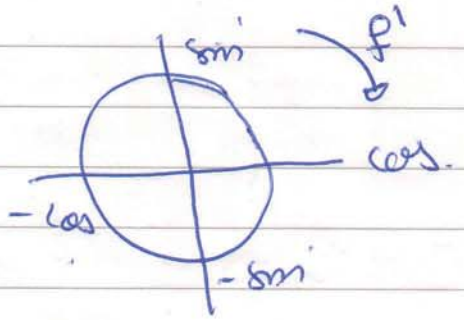
<u>EXO 1</u>



① $f(x) = \sin(x) \Rightarrow f^{(n)} = ?$

$f^{(1)} = \cos(x).$
$f^{(2)} = -\sin(x).$
$f^{(3)} = -\cos(x).$
$f^{(4)} = \sin(x)$

on constate que.

$$f^{(n)}(x) = \begin{cases} \sin(x), & \text{si } n = 4p \;/\; p \in \mathbb{N} \\ \cos(x), & \text{si } n = 4p+1 \;/\; p \in \mathbb{N} \\ -\sin(x), & \text{si } n = 4p+2 \;/\; p \in \mathbb{N} \\ -\cos(x), & \text{si } n = 4p+3 \;/\; p \in \mathbb{N}. \end{cases}$$

② $f(x) = \cos(x) \Rightarrow f^{(n)} = ?$

$$f^{(n)}(x) = \begin{cases} \cos(x); & \text{si } n = 4p \;/\; p \in \mathbb{N} \\ -\sin(x); & \text{si } n = 4p \;/\; p \in \mathbb{N} \\ -\cos(x); & \text{si } n = 4p \;/\; p \in \mathbb{N} \\ \sin(x); & \text{si } n = 4p \;/\; p \in \mathbb{N}. \end{cases}$$

③ $f(x) = \ln(x+1) \Rightarrow f^{(n)} = ?$

$f^{(1)}(x) = \frac{1}{x+1}$

$f^{(2)}(x) = \frac{-1}{(x+1)^2}.$

$f^{(3)}(x) = \frac{+2\times 1}{(x+1)^2} = \frac{2!}{(x+1)^2}$

$f^{(4)}(x) = \frac{-3\times 2\times 1}{(x+1)^4} = \frac{-3!}{(x+1)}$

$f^{(5)} = \frac{+4\times 3\times 2\times 1}{(x+1)^5} = \frac{5!}{(x+1)^5}.$

$\boxed{\cancel{f^{(n)} = \frac{(-1)^{n+1}(n-1)!}{(x+1)^n} \quad \forall n \in \mathbb{N}^*}}$

$f^{(n)} = \frac{(-1)^{n-1}(n-1)!}{(x+1)^n} \quad \forall n \in \mathbb{N}^*$

EXO2: $f : \mathbb{R} \to \mathbb{R}$

$x \longmapsto \ln\left(\frac{x+1}{1-x}\right)$

① $D_f = ?$

$D_f = \{x \in \mathbb{R} / (x+1)(1-x) > 0\}$.

$\Rightarrow D_f =]-1, 1[$.

tableau -1-

x		-1		1	
$1+x$	$-$	0	$+$		$+$
$1-x$	$+$		$+$	0	$-$
$(x+1)(1-x)$	$-$	||	$+$	||	$-$

② $f(x)$ est paire ou impaire

$\begin{cases} f(-x) = f(x) \Rightarrow f \text{ est paire ssi} \\ f(-x) = -f(x) \Rightarrow f \text{ est impaire ssi} \end{cases}$ (*)

$f(-x) = \ln\left(\frac{-x+1}{1+x}\right) = \ln\left[\left(\frac{1+x}{1-x}\right)^{-1}\right]$

$= -1 * \ln\left(\frac{1+x}{1-x}\right)$ / car $\ln(x^a) = a\ln(x)$.

$f(-x) = -f(x)$ —— (**)

de (*) et (**) $\Rightarrow f$ est ~~paire~~ impaire ssi

③ f est bijective ?

$f(D_f) = f(]-1,1[) = ?$

$f'(x) = 2 * \left(\frac{1-x}{1+x}\right) > 0 \quad \forall x \in D_f$ / voir le tableau 1.

$\Rightarrow f$ croissante.

$\Rightarrow f(]-1,1[) =]\lim\limits_{x \to -1} f(x) ; \lim\limits_{x \to 1} f(x)[$

$=]-\infty, +\infty[$

f bijective $\Leftrightarrow$ f est injective + f est surjective

① f est injective car f est strictement croissante (monotone)
② f est surjectif car f est continue sur D_f.

de ① et ② $\Rightarrow$ f est bijective.

$f^{-1} = ?$

on a $y = \ln\left(\frac{1+x}{1-x}\right) \Rightarrow e^y = \frac{1+x}{1-x}$.

$\Rightarrow e^y - xe^y - x = 1 \Rightarrow x(e^y+1) = e^y - 1$

$$\Rightarrow \boxed{x = \frac{e^y - 1}{e^y + 1}}$$

$$\Rightarrow \boxed{\begin{aligned} f^{-1} : \mathbb{R} &\longrightarrow]-1, 1[\\ x &\longmapsto y = \frac{e^x - 1}{e^x + 1} \end{aligned}}$$

EXO3. $f(x) = \arcsin\left(\frac{1+x}{1-x}\right)$

$D_f = \{x \in \mathbb{R} / -1 \leq \frac{1+x}{1-x} \leq 1 \text{ et } 1 - x \neq 0\}$

~~on a 1+~~

soit $1 - x \geq 0 \Rightarrow x \leq 1$.

$-1 \leq \frac{1+x}{1-x} \leq 1 \Rightarrow \begin{cases} \frac{1+x}{1-x} \leq 1 \\ \text{et} \\ \frac{1+x}{1-x} \geq -1 \end{cases} \Rightarrow \begin{cases} 1 + x \leq 1 - x. \\ \text{et} \\ 1 + x \geq x - 1. \end{cases} \Rightarrow \begin{cases} 2x \leq 0 \\ \text{et} \\ 2 \geq 0 \end{cases}$

$\Rightarrow x \in]-\infty, 0] \cap]-\infty, +\infty[\cap]-\infty, 1] \Rightarrow \boxed{x \in]-\infty, 0].}$ ①

soit $1-x \leq 0 \Rightarrow \boxed{x \geq 1}$

$$-1 \leq \frac{1+x}{1-x} \leq 1 \Rightarrow \begin{cases} \frac{1+x}{1-x} \leq 1 \\ \frac{1+x}{1-x} \geq -1 \end{cases} \Rightarrow \begin{cases} 1+x \geq 1-x. \\ 1+x \leq x-1. \end{cases} \Rightarrow \begin{cases} 2x \geq 0 \\ 2 \leq 0 \quad (\text{impossible}) \end{cases}$$

$$\Rightarrow x \in [0,+\infty[\cap \emptyset \cap [1,+\infty[\Rightarrow x \in \emptyset. \quad \text{--- } ②$$

Donc de ① et ② on déduit que:

$$\boxed{D_f =]-\infty, 0]}$$

② $f'(x) = ?$

on a $(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$ et $(f \circ g)' = g' * f' \circ g$.

$$f(x) = \arcsin(g(x)) \quad / \quad g(x) = \frac{x+1}{1-x}.$$

$$\Rightarrow f'(x) = \left(\frac{x+1}{1-x}\right)' \arcsin'(g(x))$$

$$= \frac{2}{(1-x)^2} \frac{1}{\sqrt{1-\left(\frac{x+1}{x-1}\right)^2}} = \frac{2}{(1-x)} * \frac{1}{\sqrt{(1-x)^2\left(1-\left(\frac{x+1}{x-1}\right)^2\right)}}.$$

$$= \frac{2}{1-x} \times \frac{1}{\sqrt{\underbrace{(1-x)^2-(x+1)^2}_{a^2-b^2=(a-b)(a+b)}}} = \boxed{\frac{1}{(1-x)\sqrt{-x}} = f'(x)}$$

<u>Exo4</u> on a : $ch(x) = \frac{e^{x} + e^{-x}}{2}$ et $sh(x) = \frac{e^{x} - e^{-x}}{2}$.

alors $3\,ch(x) - sh(x) - 3 = 0 \Longleftrightarrow 3\left(\frac{e^{x} + e^{-x}}{2}\right) - \left(\frac{e^{x} - e^{-x}}{2}\right) - 3 = 0$

$\Rightarrow \frac{2e^{x} + 4e^{-x}}{2} - 3 = 0 \Rightarrow e^{x} + \frac{2}{e^{x}} - 3 = 0$

on pose $z = e^{x}$.

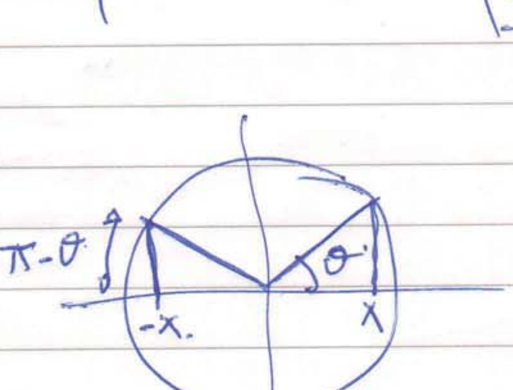
$\Rightarrow e^{2x} + 2 - 3e^{x} = 0$ — (*)

(*) $\Longleftrightarrow z^{2} - 3z + 2 = 0$

$\Delta = 1 \Rightarrow \begin{cases} z_1 = 2 \\ z_2 = 1 \end{cases} \Rightarrow \begin{cases} e^{x_1} = 2 \\ e^{x_2} = 1 \end{cases} \Rightarrow \begin{cases} x_1 = \ln(2) \\ x_2 = \ln(1) = 0. \end{cases}$

Donc les solution de l'équation soit : $\boxed{x_1 = \ln(2) \text{ et } x_2 = 0.}$

<u>Exo5</u> :



~~on a cos(~~

soit : $\begin{cases} \cos(\theta) = x. \\ \cos(\pi - \theta) = -x \end{cases}$

$\Rightarrow \begin{cases} \arccos(x) = \theta \\ \arccos(-x) = \pi - \theta \end{cases} \Rightarrow \arccos(x) + \arccos(-x) = \theta + (\pi - \theta)$

$\Rightarrow \boxed{\arccos(x) + \arccos(-x) = \pi}$ ✓