Correction de l'examen.

EXO 1 6pt

$$h = \frac{b-a}{n} = \frac{\pi/2 - 0}{6} = 0{,}26 \quad 1$$

$x_i = x_0 + ih$, avec $x_0 = 0$ 1

$\Rightarrow x_1 = x_0 + h$; $x_2 = x_0 + 2h$; $x_3 = x_0 + 3h$

$x_4 = x_0 + 4h$; $x_5 = x_0 + 5.h$; $x_6 = x_0 + 6.h$

Ainsi : $x_1 = 0{,}26$; $x_2 = 0{,}52$; $x_3 = 0{,}78$

$x_4 = 1{,}04$; $x_5 = 1{,}30$; $x_6 = 1{,}56$

En appliquant la méthode des Trapèzes, il vient :

$$I_1 = \frac{h}{2}\left[f(0) + f(0{,}26)\right] = 0{,}03 \quad 0{,}5$$

$$I_2 = \frac{h}{2}\left[f(0{,}26) + f(0{,}52)\right] = 0{,}08 \quad 0{,}5$$

$$I_3 = \frac{h}{2}\left[f(0{,}52) + f(0{,}78)\right] = 0{,}10 \quad 0{,}5$$

$$I_4 = \frac{h}{2}\left[f(0{,}78) + f(1{,}04)\right] = 0{,}07 \quad 0{,}5$$

$$I_5 = \frac{h}{2}\left[f(1{,}04) + f(1{,}30)\right] = 0{,}04 \quad 0{,}5$$

$$I_6 = \frac{h}{2}\left[f(1{,}30) + f(1{,}56)\right] = 0{,}01 \quad 0{,}5$$

$$\Rightarrow I(f) = \sum_{i=1}^{6} I_i \simeq 0{,}33. \quad 1$$

1

Exo 2 (4 pts)

La méthode de Newton :

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ 0,5

$f(x_n) = 1 + x_n + e^{x_n}$ 0,5

$f'(x_n) = 1 + e^{x_n}$ 0,5

$$\Rightarrow x_{n+1} = x_n - \frac{1 + x_n + e^{x_n}}{1 + e^{x_n}}$$ 0,5

$$\Rightarrow x_{n+1} = x_n - \left[\frac{x_n}{1+e^{x_n}} + \frac{1+e^{x_n}}{1+e^{x_n}}\right]$$

$$\Rightarrow x_{n+1} = x_n - \left[\frac{x_n}{1+e^{x_n}} + 1\right]$$

La valeur initiale $x_0 = -1/2$

$$x_1 = -1/2 - \left[\frac{(-1/2)}{1+e^{(-1/2)}} + 1\right] = -0,67$$ 0,5

$\| -0,67 + 0,5 0 \| = 0,17 > \varepsilon$

$$x_2 = -0,67 - \left[\frac{-0,67}{1+e^{(-0,67)}} + 1\right] = -1,22$$ 0,5

$\| -1,22 + 0,67 \| = 0,55 > \varepsilon$

$$x_3 = -1{,}22 - \left[\frac{-1{,}22}{1+e^{(-1{,}22)}} + 1\right] = -1{,}2783$$

0,1

$\Rightarrow \| x_3 - x_2 \| > \varepsilon$

$x_4 = -1{,}2785$ et $\| x_4 - x_3 \| \simeq 10^{-4}$.

0,1

$\Rightarrow$ La racine est $x^* = x_4 = -1{,}278$.

Exo 3

Dans cet exercice, il est question uniquement de la méthode d'Euler.

$$\begin{cases} c'(t) = -k\, c^2(t), & t\,[0,1] \\ c(0) = 1. \end{cases}$$

$$\Rightarrow \frac{u_{n+1} - u_n}{h} = -k\, u_n^2$$

$$\Rightarrow u_{n+1} = u_n - k\,h\,u_n^2.$$

2

ainsi : $u_1 = 1 - 1 \times 0{,}1 \times [1]^2 = 0{,}90$

$u_2 = 0{,}90 - 1 \times 0{,}1 \times [0{,}9]^2 = 0{,}81.$

$u_3 = 0{,}74$; $u_4 = 0{,}68$. $u_5 = 0{,}63$

$u_6 = 0{,}59$; $u_7 = 0{,}55$; $u_8 = 0{,}51$.

8 pts

$u_9 = 0{,}48$; $u_{10} = 0{,}46$.