Rigid body equilibrium: Conditions

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Objective :

To introduce the general conditions for 2D and 3D rigid body equilibrium problems.

Particle equilibrium

First let's recall the conditions for **particle equilibrium**.

According to **Newton's first law**, an object will have **a linear acceleration of zero** when there is **no unbalanced force** acting on it.

$$\mathbf{F}_R = \sum \mathbf{F} = \mathbf{0}$$

Particle equilibrium $\mathbf{F}_R = \sum \mathbf{F} = \mathbf{0}$

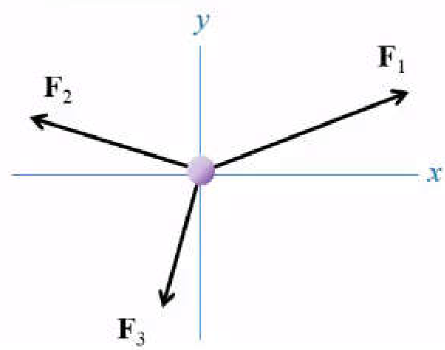
Since **Particle** is an idealized object with **no size** or **shape,** and is only represented by **a dot** in space, the forces acting on the particle will be **concurrent**.

For a 2D problem, the vector equation can be written as **2 scalar equations**.

For a 3D problem, the vector equation becomes **3 scalar equations**.

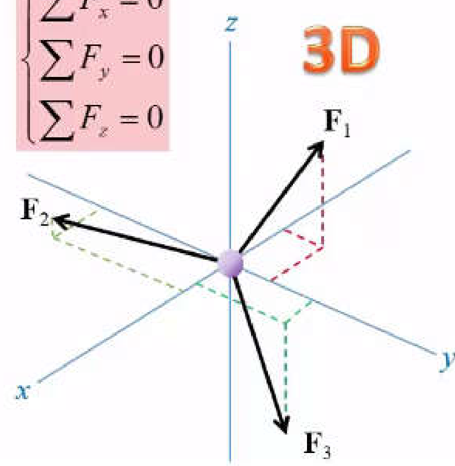
2D

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$$



3D

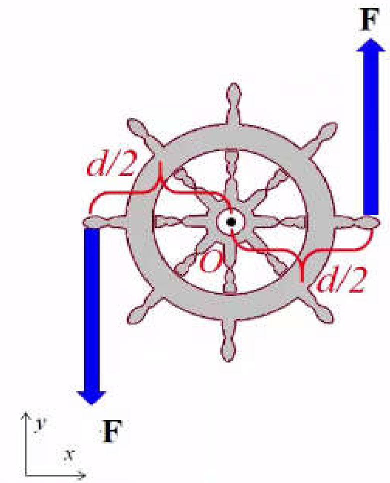
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$



2D probem = Solve for 2 unknowns

3D probem = Solve for 3 unknowns

Recall: moment of a couple



However, **a rigid body** has **shape** and **size** and it is **not necessarily static** even if **the resultant force** acting on it is indeed **zero**.

The two forces acting on this wheel are indeed in **equilibrium**.

$$\sum F_y = F + (-F) = 0$$

But this only means that they don't cause **translational motion**. We already learned that these two forces make **a couple moment**.

$$\sum M_O = F \cdot \frac{d}{2} + F \cdot \frac{d}{2}$$
$$= F \cdot d$$

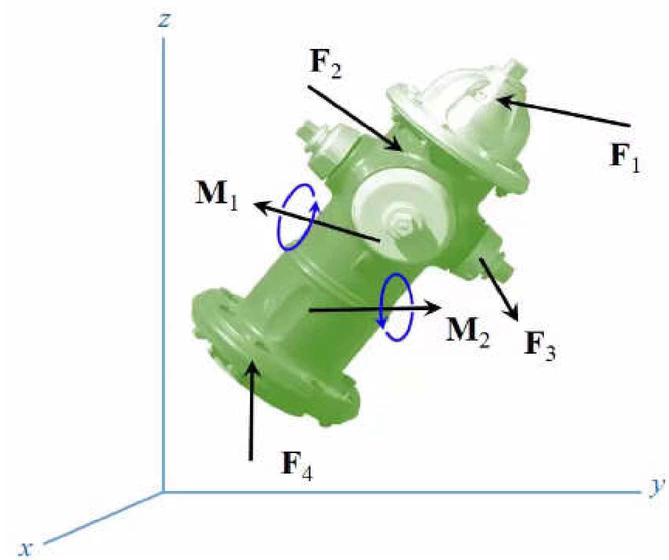
This moment causes **rotational effect** on this wheel.

Therefore, for **a rigid body** to be **static**, it is not enough to only have **unbalanced force**, but the **resultant moment** summarized about **any arbitrary point** must be **Zero** as well.

Otherwise, the object will rotate.

For **a rigid body** that is subjected to multiple forces and couple moments, **the first condition for equilibrium is:**

$$\mathbf{F}_R = \sum \mathbf{F} = \mathbf{0}$$



Then, **the resultant moment** summarized about any point, must also be **zero**, includes both the **total moment** caused **by the forces** and the **total couple moments.**

$$\mathbf{M}_{R,O} = \sum \mathbf{M}_{F,O} + \sum \mathbf{M} = \mathbf{0}$$

Conditions for rigid body equilibrium

As a summary, for rigid body equilibrium, we can have **two vector equations**, one for force and one for moment.

$$\begin{cases} \mathbf{F}_R = \sum \mathbf{F} = \mathbf{0} \\ \mathbf{M}_{R,O} = \sum \mathbf{M}_{F,O} + \sum \mathbf{M} = \mathbf{0} \end{cases}$$

For a 2D problem, based on one free body diagram, we can write a maximum of **3 independent scalar equations** and then solve for **3 unknowns**.

2-D problems:

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_O = 0 \end{cases}$$

2-D problems:

$$\begin{cases} \sum F_x = 0 \\ \sum M_A = 0 \\ \sum M_B = 0 \end{cases}$$

2-D problems:

$$\begin{cases} \sum M_A = 0 \\ \sum M_B = 0 \\ \sum M_C = 0 \end{cases}$$

Conditions for rigid body equilibrium

As a summary, for rigid body equilibrium, we can have **two vector equations**, one for force and one for moment.

$$\begin{cases} \mathbf{F}_R = \sum \mathbf{F} = \mathbf{0} \\ \mathbf{M}_{R,O} = \sum \mathbf{M}_{F,O} + \sum \mathbf{M} = \mathbf{0} \end{cases}$$

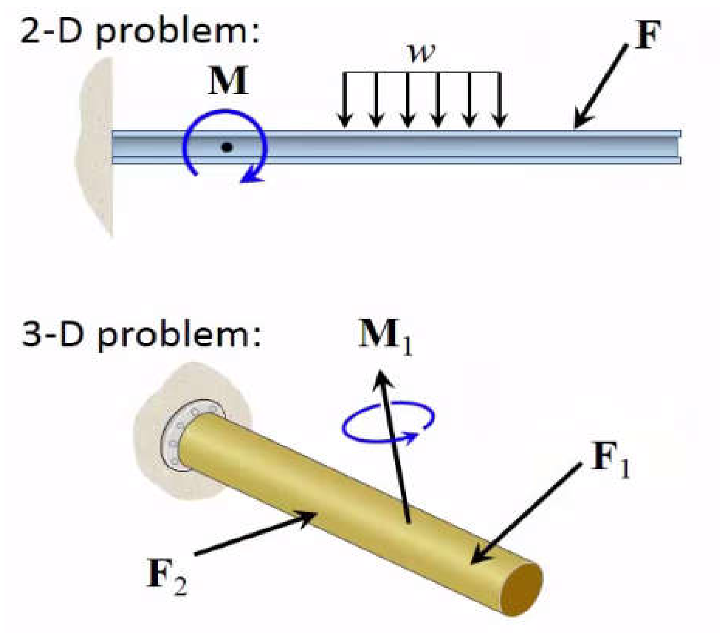
For a 3D problem, based on one free body diagram, we can write a maximum of **6 independent scalar equations** and solve for a maximum of **6 unknowns**.

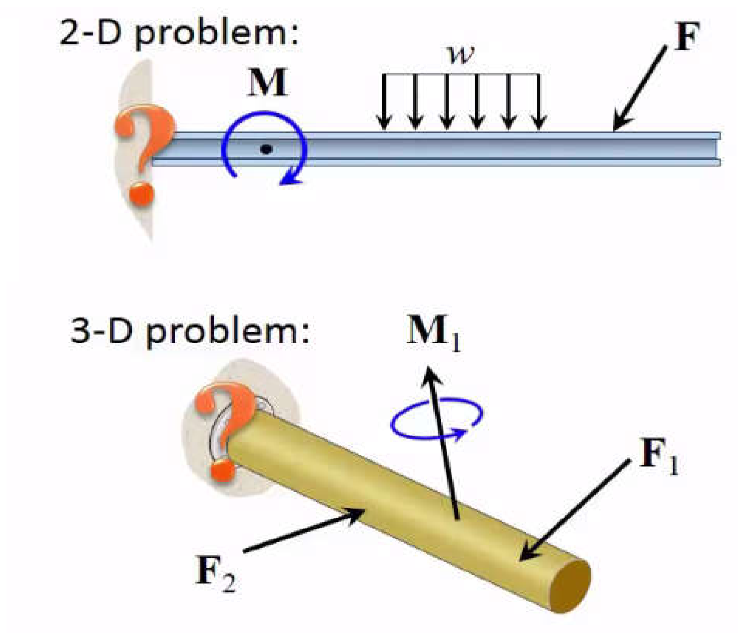
3-D problems:

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases} \quad \begin{cases} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{cases}$$

Here are **two examples** of rigid body **equilibrium problems**. Normally the **applied loadings** are **known**, and we will need to use **the equilibrium equations** to find the **unknown support reactions**.

The support reactions are also external force or moment acting on the body.





Rigid body equilibrium:
2D Supports

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Objectives :

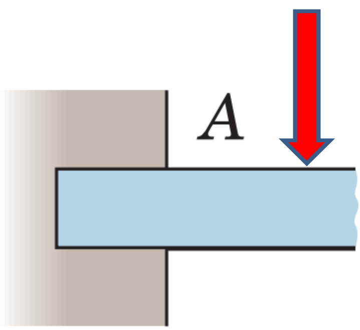
- To explain how to identify 2D support reactions.
- To demonstrate how to solve 2D rigid body equilibrium problems.

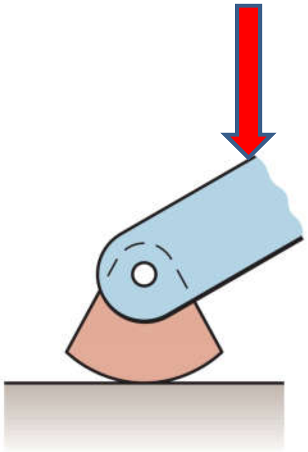
Support types:

- The reaction in the element depends on the support it self.

Question:

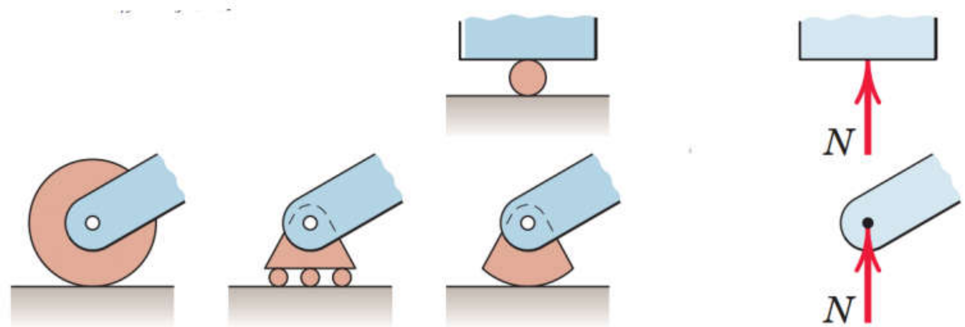
What is the difference here:





Support types:

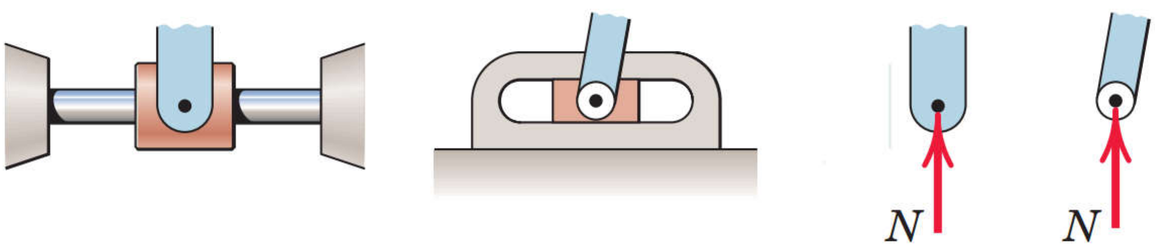
- Roller Support



Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

Support types:

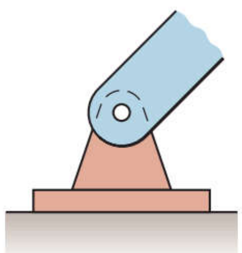
- Freely sliding guide

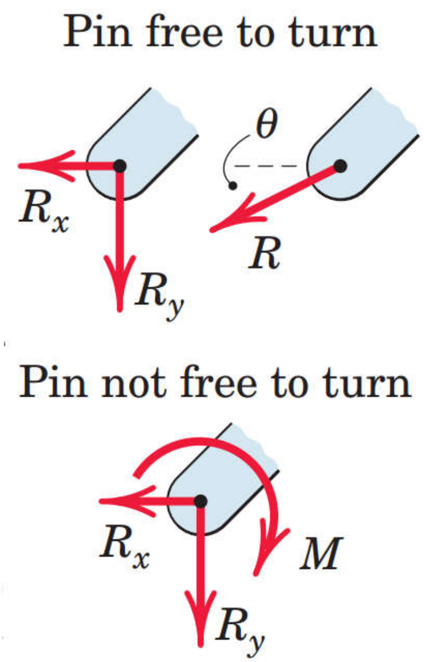


Collar or slider free to move along smooth guides; can support force normal to guide only.

Support types:

- Pin connection

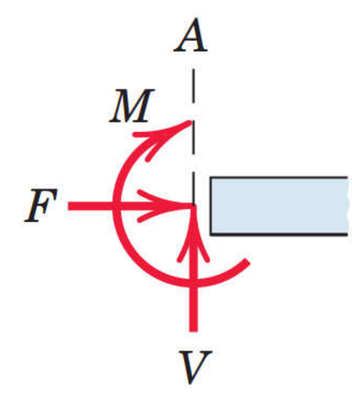




A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.

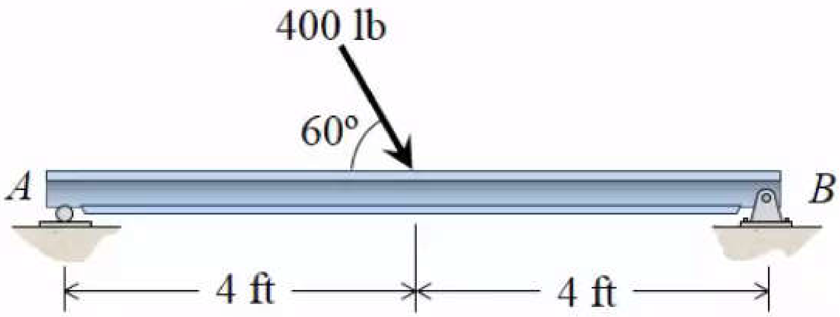
Support types:

- Pin connection



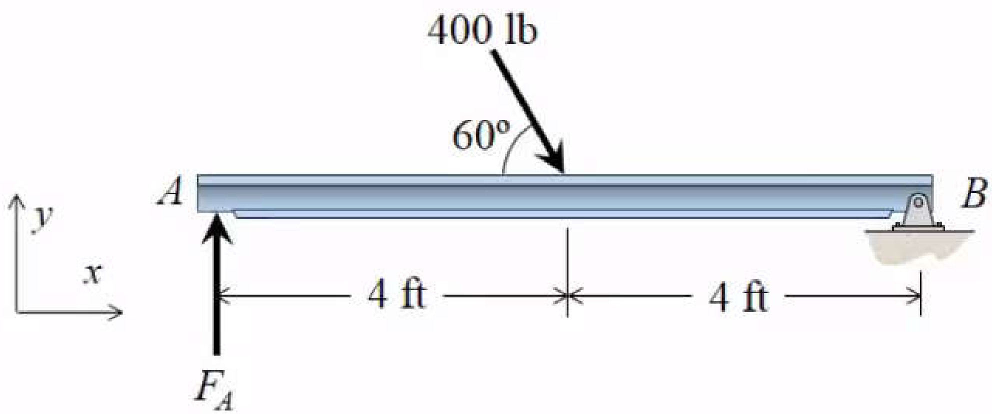
A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.

Example 3: Determine the support reactions at the roller at A and the pin at B. Neglect the weight and size of the beam.



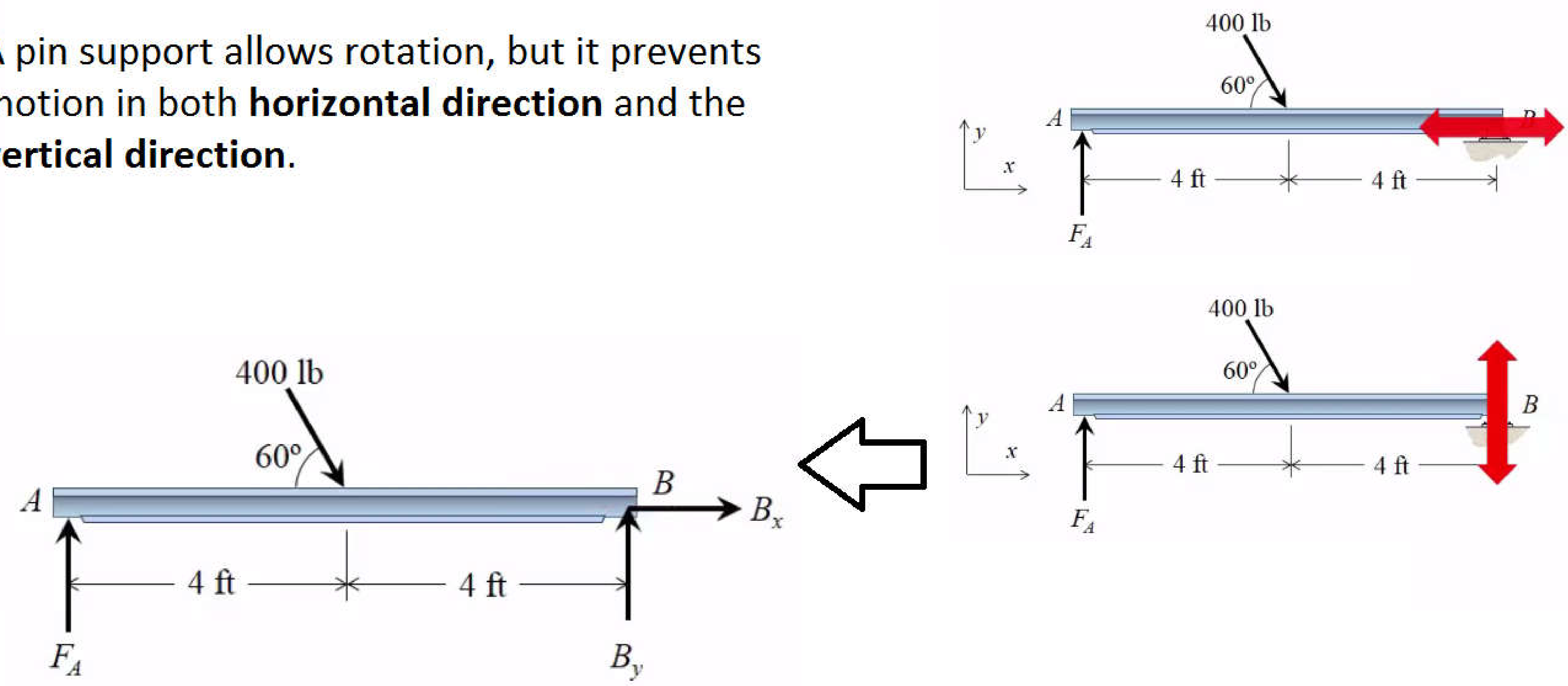
We need to determine the support reactions at the **roller** and the **pin**.

Free body diagram (FBD)

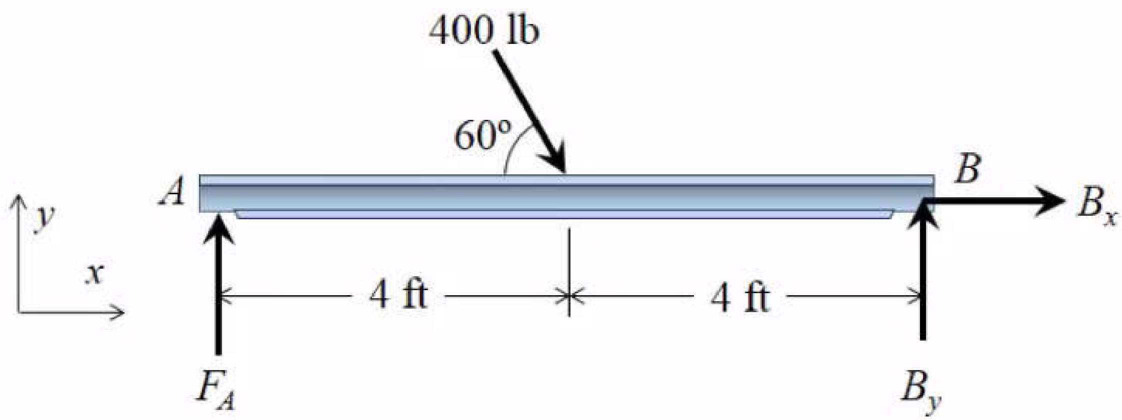


A **roller is similar to a rocker**, or **a simple contact** support. It exerts a force that is perpendicular to the contacting surface.

A pin support allows rotation, but it prevents motion in both **horizontal direction** and the **vertical direction**.



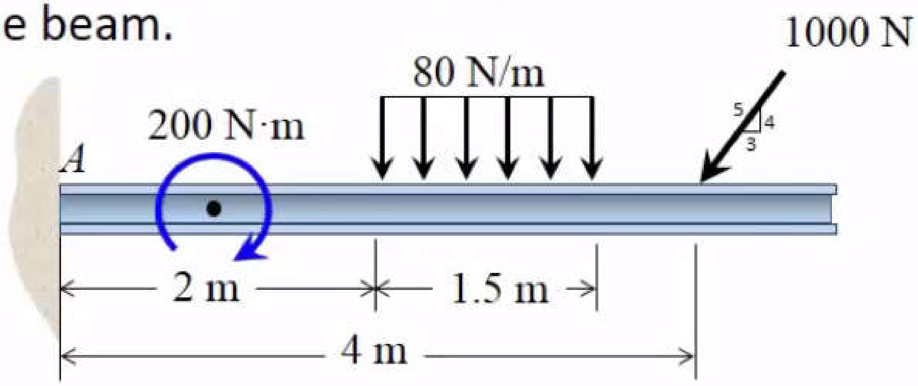
Free body diagram (FBD)



$$\sum F_x = 400\text{ lb}\cdot\cos 60^\circ + B_x = 0$$
$$\sum F_y = F_A - 400\text{ lb}\cdot\sin 60^\circ + B_y = 0$$
$$\sum M_B = -F_A\cdot 8\text{ ft} + 400\text{ lb}\cdot\sin 60^\circ\cdot 4\text{ ft} = 0$$

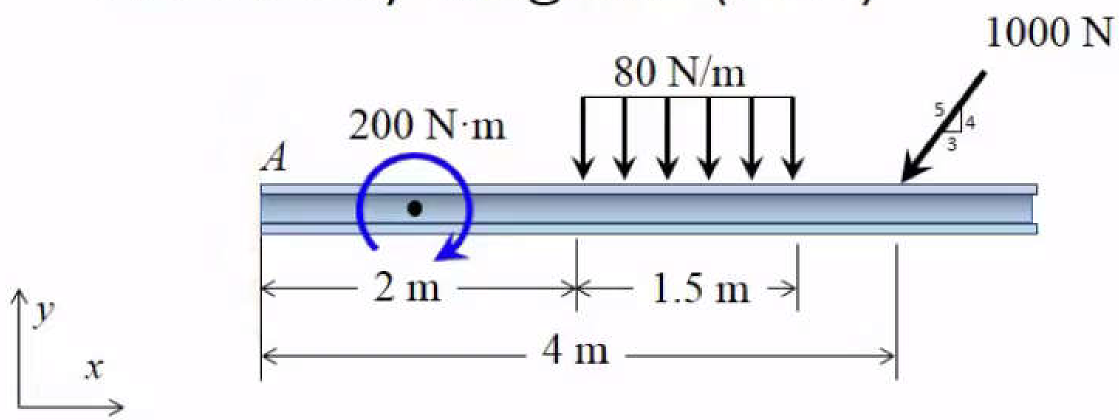
$$\therefore \begin{cases} F_A = 173\text{ lb} \\ B_x = -200\text{ lb} \\ B_y = 173\text{ lb} \end{cases}$$

Example 1: Determine the support reactions at the fixed support, *A*. Neglect the weight and size of the beam.



The applied loadings acting on this beam is known and we need to determine **the support reactions** at the fixed support A.

Free body diagram (FBD)

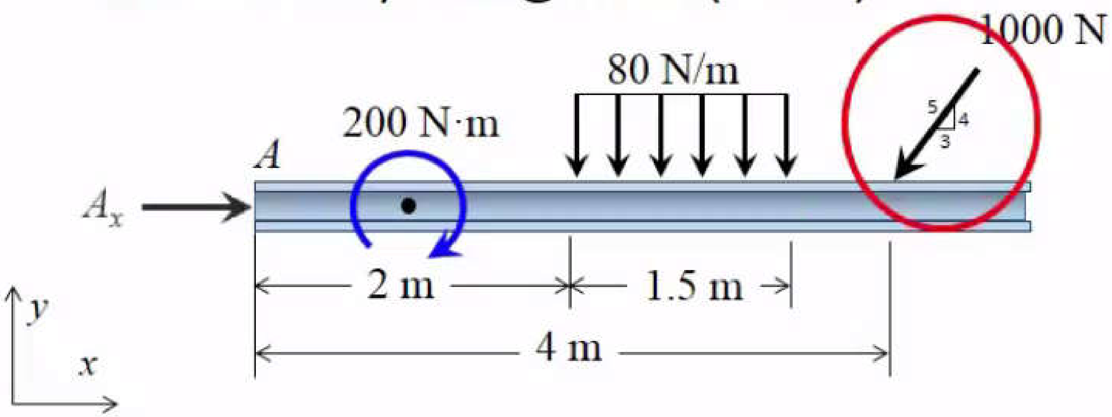


1) We need to first isolate our object. Therefore, the wall needs to disappear.

2) We need to note **all external forces** and moments on the diagram.

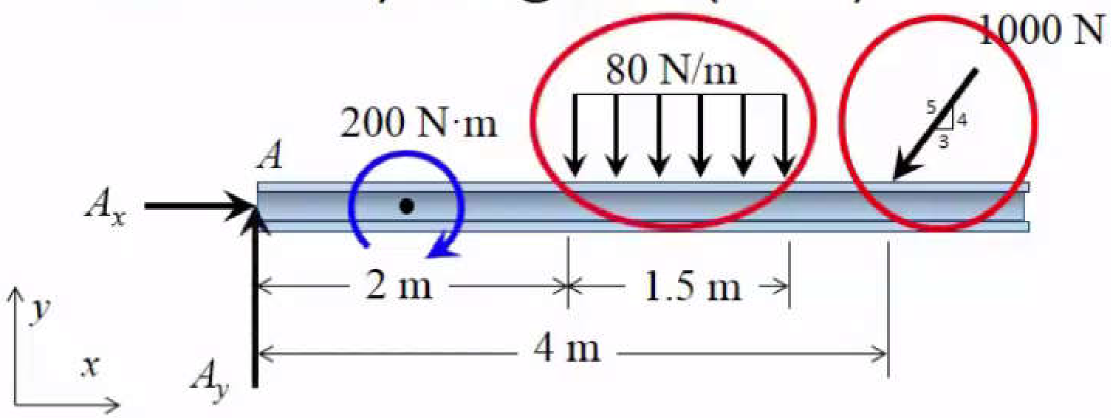
First, we need to be clear that the body is in equilibrium, in other words, **it is not moving.**

Free body diagram (FBD)



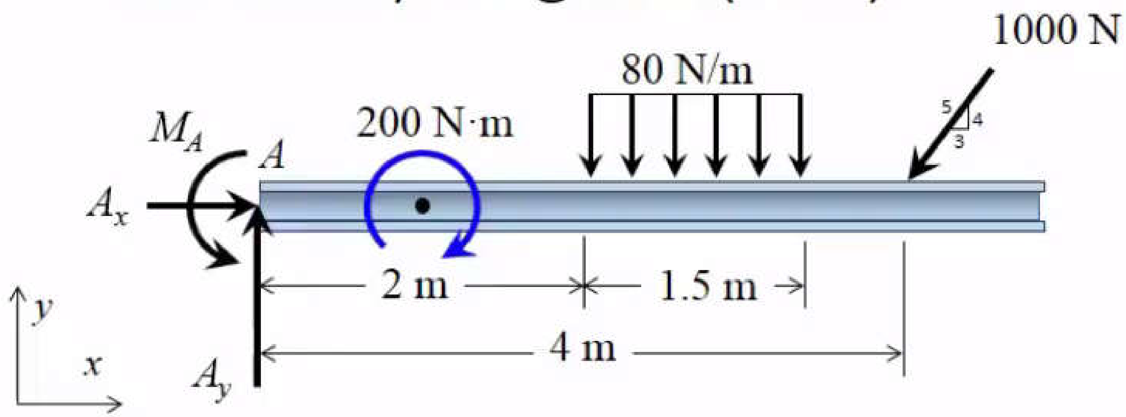
We see that there is a **horizontal component** of this force that will **push** the member to the **left.** Therefore, the wall must exert a horizontal force to the **right** to prevent this motion to the **left.**

Free body diagram (FBD)



Because of the vertical component of the **concentrated force**, as well as **the distributed force**, causing the member **to go down**. Therefore, the wall must exert **a vertical force up** in order to prevent this downward motion.

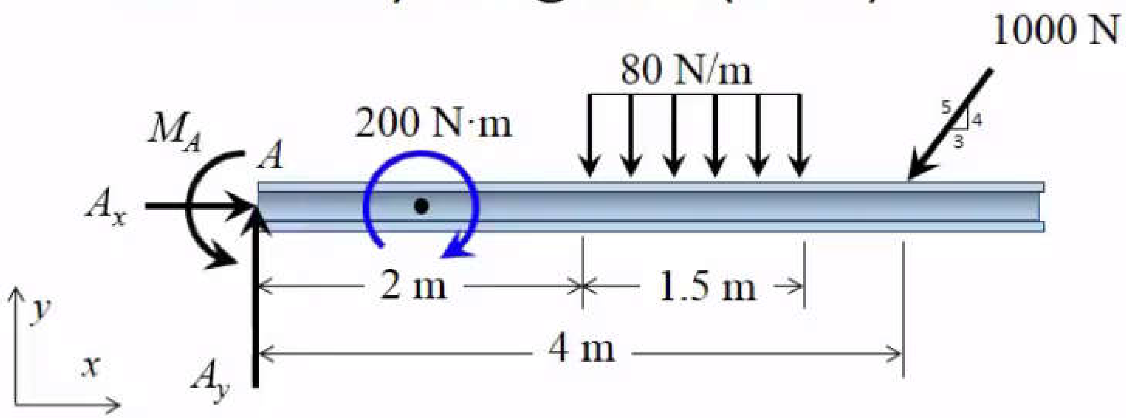
Free body diagram (FBD)



Finally, the forces and **the moment** all cause the member to **rotate** clockwise. Therefore, the wall also must exert **a moment support** that is **counterclockwise**, in order to prevent this **rotational motion**.

Since this is a **2D problem**, all motions can only accur within the **xy plane**, therefore there can not be any other type of motion, and the wall **will not provide** any other type of **support reactions**.

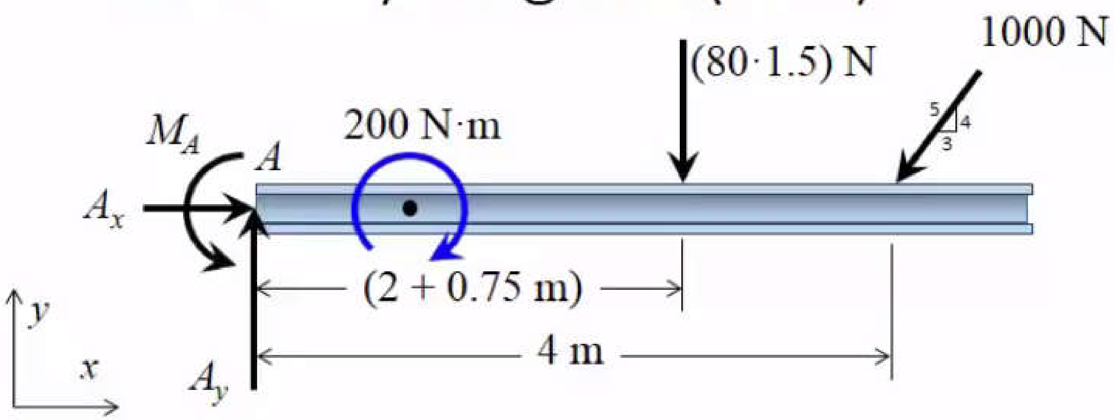
Free body diagram (FBD)



We have 3 unknowns: A_x, A_y and M_A.

We can write the three equilibrium equations to solve for all of them.

Free body diagram (FBD)



$$\sum F_x = A_x - \frac{3}{5}\cdot 1000\text{N} = 0$$

$$\sum F_y = A_y - 80\cdot 1.5\text{N} - \frac{4}{5}\cdot 1000\text{N} = 0$$

$$\sum M_A = M_A - 200\text{N}\cdot\text{m} - 80\cdot 1.5\text{N}\cdot 2.75\text{m}$$
$$-\frac{4}{5}\cdot 1000\text{N}\cdot 4\text{m} = 0$$

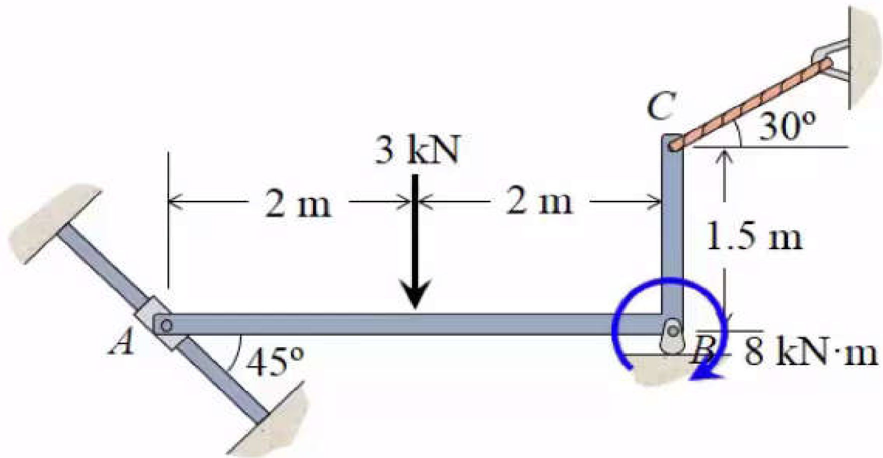
$$\therefore \begin{cases} A_x = 600\text{ N} \\ A_y = 920\text{ N} \\ M_A = 3730\text{ N}\cdot\text{m} \end{cases}$$

Identify all the support reactions

- If a support prevents translational effect in a direction – exerts a **force**.

- If a support prevents rotational effect about an axis – exerts a **couple moment**.

Example 2: Determine the support reactions at the smooth collar at A, rocker at B and cable at C. Neglect the weight and size of member ABC.



- It is supported by a **smooth collar** at point A.
- A **rocker** at point B.
- A **cable** at point C.

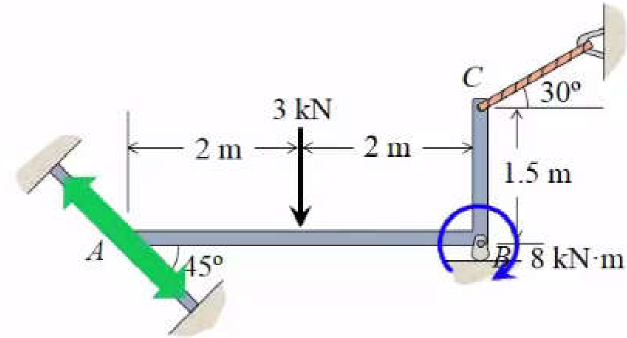
We are asked to determine the support reactions at these points A, B and C.

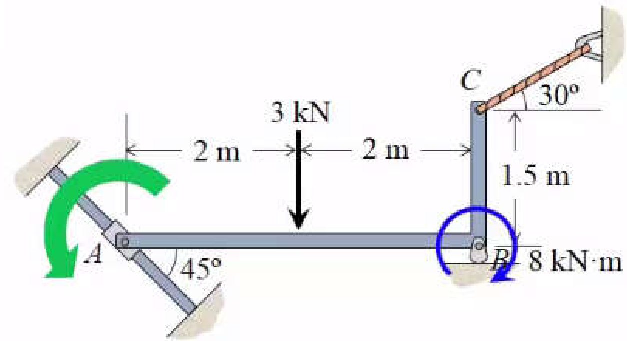
Smooth collar:

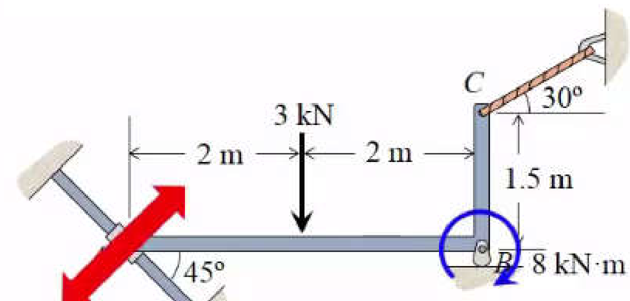
1) It does allow **movement** in this direction.

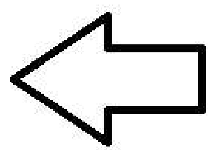
2) Because **the member is pinned** to the **coller**, the member is allowed **to rotate**.

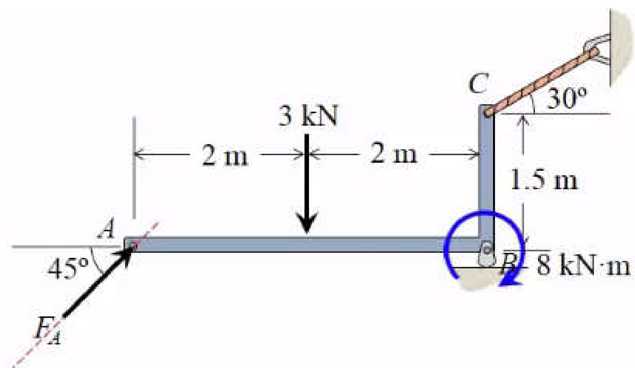
3) The only motion **it prevents** is along the direction **perpendicular** to the rod.











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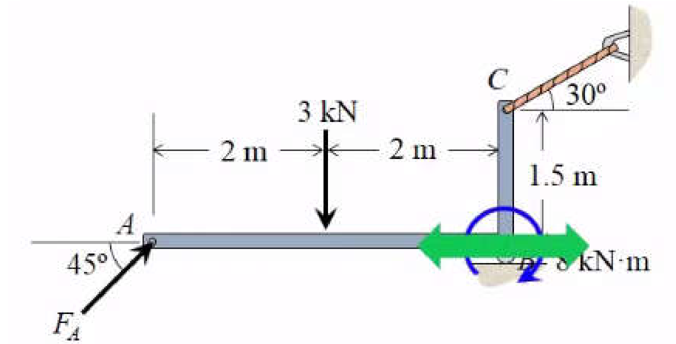
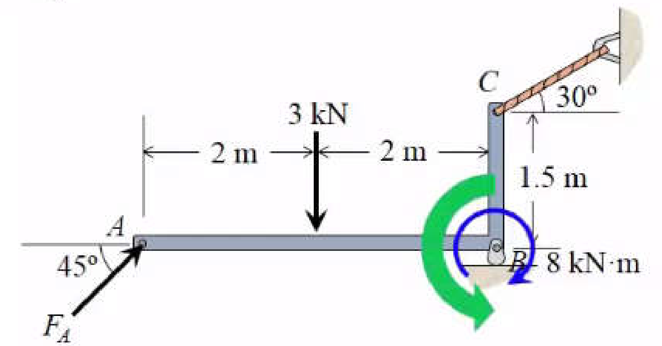
Rocker:

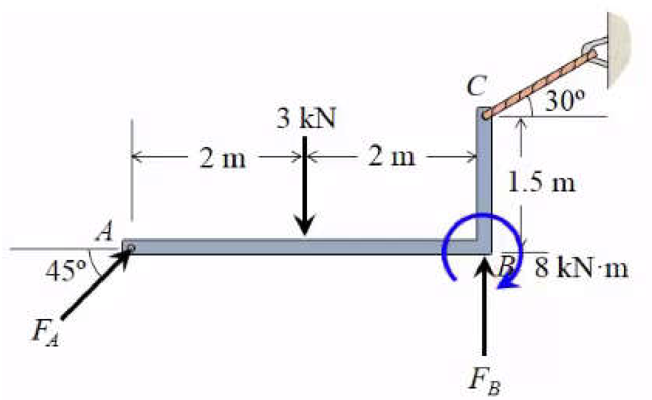
1) It allows motion along the **horizontal direction**.

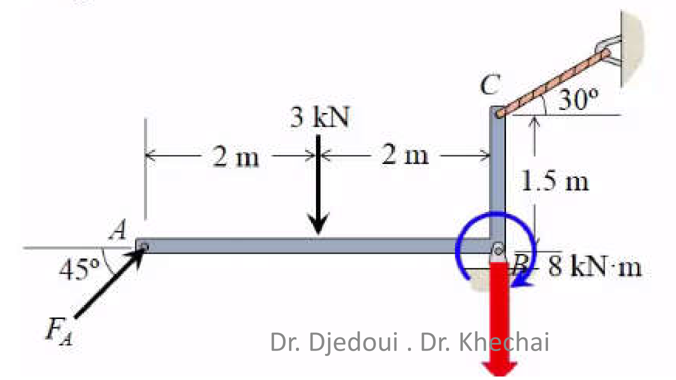
2) It allows **rotations** as well.

3) The only motion **it does not allow** is for the member **to move down**.



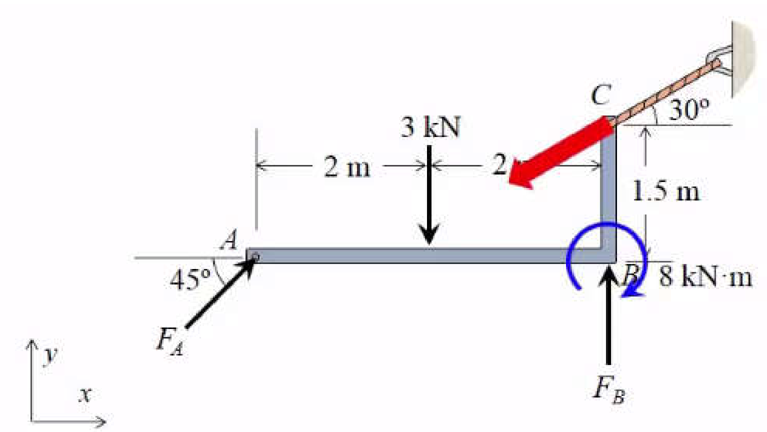


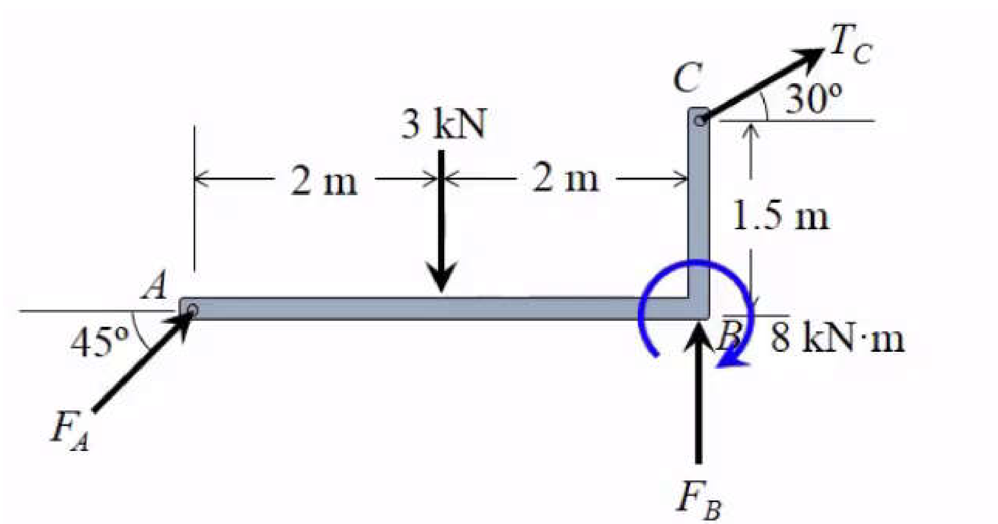


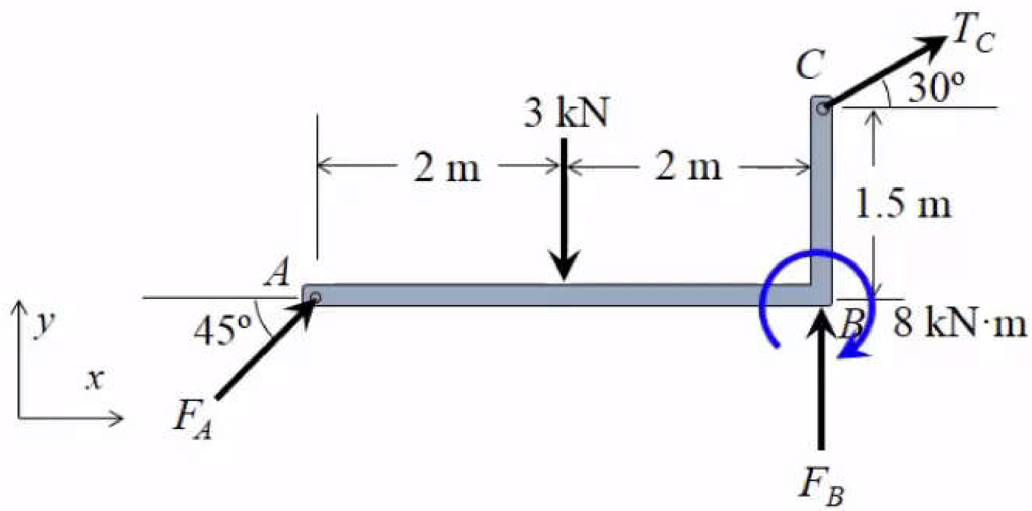


Cable:

Cable at point C allows all motions except for when the member wants **to move away**.







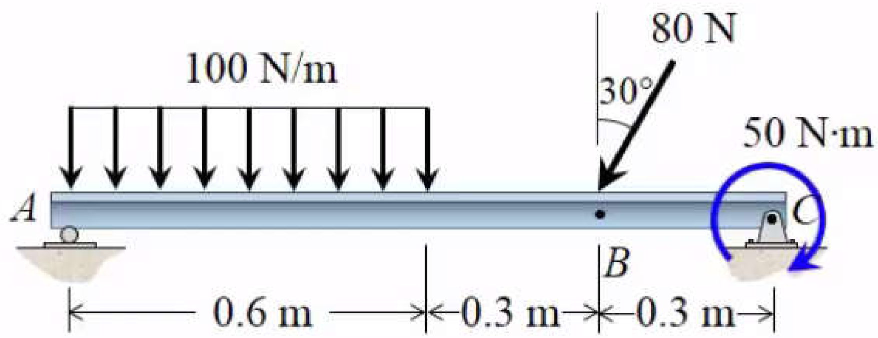
$$\sum F_x = F_A \cdot \cos 45^\circ + T_C \cdot \cos 30^\circ = 0$$

$$\sum F_y = F_A \cdot \sin 45^\circ - 3\text{kN} + F_B + T_C \cdot \sin 30^\circ = 0$$

$$\sum M_C = -F_A \cdot \sin 45^\circ \cdot 4\text{m} + F_A \cdot \sin 45^\circ \cdot 1.5\text{m} + 3\text{kN} \cdot 2\text{m} - 8\text{kN} \cdot \text{m} = 0$$

$$\therefore \begin{cases} F_A = -1.13 \text{ kN} \\ F_B = 3.34 \text{ kN} \\ T_C = 0.924 \text{ kN} \end{cases}$$

Question 1: For the beam as shown, what is the magnitude of the support force at roller support A?



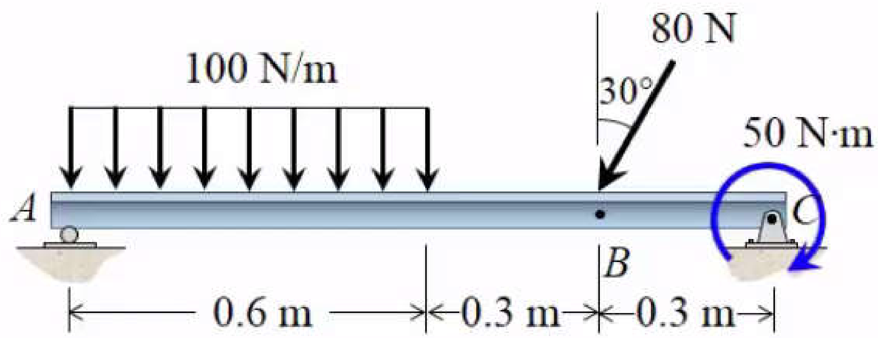
(a) 109 N

(b) 40 N

(c) 116 N

(d) 20.7 N

Question 2: For the beam as shown, what is the magnitude of the support force at pin support *C*?



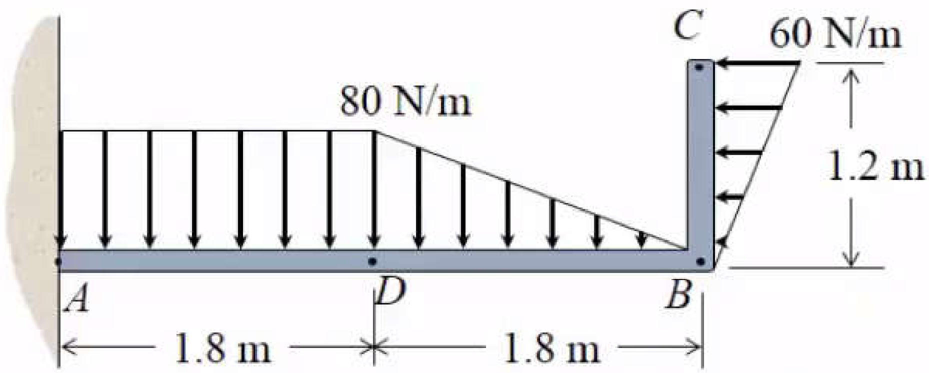
(a) 109 N

(b) 40 N

(c) 116 N

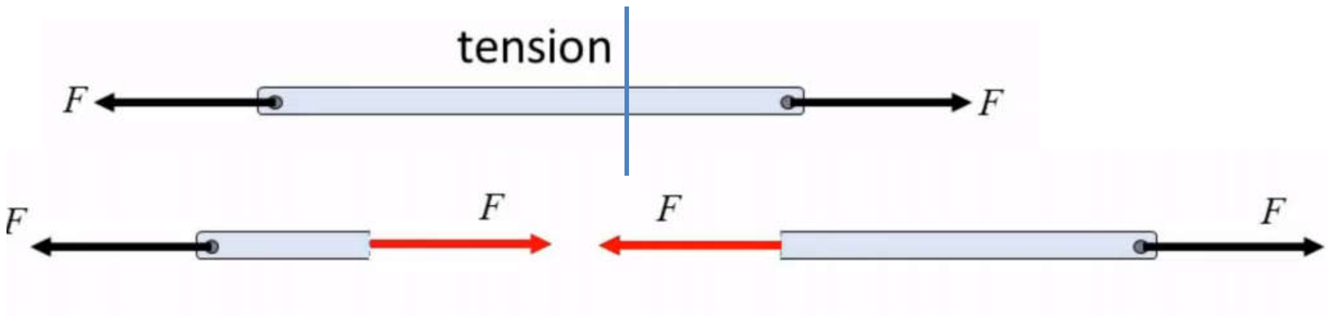
(d) 20.7 N

Question 3: For the beam as shown, determine the support reactions at the fixed support at point A.

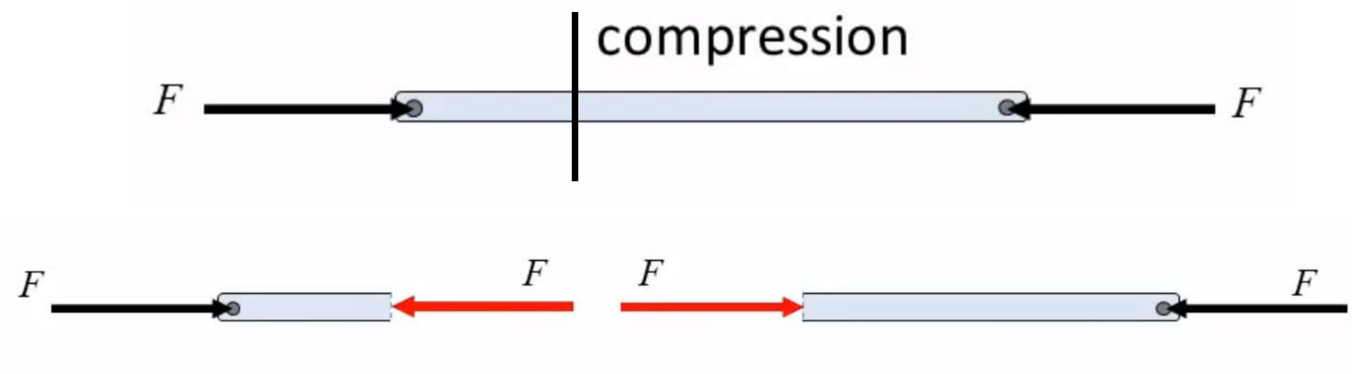


Internal forces:

- Method of Section (introduction)



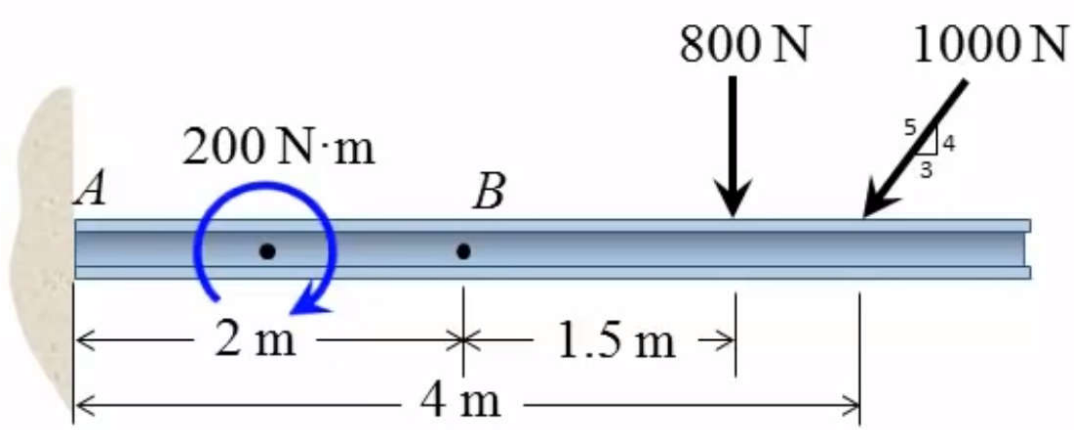
$$\sigma = \frac{F}{A} \le \sigma_{allow}$$



Internal forces:

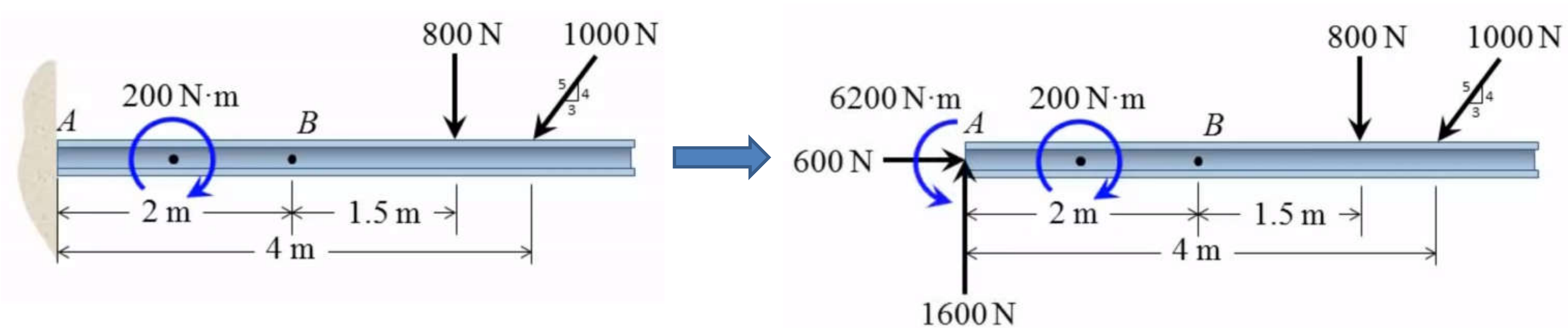
- Method of Section: example

Determine the internal forces at point *B* of the cantilever beam.



Internal forces:

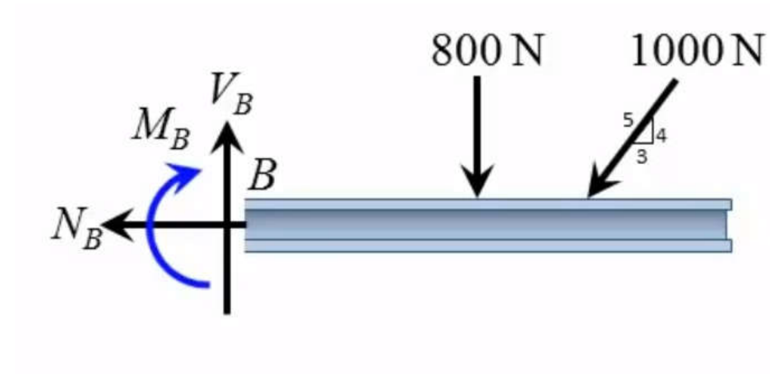
- Step 1: if necessary, determine the external support reactions(s)

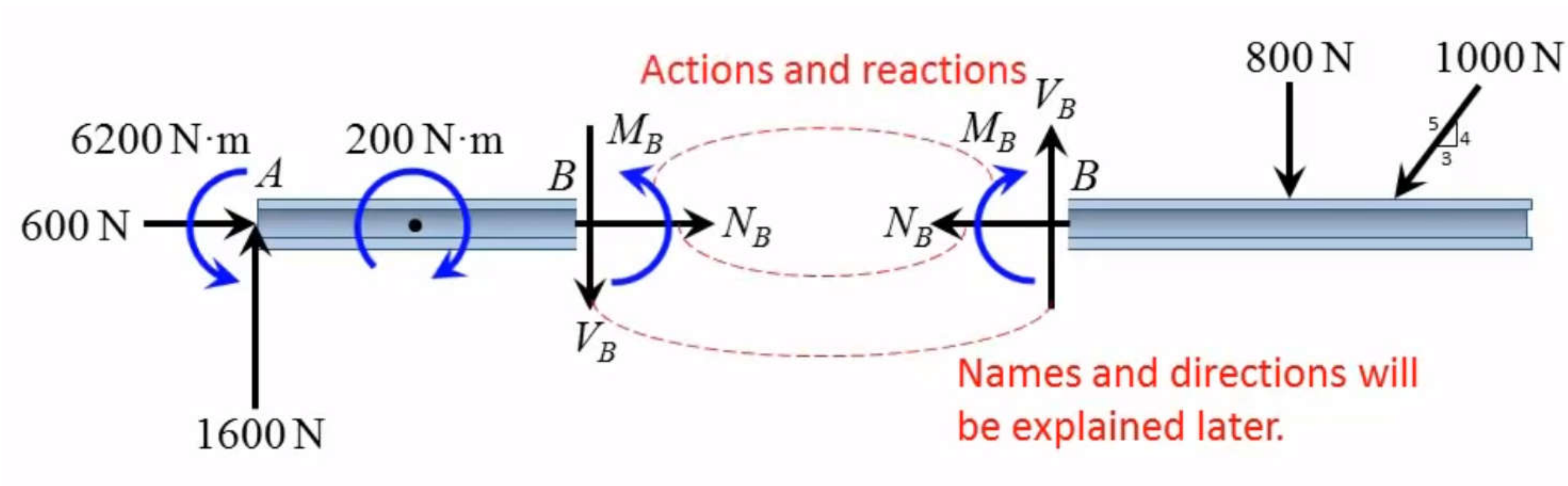


- Step 2: 'cut' the member at the specified point and not the unknowns

Internal forces:

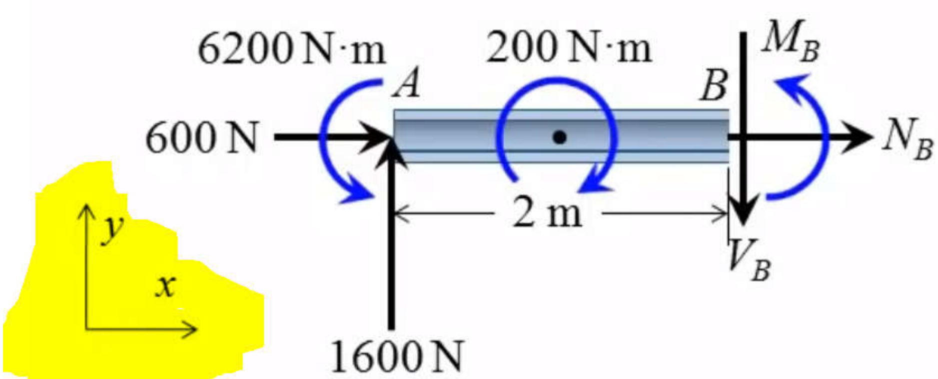
The right segment





Internal forces:

Solving for the left segment:



$$\sum F_x = 600\text{N} + N_B = 0$$

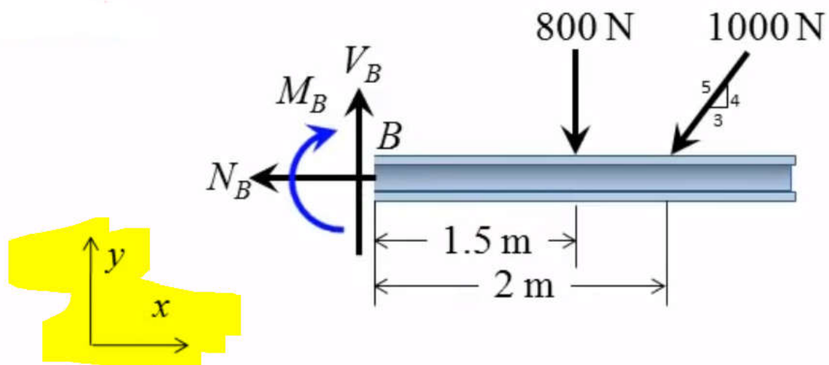
$$\sum F_y = 1600\text{N} - V_B = 0$$

$$\sum M_B = 6200\text{N}\cdot\text{m} - 200\text{N}\cdot\text{m} - 1600\text{N}\cdot 2\text{m} + M_B = 0$$

$$\begin{cases} N_B = -600\ \text{N} \\ V_B = 1600\ \text{N} \\ M_B = -2800\ \text{N}\cdot\text{m} \end{cases}$$

Internal forces:

Solving for the right segment:



$$\sum F_x = -N_B - \frac{3}{5}\cdot 1000\text{N} = 0$$

$$\sum F_y = V_B - 800\text{N} - \frac{4}{5}\cdot 1000\text{N} = 0$$

$$\sum M_B = -M_B - 800\text{N}\cdot 1.5\text{m} - \frac{4}{5}\cdot 1000\text{N}\cdot 2\text{m} = 0$$

$$\therefore \begin{cases} N_B = -600\ \text{N} \\ V_B = 1600\ \text{N} \\ M_B = -2800\ \text{N}\cdot\text{m} \end{cases}$$

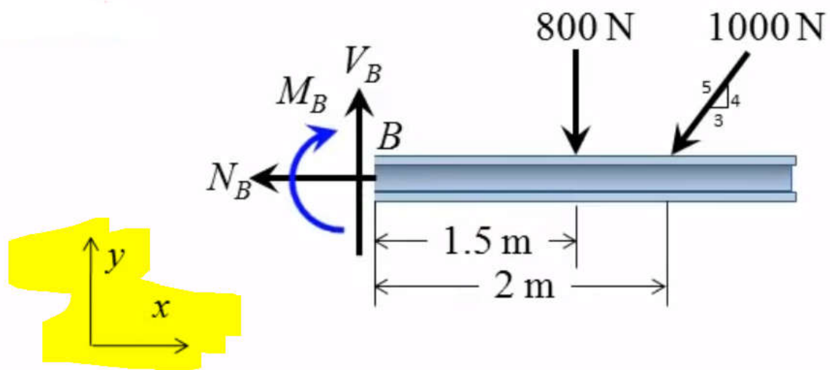
N: Normal Force

V: shear force

M: Bending Moment

Internal forces:

Solving for the right segment:



$$\sum F_x = -N_B - \frac{3}{5} \cdot 1000\text{N} = 0$$

$$\sum F_y = V_B - 800\text{N} - \frac{4}{5} \cdot 1000\text{N} = 0$$

$$\sum M_B = -M_B - 800\text{N} \cdot 1.5\text{m} - \frac{4}{5} \cdot 1000\text{N} \cdot 2\text{m} = 0$$

$$\therefore \begin{cases} N_B = -600\ \text{N} \\ V_B = 1600\ \text{N} \\ M_B = -2800\ \text{N} \cdot \text{m} \end{cases}$$

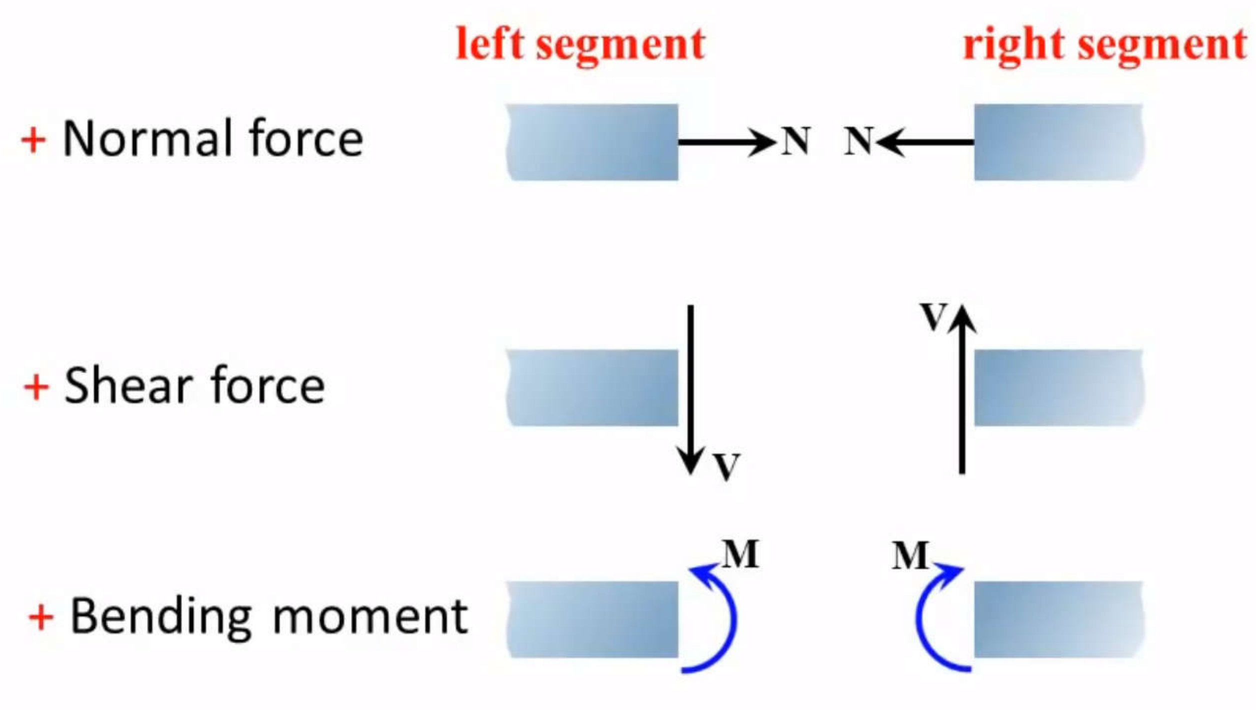
N: Normal Force

V: shear force

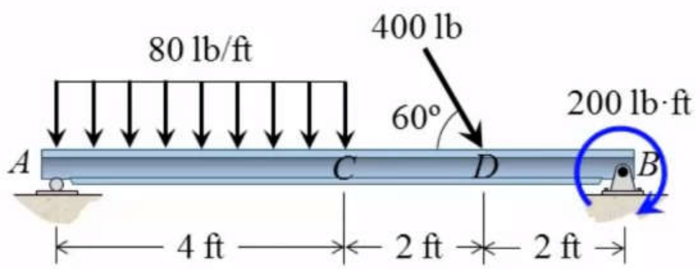
M: Bending Moment

Internal forces:

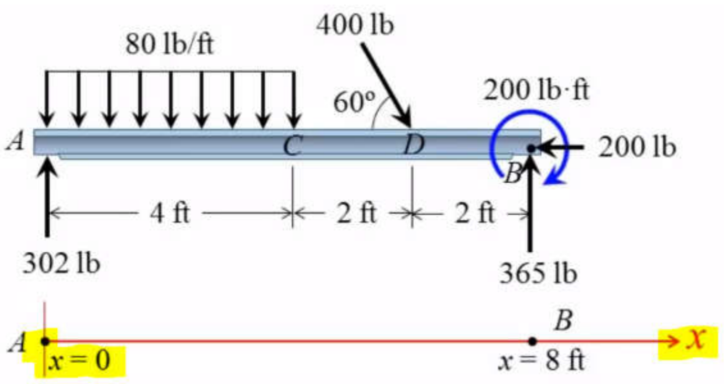
Sign convention:

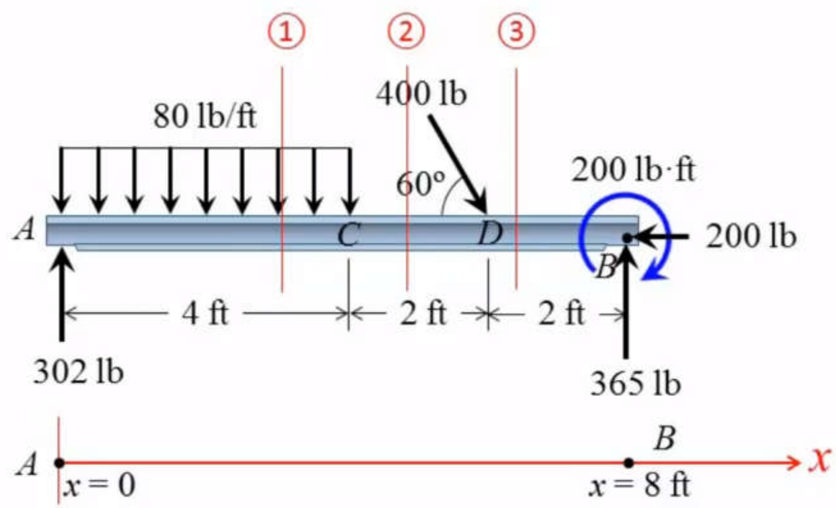


Internal forces: Shear fore and bending moment functions and diagrams



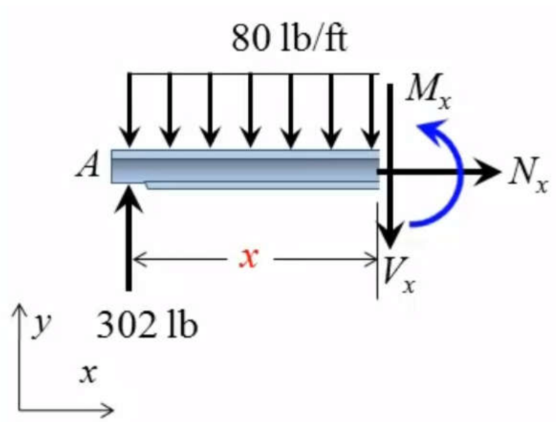
Free Body diagram (FBD):

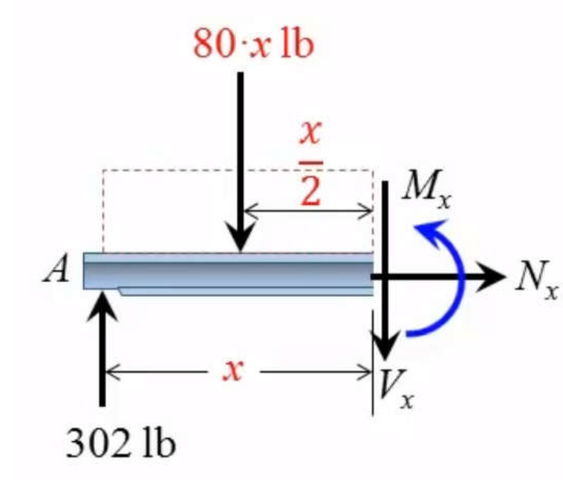




Internal forces: Shear fore and bending moment functions and diagrams

Section 1:





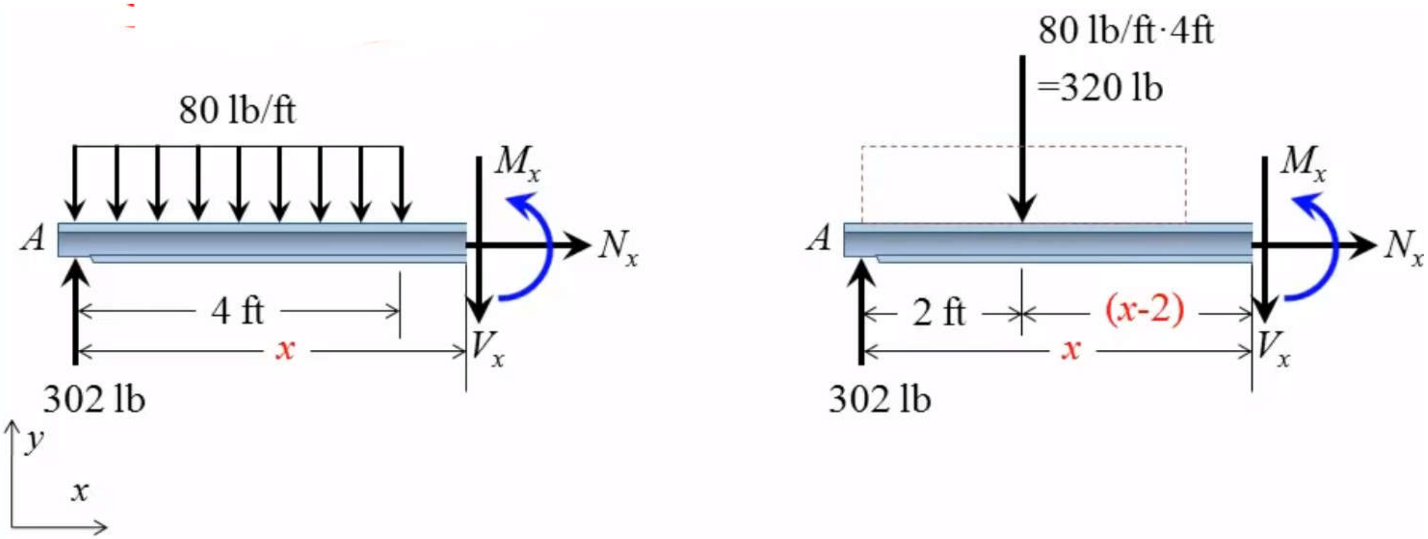
$$\sum F_y = 302 - 80 \cdot x - V_x = 0$$

$$\sum M_x = -302 \cdot x + 80x \cdot \frac{x}{2} + M_x = 0$$

$$\begin{cases} V_x = -80x + 302 \ (\text{lb}) \\ M_x = -40x^2 + 302x \ (\text{lb} \cdot \text{ft}) \end{cases}$$

Internal forces: Shear fore and bending moment functions and diagrams

Section 2:



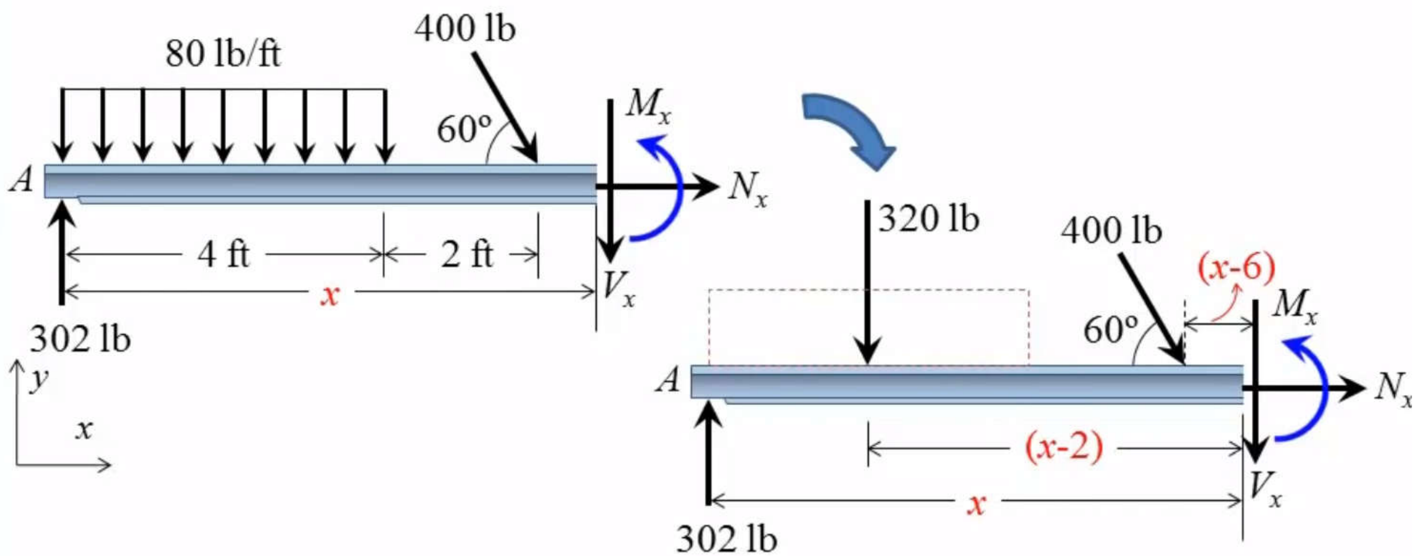
$$\sum F_y = 302 - 320 - V_x = 0$$

$$\sum M_x = -302 \cdot x + 320 \cdot (x-2) + M_x = 0$$

$$\therefore \begin{cases} V_x = -18 \text{ (lb)} \\ M_B = -18x + 640 \text{ (lb} \cdot \text{ft)} \end{cases}$$

Internal forces: Shear fore and bending moment functions and diagrams

Section 3:



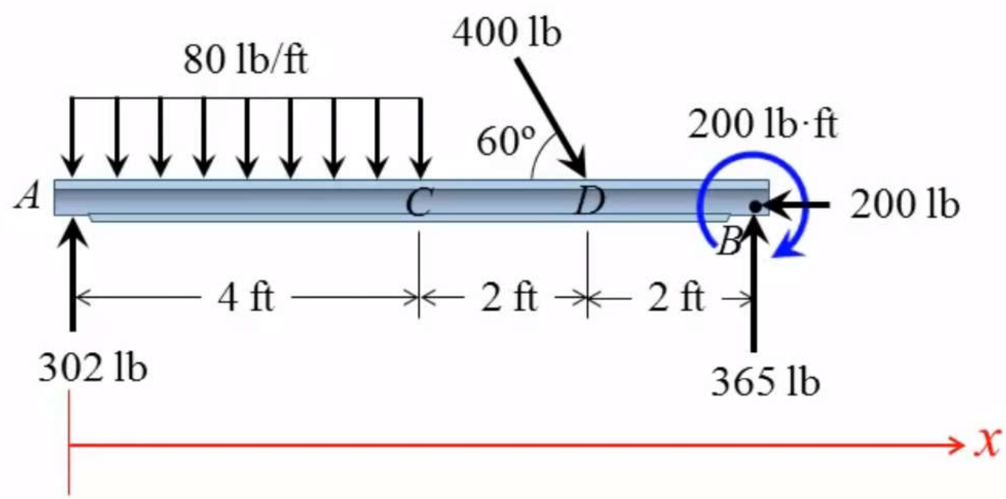
$$\sum F_y = 302 - 320 - 400\cdot\sin 60^\circ - V_x = 0$$

$$\sum M_x = -302\cdot x + 320\cdot(x-2)$$

$$400\cdot\sin 60^\circ\cdot(x-6) + M_x = 0$$

$$\therefore \begin{cases} V_x = -365 \ (\text{lb}) \\ M_B = -365x + 2718 \ (\text{lb}\cdot\text{ft}) \end{cases}$$

Internal forces: Shear fore and bending moment functions and diagrams



$$V_x \text{ (lb)} = \begin{cases} -80x + 302, & 0 < x < 4 \text{ ft} \\ -18, & 4 \text{ ft} < x < 6 \text{ ft} \\ -365, & 6 \text{ ft} < x < 8 \text{ ft} \end{cases}$$

$$M_x \text{ (lb·ft)} = \begin{cases} -40x^2 + 302x, & 0 < x < 4 \text{ ft} \\ -18x + 640, & 4 \text{ ft} < x < 6 \text{ ft} \\ -365x + 2718, & 6 \text{ ft} < x < 8 \text{ ft} \end{cases}$$

Internal forces: Shear fore and bending moment functions and diagrams

Representation 'Graph':