Formulaire nécessaire pour la théorie du signal

Valeurs des sin, cos et tang des angles usuelles

x en radians	$cos(x)$	$sin(x)$	$tang(x)$
$\mathbf{0\ (2\pi)}$	**1**	**0**	**0**
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	**1**
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	**0**	**1**	$+\infty$
π	**-1**	**0**	**0**

Formule trigonométriques

- $cos^2(x) + sin^2(x) = 1$
- $cos^2(x) = \frac{1+\cos(2x)}{2}$
- $sin^2(x) = \frac{1-\cos(2x)}{2}$
- $cos(2x) = cos^2(x) - sin^2(x)$
- $cos(2x) = 1 - 2sin^2(x)$
- $sin(2x) = 2\sin(x).\cos(x)$
- $cos(-x) = \cos(x)$
- $sin(-x) = -\sin(x)$
- $sin(\pi + x) = -\sin(x)$
- $cos(\pi - x) = -\cos(x)$
- $cos\left(\frac{\pi}{2} - x\right) = -\sin(x)$
- $sin\left(\frac{\pi}{2} + x\right) = \cos(x)$
- $tang(x) = \frac{sin(2x)}{1+\cos(2x)} = \frac{1-\cos(2x)}{sin(2x)}$
- $cos(a+b) =$ $\cos(a)\cos(b) - \sin(a)\sin(b)$
- $sin(a+b) =$ $\sin(a)\cos(b) + \cos(a)\sin(b)$
- $tang(a+b) = \frac{tang(a)+tang(b)}{1-tang(a).tang(b)}$
- $cos(a-b) =$ $\cos(a)\cos(b) + \sin(a)\sin(b)$
- $sin(a-b) =$ $\sin(a).\cos(b) - \cos(a).\sin(b)$
- $tang(a-b) = \frac{tang(a)-tang(b)}{1+tang(a).tang(b)}$

Nombres complexes

- $Z = a + jb$, forme algébrique.
- $Z^* = a - jb$, Z^* : le conjugué.
- $(z_1 + z_2)^* = {z_1}^* + {z_2}^*$
- $(z_1.z_2)^* = {z_1}^*.{z_2}^*$
- $|z|^2 = z.z^*$
- $|z| = \sqrt{a^2 + b^2}$, module.
- $\theta = \arg(z) = arctang(\frac{b}{a})$, l'argument ou la phase.
- $z = |z|e^{j\theta}$, forme polaire
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

Formule d'Euler

- $e^{jx} = \cos(x) + j\sin(x)$
- $\cos(x) = \frac{e^{jx}+e^{-jx}}{2}$
- $\sin(x) = \frac{e^{jx}-e^{-jx}}{2j}$

Formule de MOIVRE

- $(\cos(x) + j\sin(x))^n = \cos(nx) + j\sin(nx)$

Développements limités

- $sin(a) = a - \frac{a^3}{3!} + \frac{a^5}{5!} + \cdots + (-1)^p \frac{a^{2p+1}}{(2p+1)!}$
- $cos(a) = 1 - \frac{a^2}{2!} + \frac{a^4}{4!} + \cdots + (-1)^p \frac{a^{2p}}{(2p)!}$

Dérivées

- $(\sin(x))' = \cos(x)$
- $(cos(x))' = -\sin(x)$
- $(tang(x))' = 1 + tang^2(x) = \frac{1}{cos^2(x)}$
- $(x^n)' = nx^{n-1}$
- $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$
- $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$
- $(e^x)' = e^x$
- $\left((f(x))^n\right)' = n(f(x))^{n-1}(f(x))'$
- $\left(\frac{1}{f(x)}\right)' = -\frac{f(x)'}{f(x)^2}$
- $\left(\sqrt{f(x)}\right)' = \frac{f(x)'}{2\sqrt{f(x)}}$
- $(e^{f(x)})' = f(x)'e^{f(x)}$
- $(ln(x))' = \frac{1}{x}$
- $(ln(f(x)))' = \frac{f(x)'}{f(x)}$
- $(f(x)g(x))' = f(x)'g(x) + f(x)g(x)'$
- $\frac{f(x)}{g(x)} = \frac{f(x)'g(x) - f(x)g(x)'}{(g(x))^2}$

Primitives

- $\int x^n dx = \frac{x^{n+1}}{n+1} + Cst$
- $\int sin(ax)dx = -\frac{cos(ax)}{a} + Cst$
- $\int cos(ax)dx = \frac{sin(ax)}{a} + Cst$
- $\int \frac{dx}{(cos(x))^2} = tang(x) + Cst$
- $\int e^{ax} dx = \frac{e^{ax}}{a} + Cst$
- $\int \frac{dx}{x} = ln|x| + Cst$
- $\int tang(x)dx = -ln|cos(x)| + Cst$
- $\int \frac{dx}{a^2+x^2} = \frac{1}{a} arctang\left(\frac{x}{a}\right) + Cst$
- $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} ln\left|\frac{a+x}{a-x}\right| + Cst$

Infiniment petits ($x \rightarrow 0$)

- $(1+x)^n = 1 + nx$
- $(1-x)^n = 1 - nx$
- $\frac{1}{1+x} = 1 - x$
- $\frac{1}{1-x} = 1 + x$
- $\sqrt{1+x} = 1 + \frac{x}{2}$
- $\sqrt{1-x} = 1 - \frac{x}{2}$
- $ln(1+x) = x$
- $e^x = 1 + x$
- $sin(x) = x$, (x en radian)
- $tang(x) = x$, (x en radian)
- $cos(x) = 1 - \frac{x^2}{2}$, ($x$ en radian)