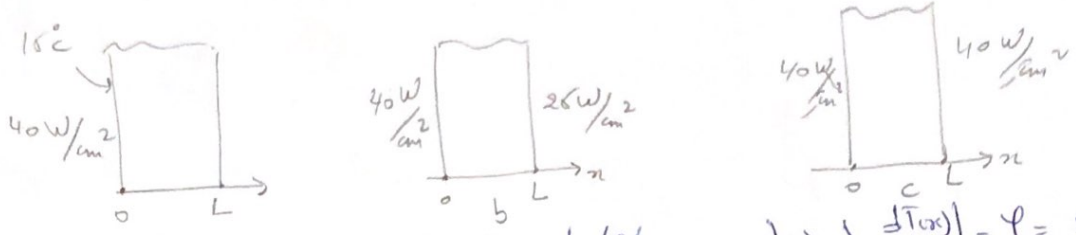
Corrigé type Interrogation écrite Groupe 1/

1/ types des conditions aux limites pour chaque cas:



a/ pour $x=0$ densité de flux imposée (Neuman) $\Rightarrow -\lambda \left.\frac{dT(x)}{dx}\right|_{x=0} = \varphi_0 = 40\ W/cm^2$ (0,5)

pour $x=0$ température imposée (Dirichlet) $\Rightarrow T(0) = T = 15°C$ (0,5)

b/ pour $x=0$ densité de flux imposée (Neuman) $\Rightarrow -\lambda \left.\frac{dT(x)}{dx}\right|_{x=0} = \varphi_0 = 40\ W/cm^2$ (0,5)

pour $x=L$ " " " " (Neuman) $\Rightarrow -\left.\frac{\lambda\, dT(x)}{dx}\right|_{x=L} = \varphi_0 = 25\ W/cm^2$ (0,5)

c/ pour $x=0$ densité de flux imposé (Neuman) $\Rightarrow \left.\frac{-\lambda\, dT(x)}{dx}\right|_{x=0} = 40\ W/cm^2$ (0,5)

pour $x=L$ " " " (Neuman) $\Rightarrow -\left.\frac{\lambda\, dT(x)}{dx}\right|_{x=L} = 40\ W/cm^2$ (0,5)

2/ Détermination de l'expression de la variation de Température (a).

On a pour ce cas: (régime permanent, sans source interne, unidimensionnel) (0,5)

$$\frac{d^2T(x)}{dx^2} = 0 \quad --(1) \quad (1)$$

pour $x=0 \Rightarrow -\lambda \left.\frac{dT(x)}{dx}\right|_{x=0} = \varphi_0$

$x=0 \Rightarrow T(0) = T = 15°C$

$(1) \Rightarrow \frac{dT(x)}{dx} = A \Rightarrow T(x) = Ax + B$ (0,5) (0,5)

$x=0 \Rightarrow \frac{dT(x)}{dx} \Rightarrow -\lambda . A = \varphi_0 \Rightarrow A = -\frac{\varphi_0}{\lambda}$ (1)

$x=0\ T(0) = T_1 = B = 15°C$ (1) $\Rightarrow \boxed{T(x) = -\frac{\varphi_0}{\lambda} + 15}$ (1)

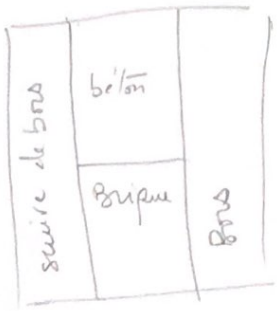
Corrigé type interrogation groupe 1

a) le taux de transfert de chaleur

$$\frac{\Phi}{S} = \varphi = \frac{\Delta T}{R_{eq}} \quad (1)$$

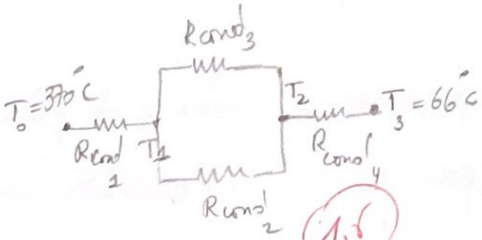
$$R_{eq} = R_{\grave{e}q} + R_{Cond_1} + R_{Cond_4} \quad (1)$$

$$\frac{1}{R_{\grave{e}q}} = \frac{1}{R_{Cond_2}} + \frac{1}{R_{Cond_3}} \quad (1)$$



$$R_{Cond_2} = \frac{e_2}{\lambda_2} = \frac{7,5.10^{-2}}{0,76} = 9,86.10^{-2}\ ^\circ C\,m^2/W \quad (0,5)$$

$$R_{Cond_3} = \frac{e_3}{\lambda_3} = \frac{7,5.10^{-2}}{0,69} = 10,86.10^{-2}\ ^\circ C\,m^2/W \quad (0,5)$$

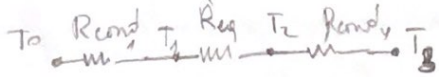


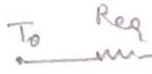
(1,5)

$$\frac{1}{R_{\grave{e}q}} = 29,72 \Rightarrow R'_{eq} = 0,0482\ ^\circ C\,m^2/W \quad (0,1)$$

$$R_{Cond_1} = \frac{e_1}{\lambda_1} = \frac{2,5.10^{-2}}{0,11} = 22,72.10^{-2}\ ^\circ C\,m^2/W \quad (0,5)$$

$$R_{Cond_4} = \frac{e_4}{\lambda_4} = \frac{5.10^{-2}}{0,147} = 34,01.10^{-2}\ ^\circ C\,m^2/W \quad (0,5)$$





$$R_{eq} = 0,0482 + 34,01.10^{-2} + 22,72.10^{-2} = 0,6155\ ^\circ C\,m^2/W \quad (0,5)$$

$$\varphi = \frac{\Delta T}{R_{eq}} = \frac{(370-66)}{0,6155} = 493,90\ W/m^2 \quad (0,5)$$

b) calcul des Température T_1 et T_2

$$\varphi = \frac{\Delta T_1}{R_{Cond_1}} \Rightarrow \Delta T_1 = \varphi . R_{Cond_1} \Rightarrow T_1 = 370 - 493,9 \cdot 22,72.10^{-2} = 257,78^\circ C$$

(0,5)

$$T_1 = 257,78^\circ C \quad (0,5)$$

$$\varphi = \frac{\Delta T_2}{R_{Cond_2}} \Rightarrow \Delta T_2 = \varphi . R_{Cond_2} \Rightarrow T_2 = 66 + \varphi . R_{Cond_4} = 66 + 493,9 . 34,01.10^{-2}$$

(0,5)

$$T_2 = 233,97^\circ C \quad (0,5)$$

Corrigé type interrogation groupe 3'

$\lambda_{verre} \sim 0{,}78\ w/m.°C$

$\lambda_{air} = 0{,}026\ w/m.°C$

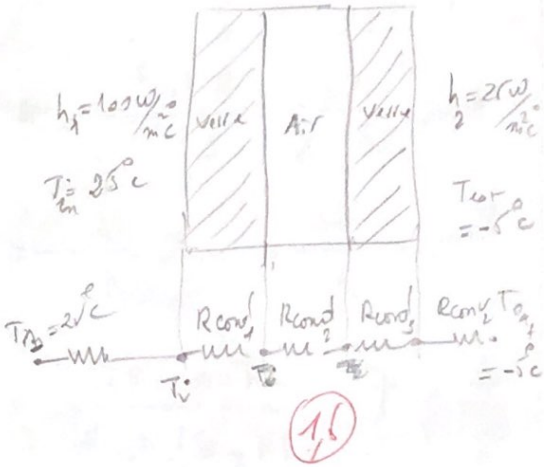
$h_1 = 10\ w/m^2.°C$, $h_2 = 25\ w/m^2.°C$

$e_{air} = 12.10^{-3}\ m$, $e_{verre} = 3.10^{-3}\ m$.

$S = 1{,}2 \times 2 = 2{,}4\ m^2$

1/ Détermination du taux de transfert de chaleur

$$\phi = \frac{\Delta T}{Req} \quad (1)$$



(1,5)

$$Req = R_{conv_1} + R_{cond_1} + R_{cond_2} + R_{cond_3} + R_{conv_2} \quad (1)$$

$$R_{conv_1} = \frac{1}{h_1 . S} = \frac{1}{10 \times (1{,}2 \times 2)} = 0{,}041\ °C/w \quad (0{,}1)$$

$$R_{conv_2} = \frac{1}{h_2 S} = \frac{1}{25\,(1{,}2 \times 2)} = 0{,}0167\ °C/w \quad (0{,}1)$$

$$R_{cond_1} = \frac{e_1}{\lambda_1 S} = \frac{3.10^{-3}}{0{,}78\,(1{,}2 \times 2)} = 0{,}0016\ °C/w \quad (0{,}1)$$

$$R_{cond_2} = \frac{e_2}{\lambda_2 . S} = \frac{12.10^{-3}}{0{,}026\,(2 \times 1{,}2)} = 0{,}192\ °C/w \quad (0{,}1)$$

$$R_{cond_3} = \frac{e_3}{\lambda_3 . S} = \frac{e_1}{\lambda_1 . S} = R_{cond_1} = 0{,}0016\ °C/w \quad (0{,}1)$$

$$Req = 0{,}2529 \quad (0{,}1)$$

$$\phi = \frac{\Delta T}{Req} = \frac{24 - (-5)}{0{,}2529} = 114{,}67\ w \quad (0{,}5)$$

2/ Calcul de la Température de la surface interne (T_i)

$$\phi = \frac{\Delta T}{R_{conv_1}} = \frac{T_1 - T_i}{R_{conv_1}} \quad (1{,}0) \Rightarrow T_i = T_1 - \phi . R_{conv_1} = 24 - 114{,}67 . 0{,}041 \quad (0{,}5)$$

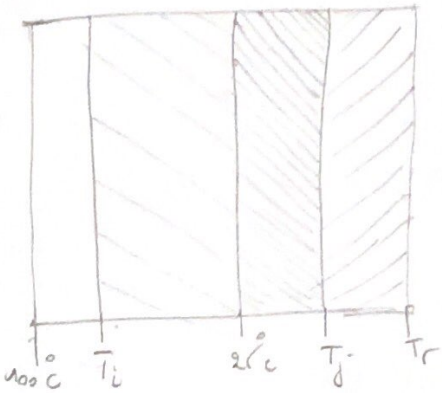
$$T_i = 19{,}30°C \quad (0{,}5)$$

Corrigé type interrogation écrite groupe 2

• Calcul du flux (densité de flux).

$$\varphi = \frac{\Phi}{S} = \frac{\Delta T}{R_{eq}} \quad (1)$$

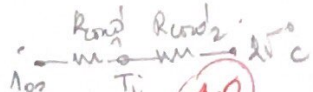
$$R_{eq} = R_{cond_1} + R_{cond_2} \quad (1)$$



$$R_{cond_1} = \frac{e_1}{\lambda_1} = \frac{2.10^{-2}}{2} = 1.10^{-2} \text{ °C m}^2/\text{W} \quad (0,5)$$

$$R_{cond_2} = \frac{e_2}{\lambda_2} = \frac{6.10^{-2}}{1} = 6.10^{-2} \text{ °C m}^2/\text{W} \quad (0,5)$$

$$\varphi = \frac{\Delta T}{R_{eq}} = \frac{100 - 25}{(1+6).10^{-2}} = 1071{,}43 \text{ W/m}^2 \quad (0,5)$$



(1,0)

Calcul des Température T_i, T_j, T_r

$T_i = ?$

$$\varphi = \Delta T_1 / R_{cond_1} = \frac{T_1 - T_i}{R_{cond_1}} \Rightarrow \varphi . R_{cond_1} = T_1 - T_i \Rightarrow T_i = T_1 - \varphi . R_{cond_1} = T_1 - \varphi . \frac{e_1}{\lambda_1} \quad (0,5)$$

$$T_i = 100 - 1071{,}43 . 1.10^{-2} = 100 - 1071{,}43.10^{-2} = \boxed{89{,}28°C} \quad (0,5)$$

$T_j = ?$

$$\varphi = \frac{\Delta T_3}{R_{cond_3}} \quad (0,5)$$

$$R_{cond_3} = \frac{e_3}{\lambda_3} = \frac{4.10^{-2}}{5} = 0{,}8.10^{-2} \text{ °C m}^2/\text{W} \quad (0,5)$$

$$T_j = T_2 - \varphi . R_{cond_3} = 25 - 1071{,}43 . 0{,}8.10^{-2} = \boxed{16{,}42°C} \quad (0,5)$$

$T_r = ?$

$$\varphi = \frac{\Delta T_4}{R_{cond_4}} \Rightarrow T_j - T_r = \varphi R_{cond_4} \Rightarrow T_r = T_j - \varphi . R_{cond_4} = 16{,}42 - 1071{,}43.10^{-2} \quad (0,5)$$

$$R_{cond_4} = \frac{4.10^{-2}}{4} = 10^{-2} \quad (0,5)$$

$$\boxed{T_r = 5{,}70°C} \quad (0,5)$$