Ex n° 1 (10 pts)

$u = 4k x_1 + 6k x_3$

$v = 4k x_2$

$w = 10k x_1 - 8k x_3$

1/ $\text{grad } u = \begin{pmatrix} 4 & 0 & 6 \\ 0 & 4 & 0 \\ 10 & 0 & -8 \end{pmatrix} k$ $\quad \text{grad}^t u = \begin{pmatrix} 4 & 0 & 10 \\ 0 & 4 & 0 \\ 6 & 0 & -8 \end{pmatrix} k$ — 1,0

$\varepsilon = \frac{1}{2}(\text{grad } u + \text{grad}^t u) = \begin{pmatrix} 4 & 0 & 8 \\ 0 & 4 & 0 \\ 8 & 0 & -8 \end{pmatrix} k$ — 1,0

$\Omega = \frac{1}{2}(\text{grad } u - \text{grad}^t u) = \begin{pmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix} k$ — 1,0

2/ $\det(\varepsilon - \lambda I) = 0 \Rightarrow \begin{vmatrix} 4-\lambda & 0 & 8 \\ 0 & 4-\lambda & 0 \\ 8 & 0 & -8-\lambda \end{vmatrix} = (4-\lambda)(\lambda^2 + 4\lambda - 96) = 0$ — 1,5

$\Delta = 400$

$\varepsilon_I = 8k \qquad \varepsilon_{II} = 4k \qquad \varepsilon_{III} = -12k$ — 1,5

$(\varepsilon - \varepsilon_i I) X_i = 0 \Rightarrow X_1 = \begin{pmatrix} 2\sqrt{5}/5 \\ 0 \\ \sqrt{5}/5 \end{pmatrix} \quad X_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad X_3 = \begin{pmatrix} -\sqrt{5}/5 \\ 0 \\ 2\sqrt{5}/5 \end{pmatrix}$

3/ $n = \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix}$ — 1,0

$\varepsilon \cdot n = \begin{pmatrix} 4 & 0 & 8 \\ 0 & 4 & 0 \\ 8 & 0 & -8 \end{pmatrix} \begin{pmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{pmatrix} = \begin{pmatrix} 6\sqrt{2} \\ 0 \\ 0 \end{pmatrix}$ — 1,0

$\varepsilon = n^t \varepsilon \cdot n = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} 6\sqrt{2} \\ 0 \\ 0 \end{pmatrix} = 6 \qquad \varepsilon = 6k$ — 1,0

$g = \sqrt{|\varepsilon \cdot n|^2 - \varepsilon^2} = \sqrt{(6\sqrt{2})^2 - 6^2} = 6 \qquad g = 6k$ — 1,0

Ex n°2 (10 pts)

$$\Sigma = \begin{pmatrix} 25 & 0 & 0 \\ 0 & -10 & 11 \\ 0 & 11 & 75 \end{pmatrix} \text{ daN/mm}^2$$

1/ $\det(\Sigma - \lambda I) = 0 \Rightarrow \begin{vmatrix} 25-\lambda & 0 & 0 \\ 0 & -10-\lambda & 11 \\ 0 & 11 & 75-11 \end{vmatrix} = (25-\lambda)(\lambda^2 - 65\lambda - 871) = 0$

$\Delta = 7709$

$\sqrt{\Delta} = 87,8$

1,5 $\sigma_I = 76,4$ daN/mm² $\quad \sigma_{II} = 25,0$ daN/mm² $\quad \sigma_{III} = -11,4$ daN/mm²

$(\Sigma - \sigma_i I) X_i = 0 \Rightarrow X_1 = \begin{pmatrix} 0 \\ 0,126 \\ 0,992 \end{pmatrix} \quad X_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad X_3 = \begin{pmatrix} 0 \\ -0,992 \\ 0,126 \end{pmatrix}$

1,5

2/ 0,5 $\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} \qquad \sigma_{kk} = 90$

0,5 $\varepsilon_{11} = \frac{1+0,3}{20000}(25) - \frac{0,3}{20000}90 = 0,275.10^{-3}$

0,5 $\varepsilon_{22} = \frac{1+0,3}{20000}(-10) - \frac{0,3}{20000}90 = -2,000.10^{-3}$

0,5 $\varepsilon_{33} = \frac{1+0,3}{20000}(75) - \frac{0,3.90}{20000} = 3,525.10^{-3}$

0,5 $\varepsilon_{23} = \frac{1+0,3}{20000}(11) - \frac{0,3.90}{20000} = 0,715.10^{-3}$

0,5 $\varepsilon = \begin{pmatrix} 0,275 & 0 & 0 \\ 0 & -2,000 & 0,715 \\ 0 & 0,715 & 3,525 \end{pmatrix} 10^{-3}$ dans $(e_1\ e_2\ e_3)$

0,5 $\varepsilon = \begin{pmatrix} 3,616 & 0 & 0 \\ 0 & 0,275 & 0 \\ 0 & 0 & -2,091 \end{pmatrix} 10^{-3}$ dans $(X_1\ X_2\ X_3)$

3/

1,0 Rankine : $\sigma_R = \max(|\sigma_I|, |\sigma_{II}|, |\sigma_{III}|) = 76,4$ daN/mm²

1,0 Tresca : $\tau_I = (\sigma_I - \sigma_{III}) = 76,4 + 11,4 = 87,8$ daN/mm²

1,5 Von Mises : $\sigma_{VM} = \sqrt{\frac{1}{2}\left[(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_I - \sigma_{III})^2\right]}$

$= \sqrt{\frac{1}{2}(76,4-25)^2 + (25+11,4)^2 + (76,4+11,4)^2} = 76,406$ daN/mm²