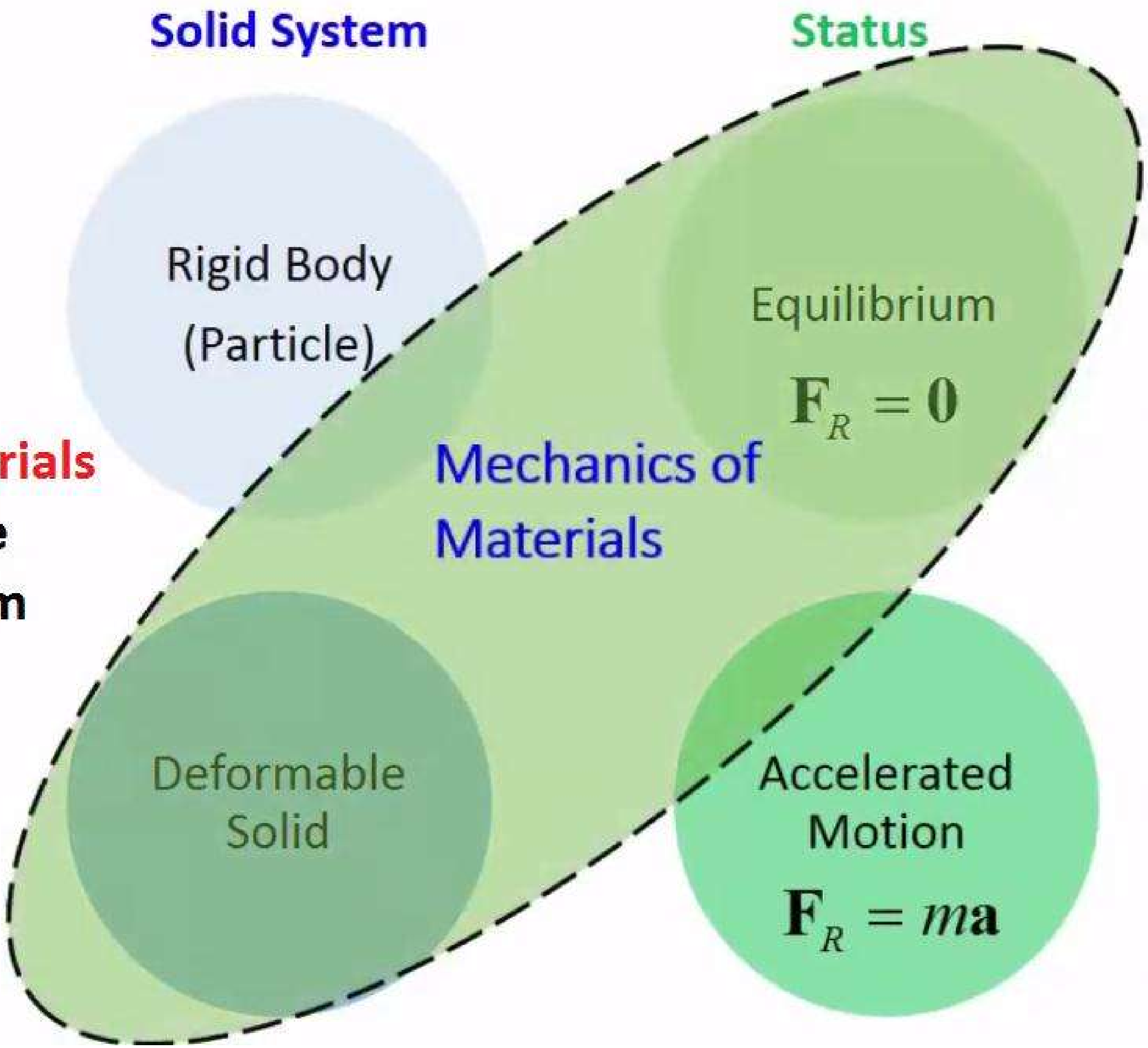


Dynamics studies rigid body with accelerated motion

It could be translation or rotation or a combination of both.

In dynamics, we learn to understand and predict the motions of rigid bodies



Mechanics of materials studies deformable solids in equilibrium

We want to know how forces cause stress, deformation and even failure in the system.

It is important in many fields such as construction or design.

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Fundamental Concepts:
Basic quantities and idealization

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Objectives :

- To introduce the basic quantities of mechanics.
- To introduce the concept of idealization commonly used in mechanics.

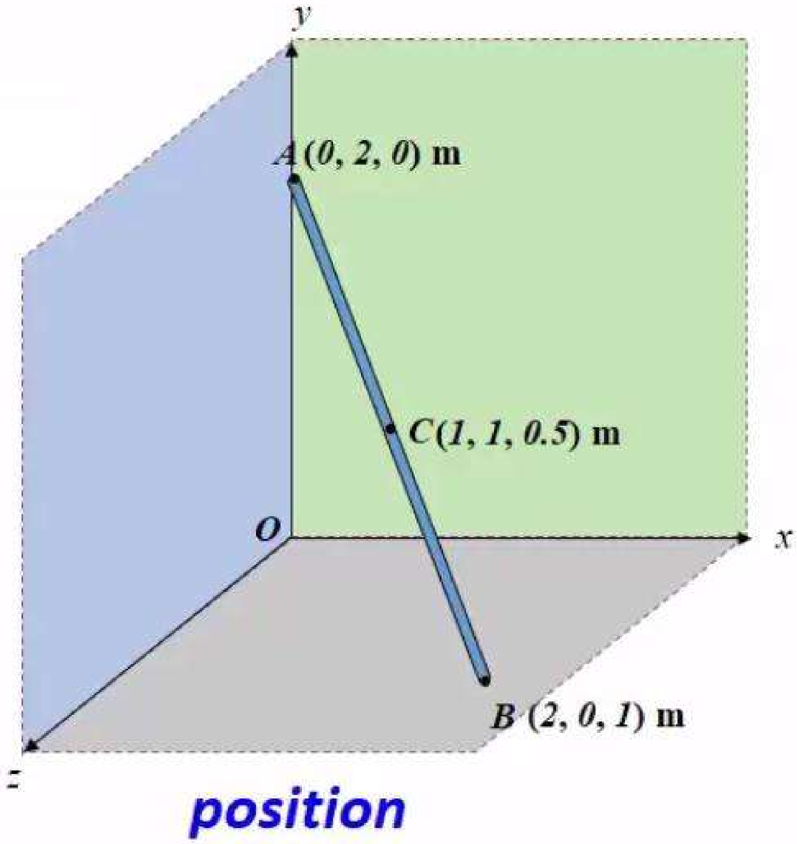
Basic quantities

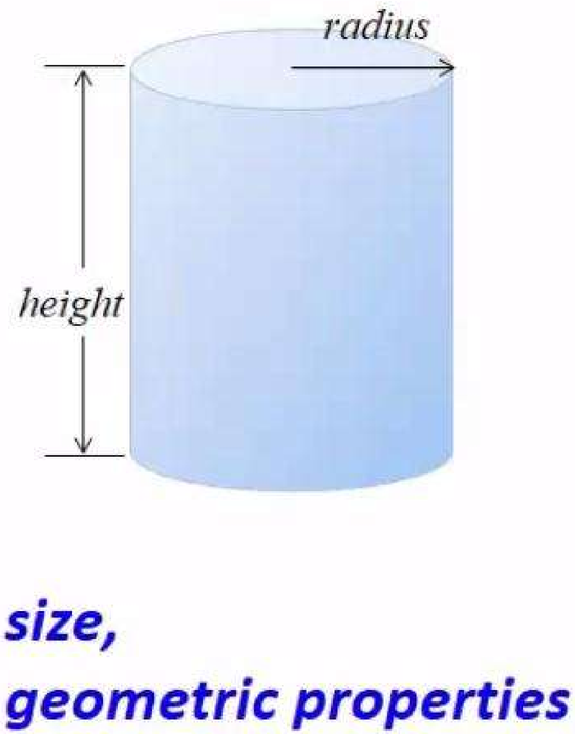
length time mass force

Question 1: In your own words, explain what *length*, *time*, *mass* and *force* are respectively.

Length

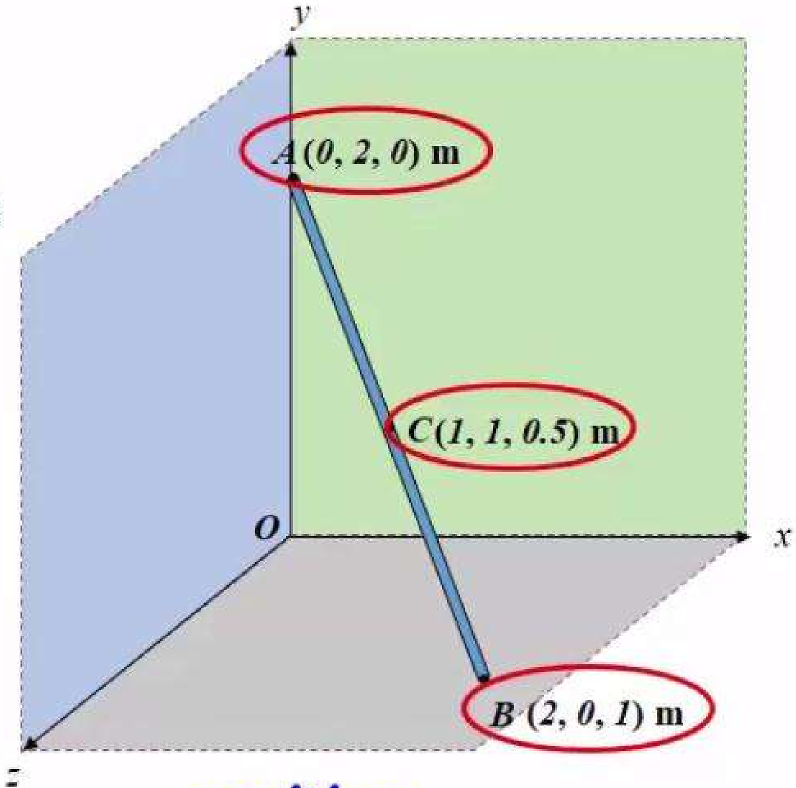
can be used to describe the position is space, the size of a physical system, and the geometric properties of a body.



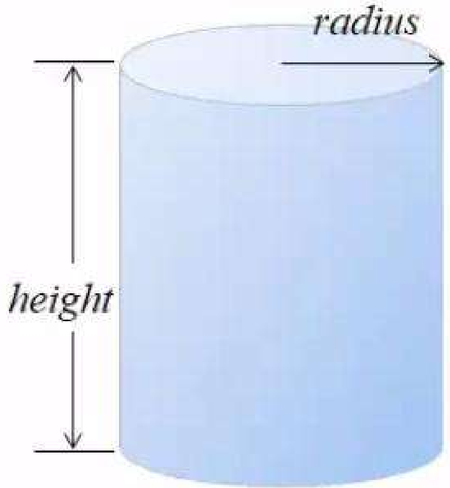


Length

Coordintes are the 3 lengths measured from the origin along *x*, *y* and *z* direction respectively in an established rectangular coordinate system.



position



size,

geometric properties

Unit: [m] or [ft]

Time

describes the succession of events.

Example: Speed describes the position of an object with respect to time.

Time is a very important concept in the subject of Dynamics, but in Statics, we mainly deal with objects that are motionless.

statics : time-independent
dynamics: time-dependent

Unit: [s]

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Question 2: A person who weighs 143 pounds is about 65 kilograms, correct? Is *weight* the same as *mass*?

Mass

is a measure of quantity of matters.

Mass is a physical property that characterizes the extent of force and object experiences in a gravitational field.

$$F = G\frac{m_1 m_2}{r^2}$$

Mass characterizes also the resistance of an object to changes in its state of motion.

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

Mass is NOT the same as weight since weight is a force

Unit: [kg] or [slug]

Force

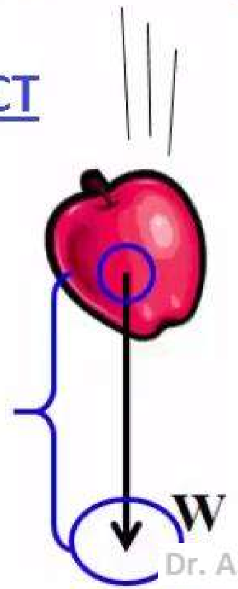
CONTACT

The concept of force characterizes the action and reaction between two bodies.

They could be CONTACT between the two bodies.

There could be NO CONTACT.

NO CONTACT



Force is a vector and it is fully described by:

magnitude
direction
point of application

Unit: [N] or [lb]

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Idealizations

means to use scientific models to represent phenomena, so that they can be simplidied to an extent.

particle

An object can be modeled as a particle when it's geometry and dimension are negligible for the interest of the study

A particle is considered to only occupy a single point in space with NO shape or size and it has NO properties except its MASS.

rigid body

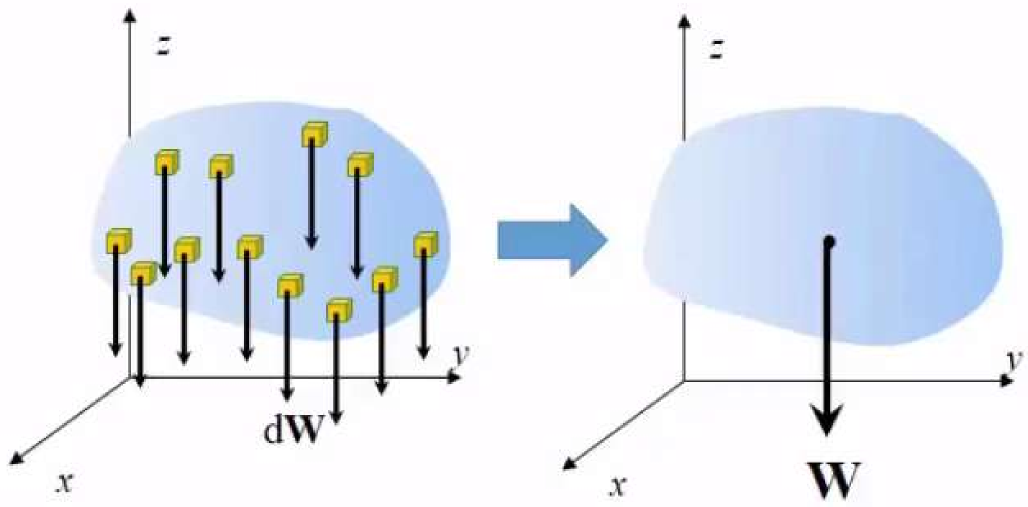
not only has mass but also has dimensions and geometry.

In other words, it has size and shape that need to be taken into consideration in our analysis.

Unlike real world objects a rigid body does NOT have any other material properties such as ELASTICITY, therefore it will not deform.

concentrated force

assumes that a force only acts on a point, although in realty, forces are applied to an AREA or a VOLUME.



Example:

The weight of an object is distributed throughout its body, but in our analysis, we often use a concentrated force that is placed at the CENTER of GRAVITY in the object to replace the distributed gravitational force.

Vector Operation:

Parallelogram Law and Triangle Rule

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Objectives :

- To revisit the concepts of scalar and vector.
- To show how to properly represent a force vector.
- To explain the parallelogram law and the triangle rule for vector addition and subtraction.

Question 1: In your own words, what is a scalar and what is a vector? List at least three examples of scalars and three examples of vectors.

Scalars and vectors

Scalar: a physical/mathematical quantity that can be completely specified by its ***magnitude***.

length (l)
mass (m)
time (t)
volume (V)

Scalars and vectors

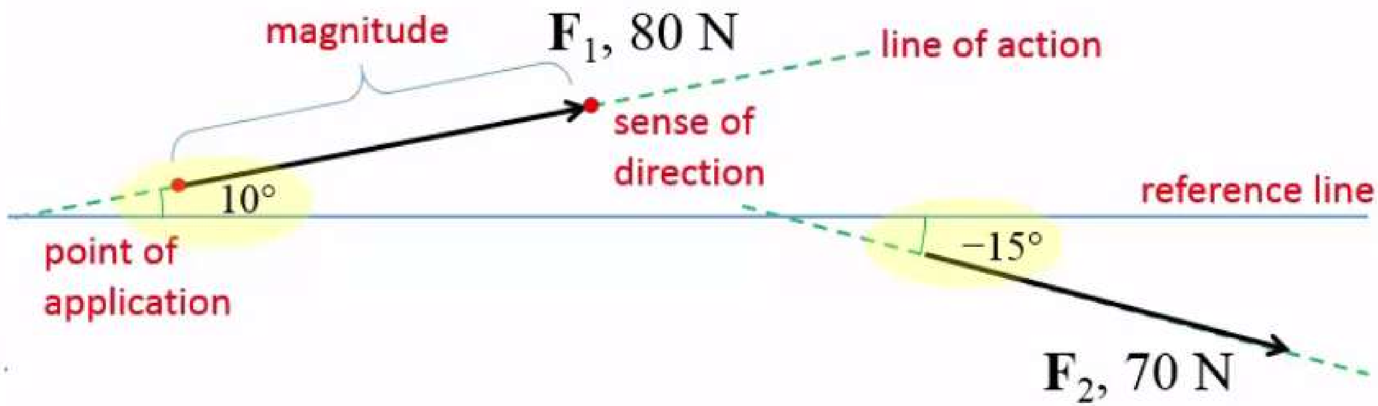
Vector: a physical/mathematical quantity that requires both a ***magnitude*** and ***direction*** for its complete description.

force (F)
velocity (v)
acceleration (a)
moment (M)

$\mathbf{F} \quad \bar{F} \quad \vec{F}$

A force can be represented by an arrow

It can be fully characterized by:
- Point of application,
- Sense of direction,
- Magnitude.



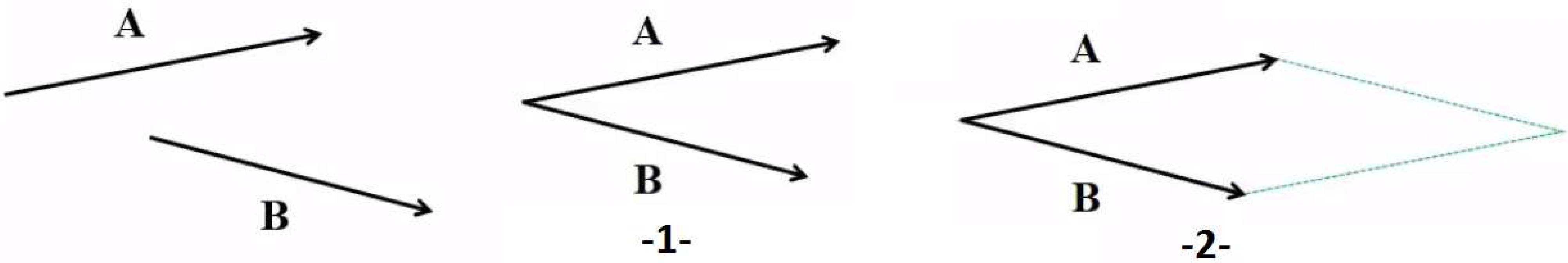
The **direction** of a force can be descibed by the **angle** made by its **line of action** and a **reference line**.

Sometimes, you might see **negative** angles, this is because by **sign convention**, **positive** angle represents **counterclockwise** rotation.

Negative angle represents clockwise rotation from the **reference line**.

Vector addition

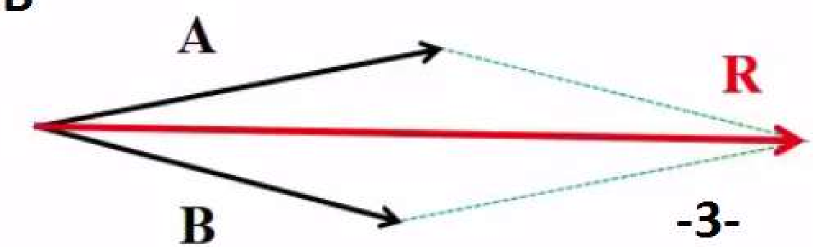
To perform vector addition, we need to follow the ***Parallelogram law***.



First step, we need to join the tails of the two vectors so that they are Concurrent.

Then, we construct a Parallelogram using A and B as the two sides.

Then, we draw an arrow that starts from the tails of A and B and points to the other end.

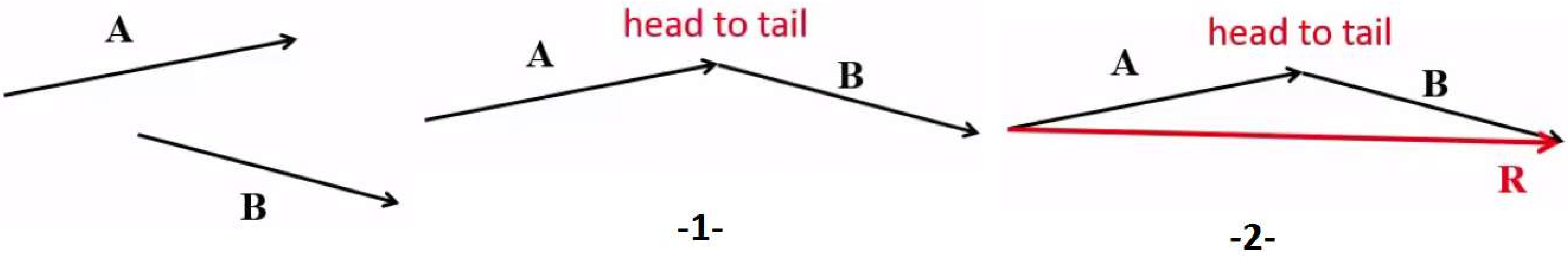


$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

R: is the resultant vector.

Vector addition

As a simplification to the parallelogram law, we can use the ***Triangle rule***



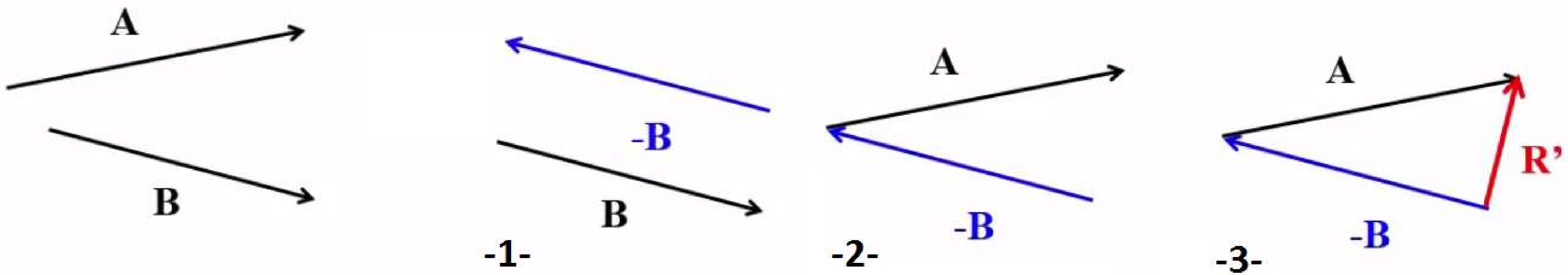
We join the vectors A and B in a head to tail fashion.

The resultant vector R can simply be represented by an arrow that starts from the tail of vector A to the head of vector B.

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

Vector subtraction

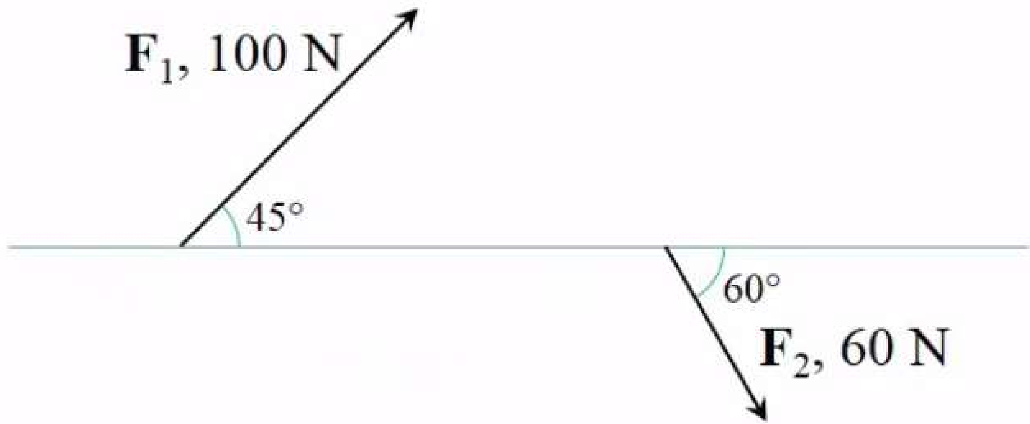
Since we know subtraction can be considered as addition with a negative quantity:
First, we can define the vector (-B), which has the same magnitude but opposite direction.



Then, we can simply add vector A to (-B) together using either Parallelogram Low or Triangle Rule.

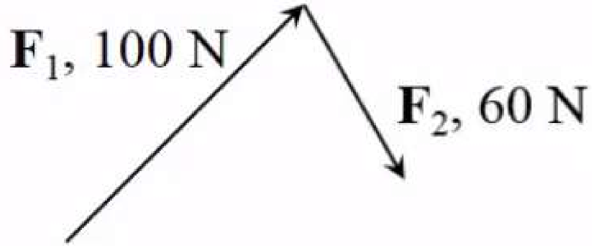
$$\mathbf{R'} = \mathbf{A} - \mathbf{B}$$

Example 1: What this the magnitude and direction (with respect to the horizontal reference line) of the resultant force of $\mathbf{F}_1$ and $\mathbf{F}_2$?



Example 1: What this the magnitude and direction (with respect to the horizontal reference line) of the resultant force of $\mathbf{F}_1$ and $\mathbf{F}_2$?

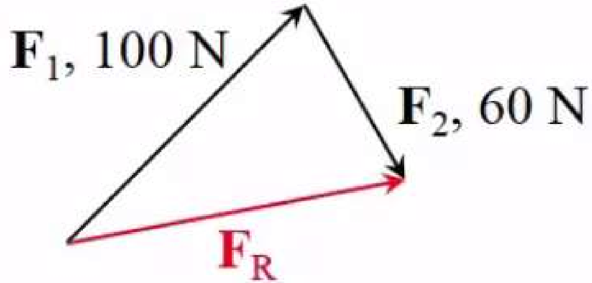
Triangle Rule



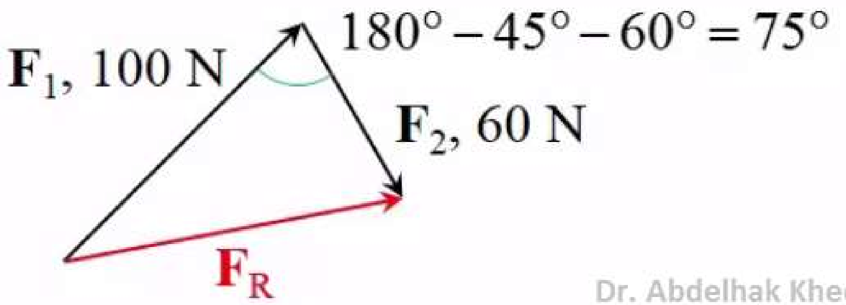
We join the two forces in a head to tail fashion.

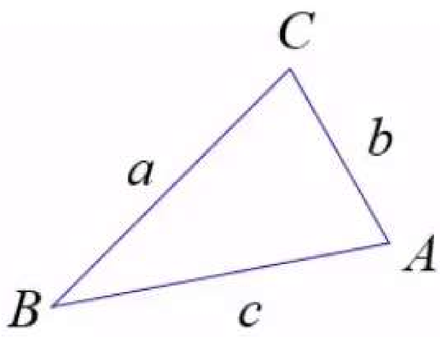
F_R is the resultant vector.

The three forces form a triangle.



Based on the geometry given in the problem statement, the angle between these two vectors is **75°**.





In a Triangle, the relation between the sides a, b, c, and the angles A, B and C, we have:

Law of sines: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines: $$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Based on the information we have:

$$a = 100$$

$$b = 60$$

$$C = 75^\circ$$

we can start with the equation:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Then, we use this equation to calculate the angle B

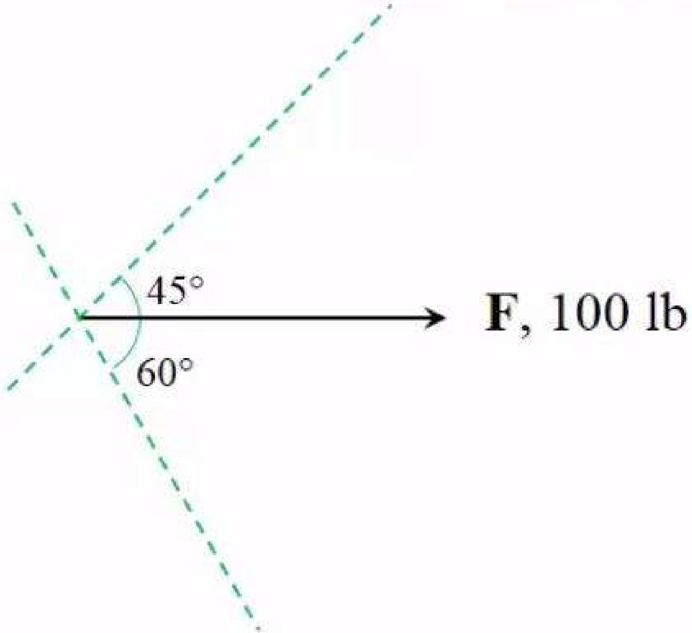
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Here are the results:

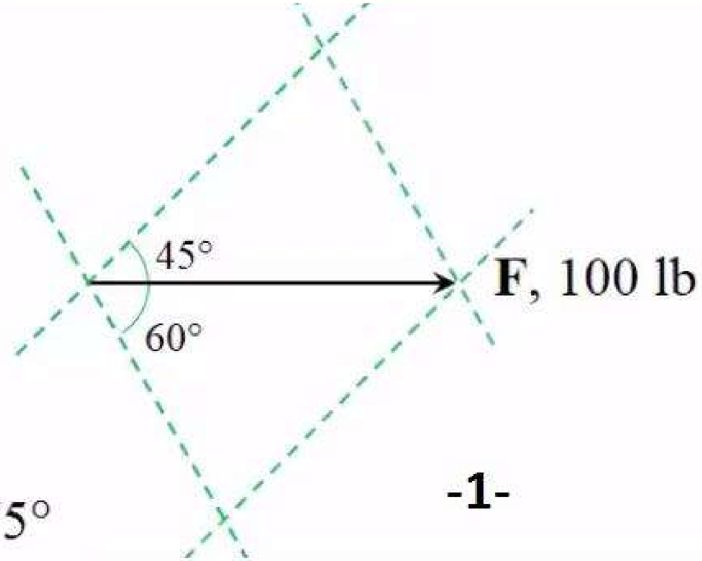
$$c = 102$$

$$B = 34.5^\circ$$

Example 2: The given force $\mathbf{F}$ is the resultant force of forces $\mathbf{F}_1$ and $\mathbf{F}_2$, for which the lines of action are given. Determine the magnitudes of these two forces.



we are going to apply the Parallelogram Law

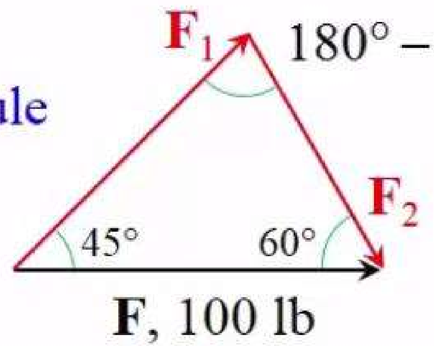


-1-

we can visualize the two component forces **F**$_1$ and **F**$_2$

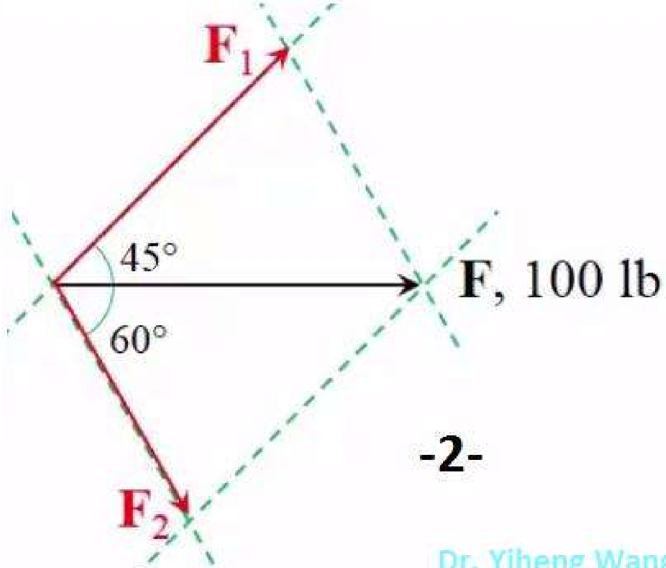
Let's make the triangle, and calculate the third angle.

Triangle Rule



$$180° - 45° - 60° = 75°$$

-3-



-2-

Then, apply law of sines directly to calculate the magnitude of these two forces

$$\frac{100}{\sin 75°} = \frac{F_1}{\sin 60°} = \frac{F_2}{\sin 45°}$$

$$\therefore \begin{cases} F_1 = 89.7 \text{ lb} \\ F_2 = 73.2 \text{ lb} \end{cases}$$

Ans.

Question 3: Which of the following is NOT a vector?

(a) Force

(b) Pressure

(c) Acceleration

(d) Energy

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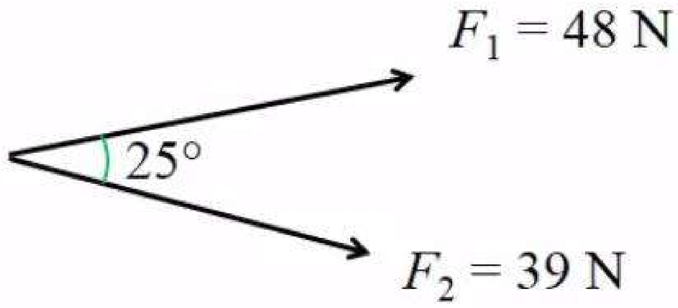
Question 4: What is the magnitude of the resultant force of forces F_1 and F_2?



(a) 85 N

(b) 87 N

(c) 21 N

(d) 11 N

Cartesian Vectors and Operation

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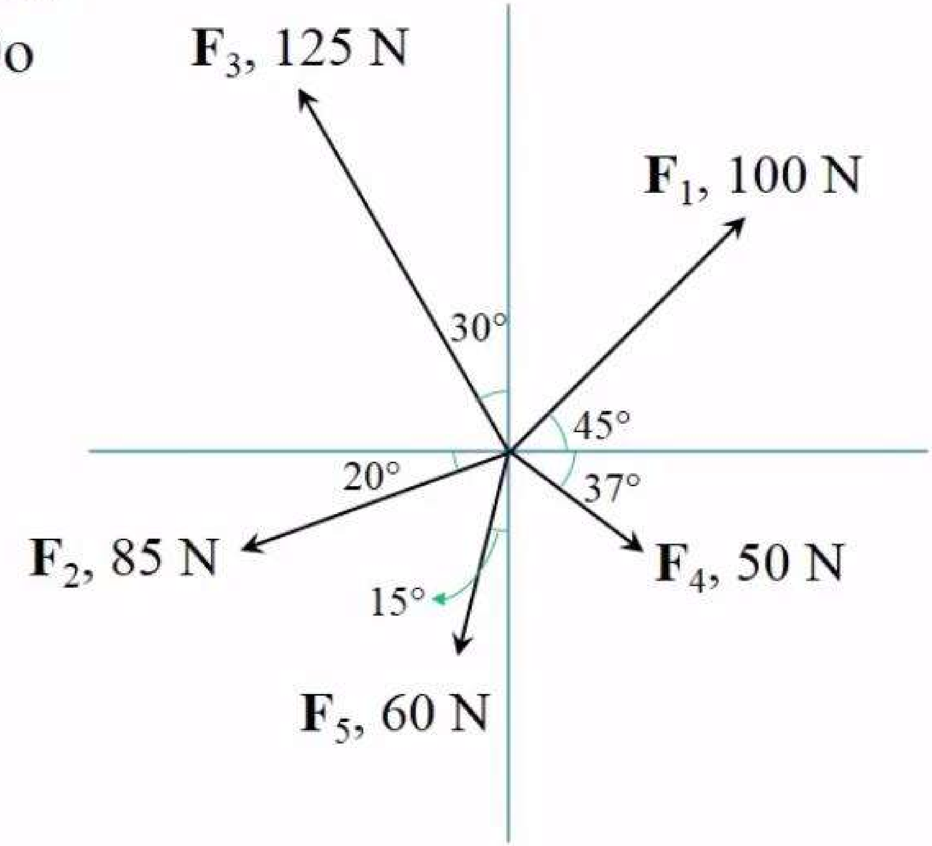
Objectives:

- To express a vector in a rectangular coordinate system and in the Cartesian vector form.
- To introduce key concepts of component vectors and unit vector.
- To determine the magnitude of a Cartesian vector and express its direction using coordinate direction angles.
- To perform vector addition of Cartesian vectors.

Question 1: If you are to use the parallelogram law or triangle rule to find the resultant force of these five forces, how do you plan to do it? Do you think there's a better way?



Right-handed
Rectangular
Coordinate System

(Cartesian
Coordinate System)

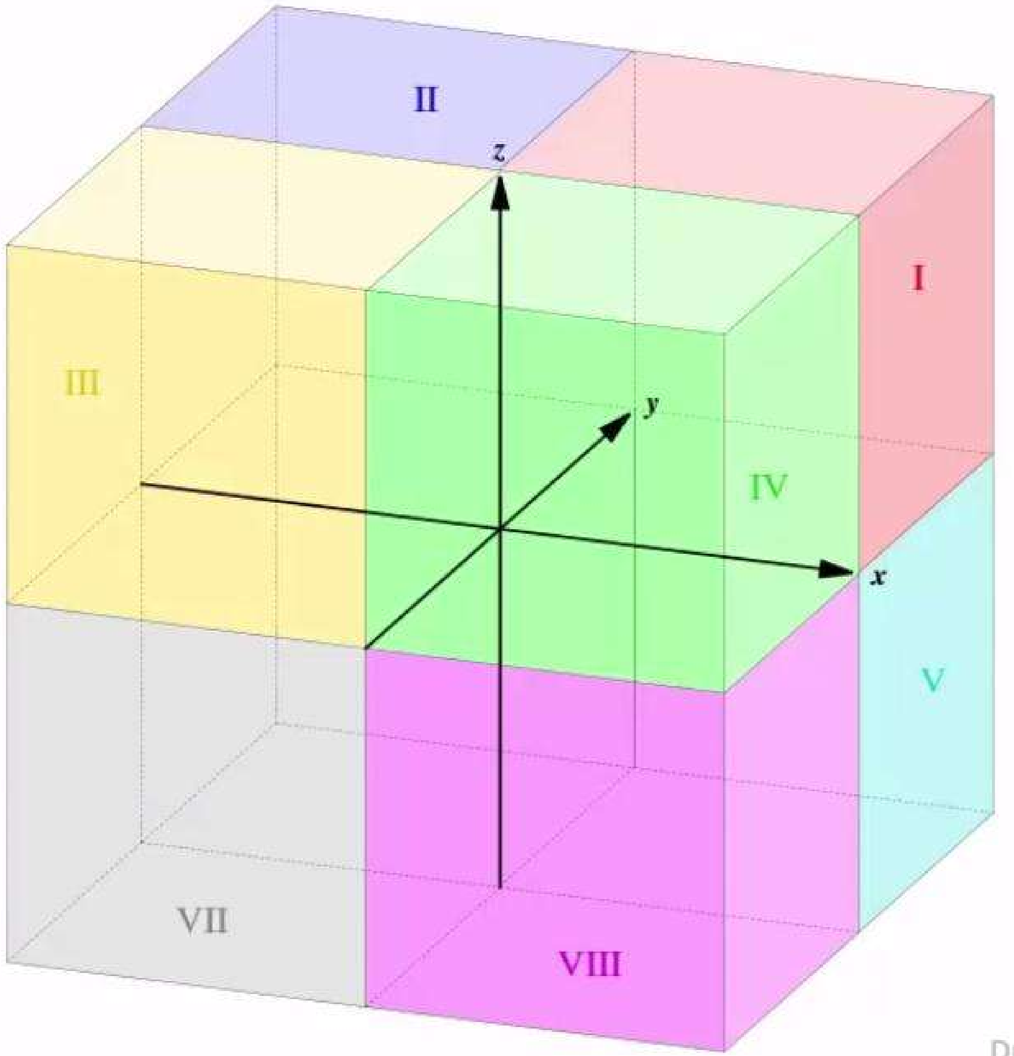
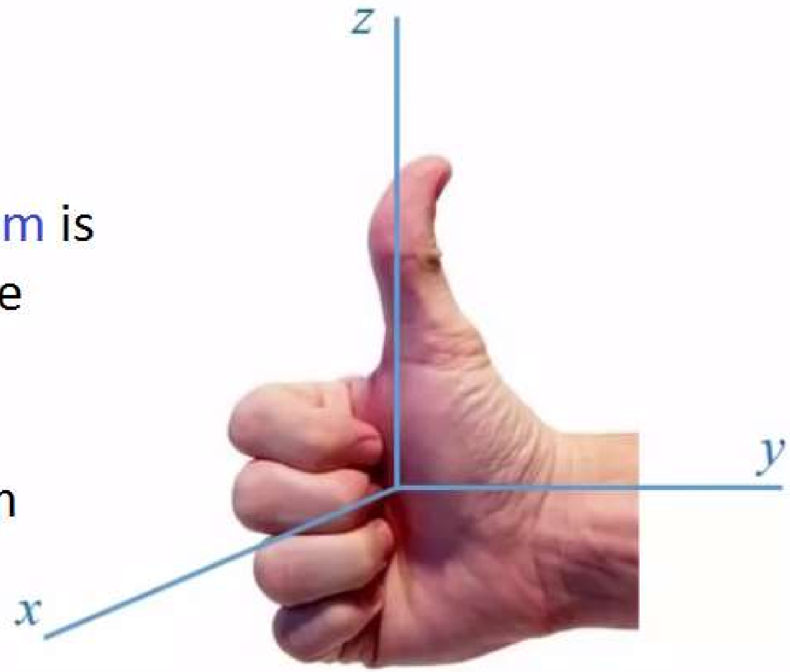


Image from wikipedia.org.

Right-handed Rectangular Coordinate System:

The reason why this is called Right-handed coordinate system is because the **POSITIVE** directions of the three axes follow the right-hand rule.

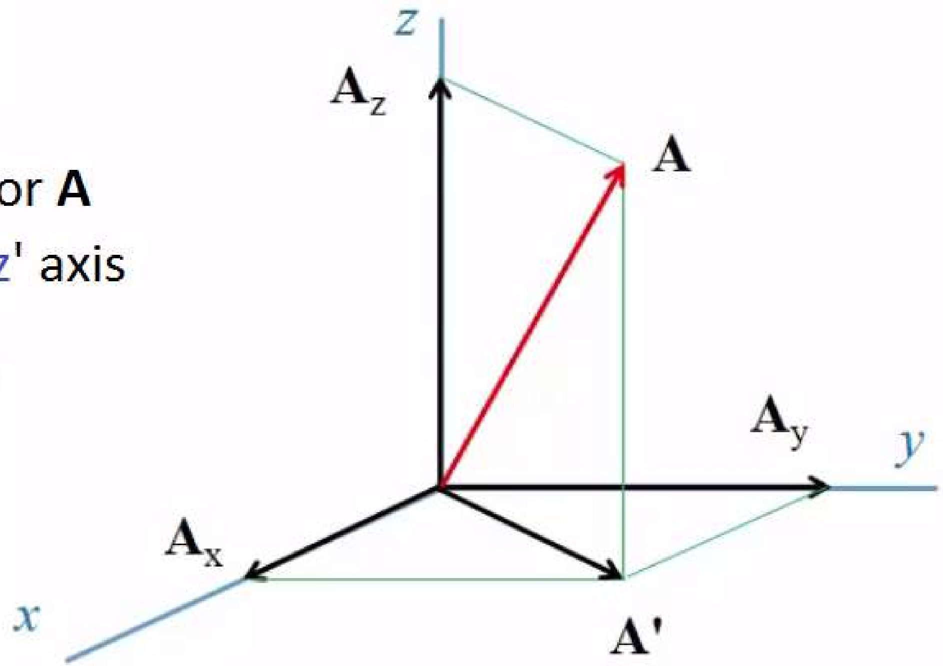
This means that if you roll the four fingers in your hand from the positive 'x' direction towards the positive 'y' direction as shown in this image, your thumb will point towards the positive 'z' direction.



Cartesian vectors

Rectangular components of a vector:

First, we apply the parallelogram law to resolve vector **A** into two component vectors **Az** that falls along the 'z' axis and **A'** that falls within the 'xy' plane. $\mathbf{A} = \mathbf{A}' + \mathbf{A}_z$



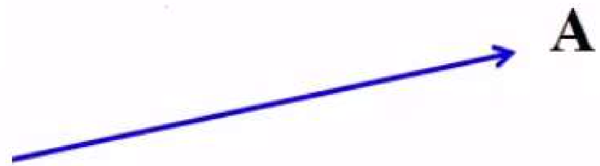
Then, we can apply the parallelogram law again to resolve **A'**.

$$\mathbf{A}' = \mathbf{A}_x + \mathbf{A}_y$$

Finally

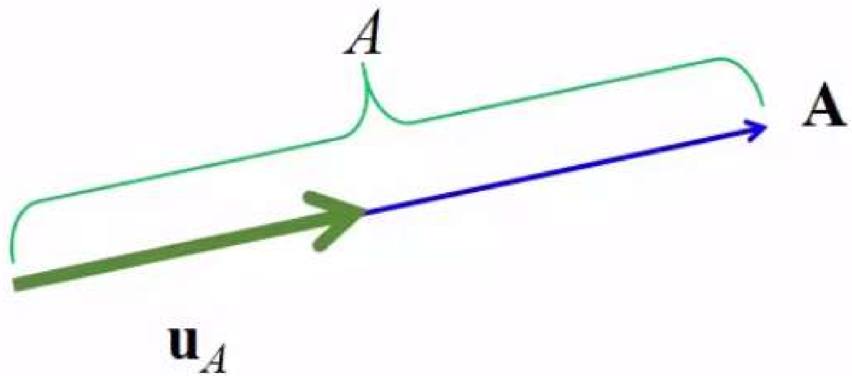
$$\therefore \quad \mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z$$

Cartesian unit vectors



Since a vector needs to be described by two parts its **magnitude** and its **direction**, we can separate these two parts by defining **unit vectors.**

For any arbitrary vector **A**, its unit vector **u_a** has the same **direction** but a **magnitude** of unit length 1.



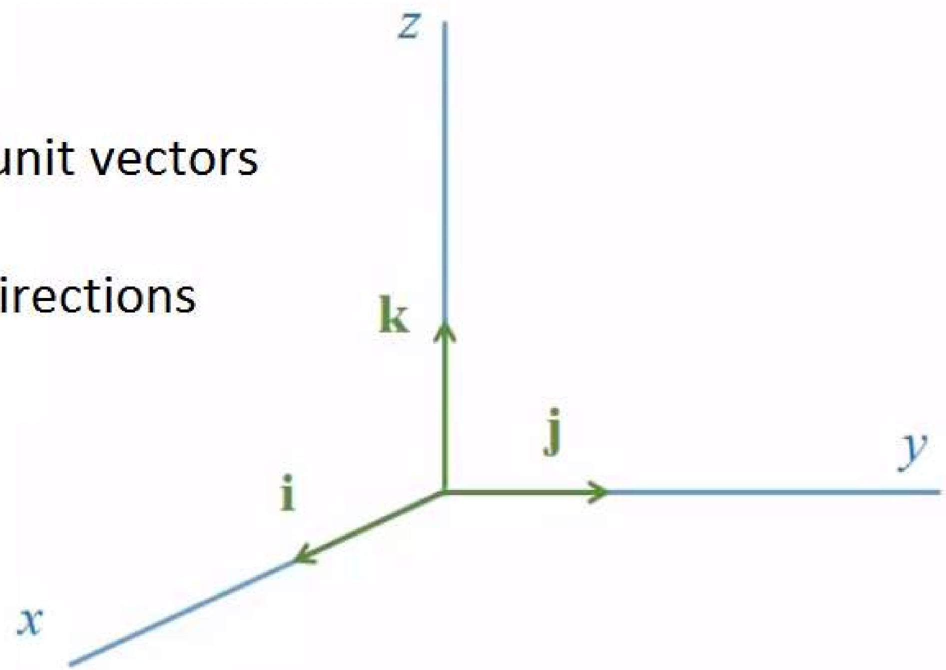
Vector **A** can be expressed by its magnitude A, which is a scalar multiplied by its unit vector **u_a**

$$\mathbf{A} = A \cdot \mathbf{u}_A$$

Cartesian unit vectors

Cartesian Unit Vectors:

In a Cartesian coordinate system, there are 3 special unit vectors **i**, **j** and **k**.
They are special because they are designated to the directions of 'x', 'y' and 'z' axis.



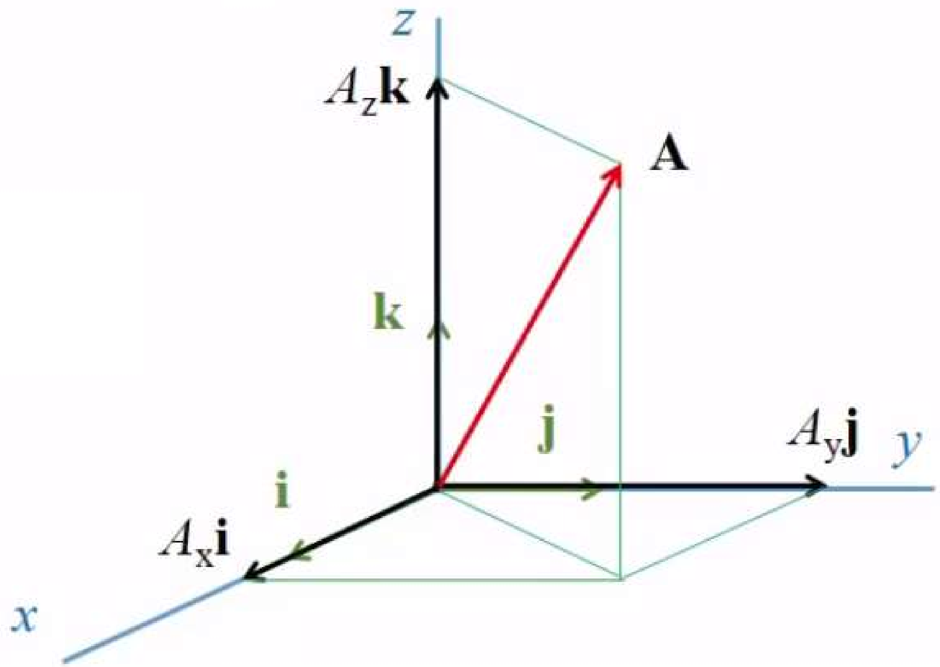
i, **j** and **k** all have magnitude of 1.

Cartesian vectors

Therefore, using the unit vectors **i**, **j** and **k**, the component vectors along the x, y and z axis can now be written as:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

A_x, A_y and A_z being the magnitudes of the component vectors.



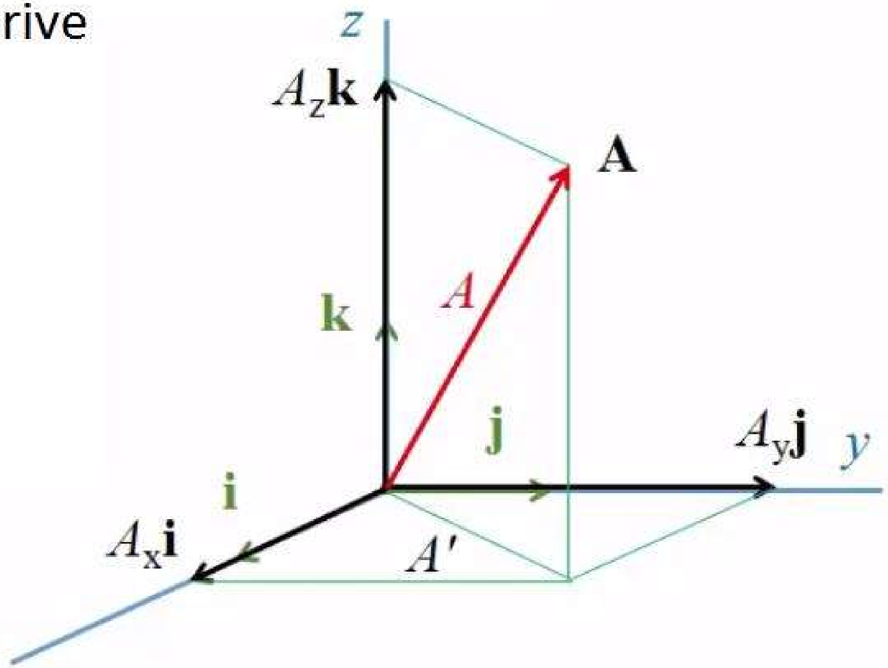
Magnitude of a Cartesian Vector

By applying the **Pythagorean theorem** twice, we can derive the magnitude of vector **A**:

$$A' = \sqrt{A_x^2 + A_y^2}$$

$$A = \sqrt{A'^2 + A_z^2}$$

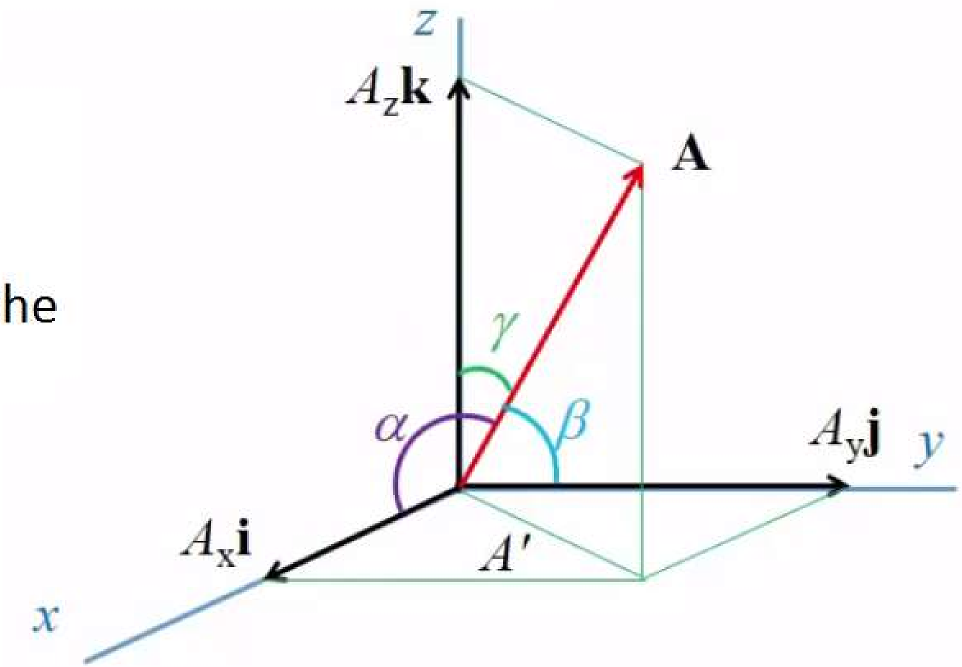
$$\therefore \quad A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



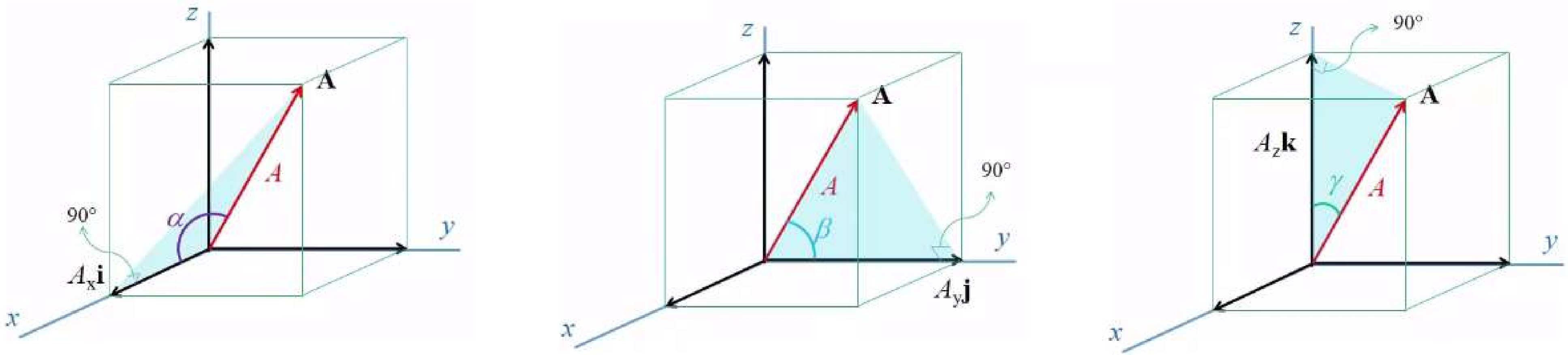
Direction of a Cartesian Vector

Coordinate direction angles α, β and γ.

To describe **the direction** of the vector, we can use the **Coordinate direction angles.**



Direction of a Cartesian Vector



According to trigonometry, we know that:

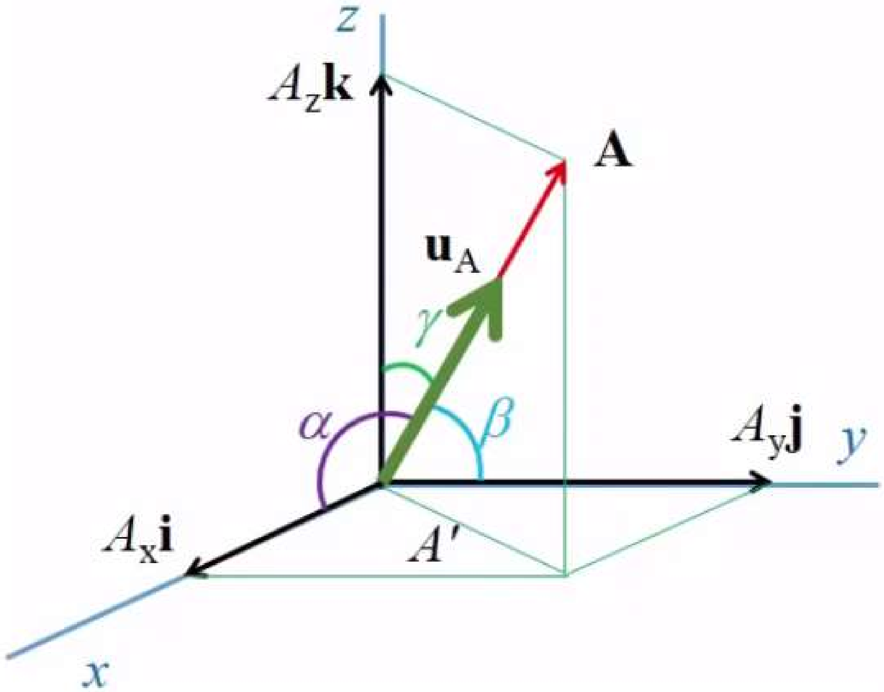
$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

Unit Vector

Because the **unit vector** of vector **A** , **u**$_A$ equals to:

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

$$\mathbf{u}_A = \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$

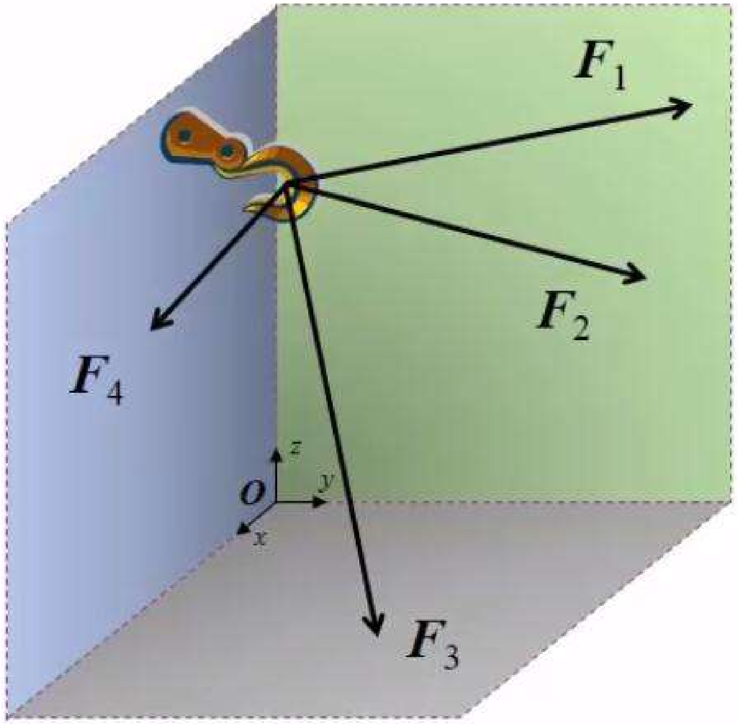


Since the **magnitude** of **unit vector** is always 1, therefore, we come to the conclusion that:

$$\therefore \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

"The cosine squared of the three coordinate direction angles for any Cartesian vector must equal 1."

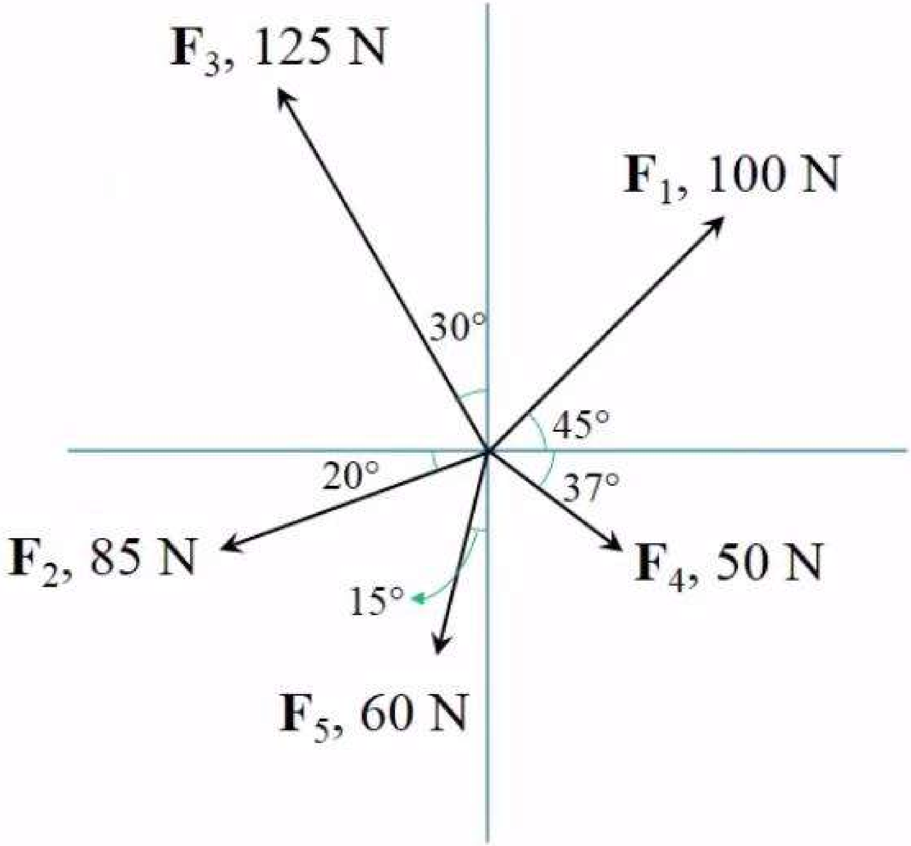
Addition of Cartesian Vectors



$$\mathbf{A}+\mathbf{B}=A_x\mathbf{i}+A_y\mathbf{j}+A_z\mathbf{k}+B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k}$$
$$=\left(A_x+B_x\right)\mathbf{i}+\left(A_y+B_y\right)\mathbf{j}+\left(A_z+B_z\right)\mathbf{k}$$

$$\mathbf{F}_R=\sum\mathbf{F}=\sum F_x\mathbf{i}+\sum F_y\mathbf{j}+\sum F_z\mathbf{k}$$

Question 2: Determine the magnitude of the resultant force of these five forces.



Example 1: For force vector $\mathbf{F} = \{1.2\mathbf{i} - 1.8\mathbf{j} + 3.6\,\mathbf{k}\}$ kN, determine its magnitude, unit vector and the three coordinate direction angles.

Example 1: For force vector $\mathbf{F} = \{1.2\mathbf{i} - 1.8\mathbf{j} + 3.6\,\mathbf{k}\}$ kN, determine its magnitude, unit vector and the three coordinate direction angles.

Magnitude:

$$F = \sqrt{1.2^2 + (-1.8)^2 + 3.6^2} = 4.2\,(\text{kN})$$

Unit vector:

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{\{1.2\mathbf{i} - 1.8\mathbf{j} + 3.6\mathbf{k}\}\ \text{kN}}{4.2\ \text{kN}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

Direction angles:

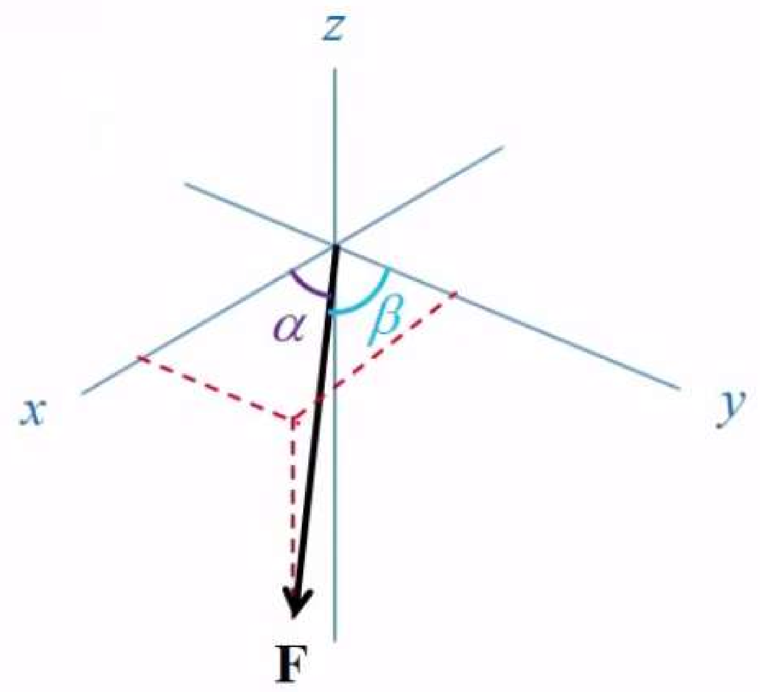
$$\alpha = \cos^{-1}\left(\frac{2}{7}\right) = 73.4°$$

$$\beta = \cos^{-1}\left(-\frac{3}{7}\right) = 115°$$

$$\gamma = \cos^{-1}\left(\frac{6}{7}\right) = 31.0°$$

Ans.

Example 2: If force $\mathbf{F}$ has a magnitude of 1200 N, and angle α is 60° and β is 45°, express the force in Cartesian vector form and determine its unit vector.



Example 2: If force **F** has a magnitude of 1200 N, and angle α is 60° and β is 45°, express the force in Cartesian vector form and determine its unit vector.

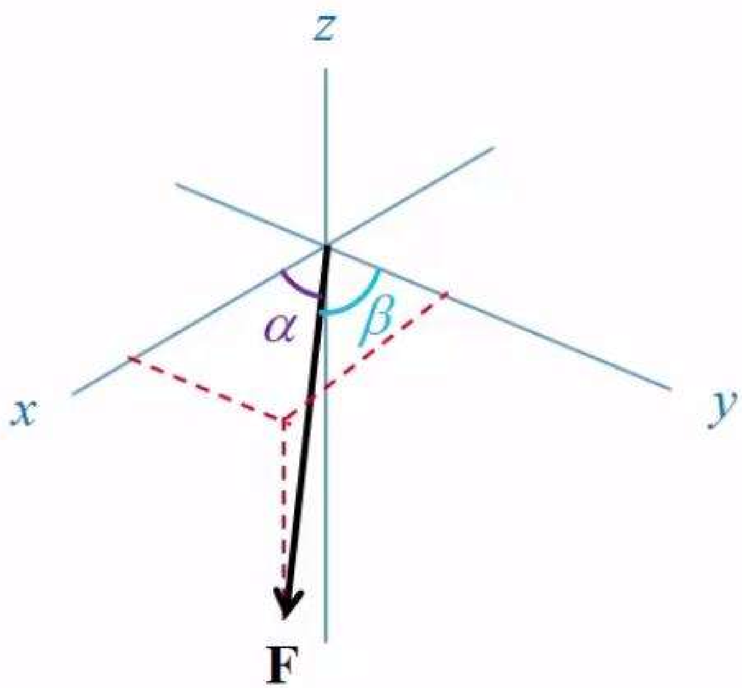
$$\because \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\gamma = 1 - \cos^2 45° - \cos^2 60° = 0.25$$

$$\cos\gamma = \pm 0.5 \quad \Rightarrow \quad \gamma = 60° \text{ or } 120°$$

we know that, the direction angle is defined as the angle made by the force with the positve part of the axis.

$$\therefore \quad \gamma = 120°$$



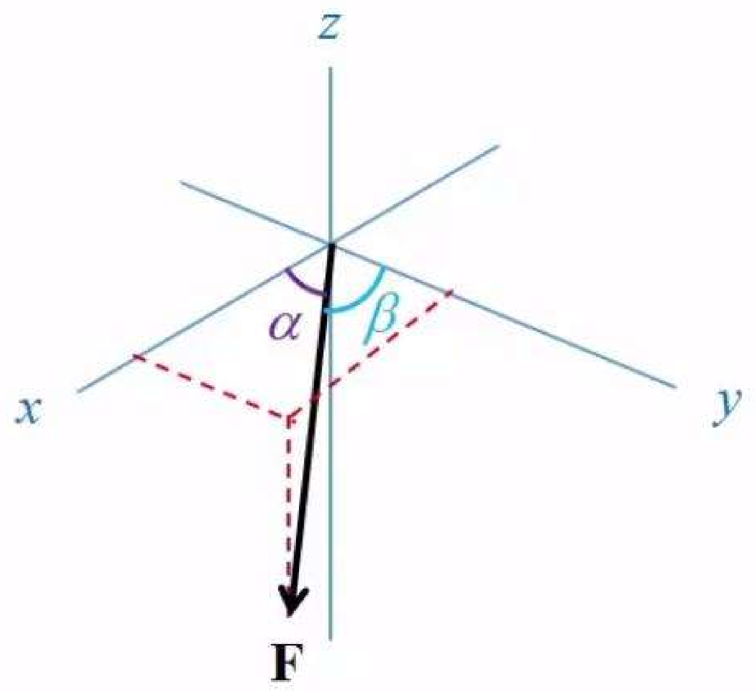
Example 2: If force **F** has a magnitude of 1200 N, and angle α is 60° and β is 45°, express the force in Cartesian vector form and determine its unit vector.

$$\mathbf{u}_F = \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$

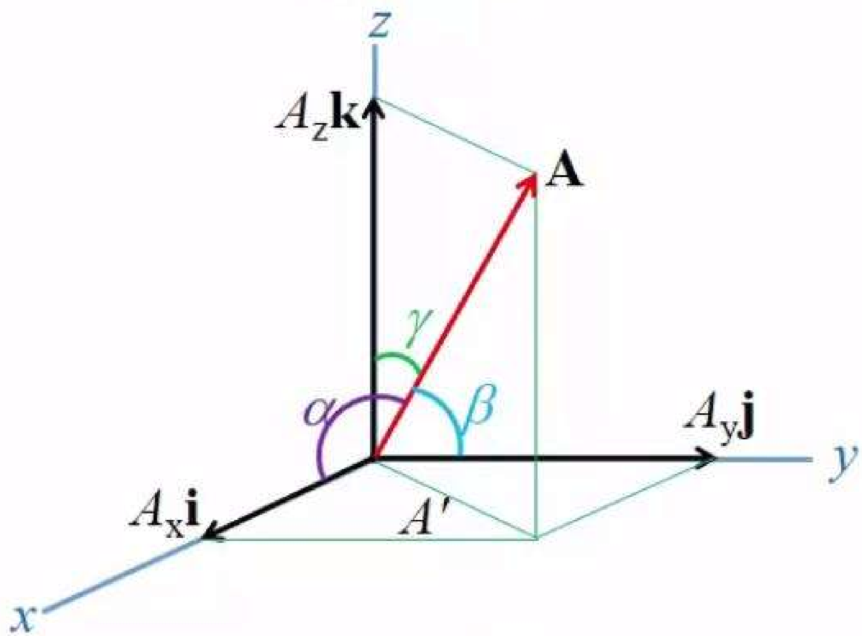
$$= 0.5\mathbf{i} + 0.707\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_F = 1200\text{ N}\cdot\{0.5\mathbf{i} + 0.707\mathbf{j} - 0.5\mathbf{k}\}$$

$$= \{600\mathbf{i} + 849\mathbf{j} - 600\mathbf{k}\}\text{ N}$$



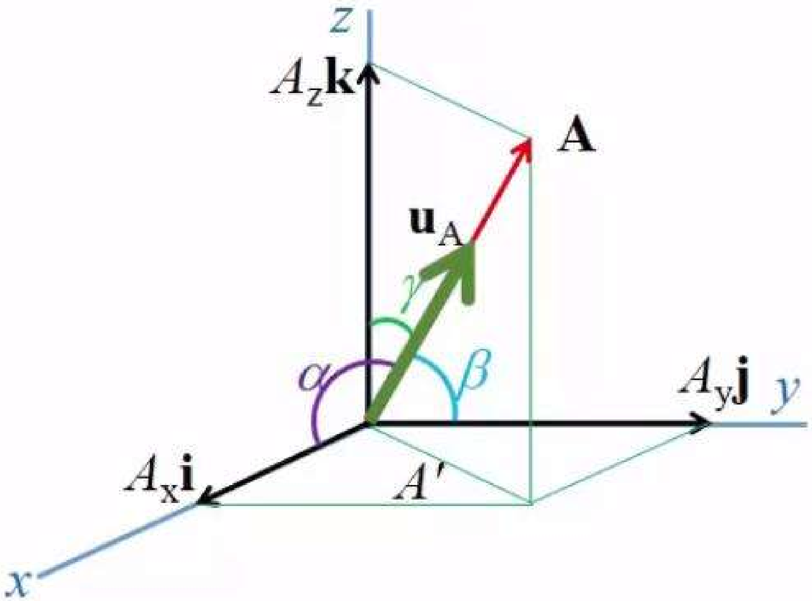
Question 3: If the coordinate direction angles $\alpha = 112°$, $\beta = 75°$ and $A_z = 5.0$ cm, determine the magnitude of vector **A**.



(a) 5.0 cm | (b) 13 cm
(c) 6.9 cm | (d) 5.6 cm

Dr. Yiheng Wang

Question 4: If the coordinate direction angles $\alpha = 112°$, $\beta = 75°$ and $A_z = 5.0$, determine the unit vector, $\mathbf{u}_A$, of $\mathbf{A}$.



(a) $-0.37\mathbf{i}+0.26\mathbf{j}+0.89\mathbf{k}$

(b) $0.37\mathbf{i}+0.26\mathbf{j}+0.89\mathbf{k}$

(c) $\{-0.37\mathbf{i}+0.26\mathbf{j}+0.89\mathbf{k}\}$cm

(d) $\{0.37\mathbf{i}+0.26\mathbf{j}+0.89\mathbf{k}\}$cm