Examen - MMC 2021/22

Ex1 (10 pts)

$u = 9kx_1 + 8kx_2$
$v = 16kx_1 - 9kx_2$
$w = 5kx_3$

1°) 0,5 $\text{grad}\,u = \begin{pmatrix} 9 & 8 & 0 \\ 16 & -9 & 0 \\ 0 & 0 & k \end{pmatrix}$ $\quad {}^t\text{grad}\,u = \begin{pmatrix} 9 & 16 & 0 \\ 8 & -9 & 0 \\ 0 & 0 & 5 \end{pmatrix} k$

$\varepsilon = \frac{1}{2}(\text{grad}\,u + {}^t\text{grad}\,u) = \begin{pmatrix} 9 & 12 & 0 \\ 12 & -9 & 0 \\ 0 & 0 & 5 \end{pmatrix} k$

0,5 $\Omega = \frac{1}{2}(\text{grad}\,u - {}^t\text{grad}\,u) = \begin{pmatrix} 0 & -4 & 0 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} k$

2°) $\det(\varepsilon - \lambda I) = 0 \Rightarrow \begin{vmatrix} 9-\lambda & 12 & 0 \\ 12 & -9-\lambda & 0 \\ 0 & 0 & 5-\lambda \end{vmatrix} k = (5-\lambda)(\lambda^2 - 225) = 0$

1,0 $= (5-\lambda)(\lambda - 15)(\lambda + 15) = 0$

1,5 $\varepsilon_I = 15k \qquad \varepsilon_{II} = 5k \qquad \varepsilon_{III} = -15k$

$(\cancel{\varepsilon}_{I} - \varepsilon_i I) X_i = 0 \qquad X_I = \begin{pmatrix} 2\sqrt{5}/5 \\ \sqrt{5}/5 \\ 0 \end{pmatrix} \qquad X_{II} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad X_3 = \begin{pmatrix} \sqrt{5}/5 \\ -2\sqrt{5}/5 \\ 0 \end{pmatrix}$

3°)
1,0 a) La sphère devient un ellipsoïde

b) Longueurs caractéristiques

0,5 $l = (1 + \varepsilon_i)\, l_0$

1,0 $l_1 = (1 + \varepsilon_I)\, l_0 = (1 + 15.10^{-3})\, 1000 = 1{,}015.10^3 = 1015$ mm

1,0 $l_2 = (1 + \varepsilon_{II})\, l_0 = (1 + 5.10^{-3})\, 1000 = 1{,}005.10^3 = 1005$ mm

1,0 $l_3 = (1 + \varepsilon_{III})\, l_0 = (1 - 15.10^{-3}).1000 = 0{,}985.10^3 = 985$ mm.

c) $\frac{\Delta V}{V} = tr(\varepsilon) \Longrightarrow \frac{V - V_0}{V_0} = tr(\varepsilon)$.

0,5 $V = [1 + tr(\varepsilon)]\, V_0 = [1 + tr(\varepsilon)]\, \frac{4}{3}\pi R^3$

0,5 $V_0 = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (500)^3 = 523{,}6 \,.10^6$ mm^3

1,0 $V = (1 + 5.10^{-3})\, \frac{4}{3}\pi (500)^3 = 526{,}22 \,.10^6$ mm^3

Ex 2 : (10 pts)

$$\Sigma = \begin{pmatrix} 25 & 0 & 0 \\ 0 & -10 & 16 \\ 0 & 16 & 75 \end{pmatrix} daN/mm^2$$

1/ $\det(\Sigma - \lambda I) = 0 \Rightarrow \begin{vmatrix} 25-\lambda & 0 & 0 \\ 0 & -10-\lambda & 16 \\ 0 & 16 & 75-\lambda \end{vmatrix} = (25-\lambda)(\lambda^2 - 65\lambda - 1006) = 0$

0,5

$\Delta = 8249$ $\sqrt{\Delta} = 94,82$

$\sigma_I = 77,91 \ daN/mm^2$ $\quad \sigma_{II} = 25,00 \ daN/mm^2$ $\quad \sigma_{III} = -12,91 \ daN/mm^2$

1,5

$\det(\Sigma - \lambda I)X_i = 0 \Rightarrow \quad X_1 = \begin{pmatrix} 0 \\ 0,179 \\ 0,984 \end{pmatrix} \quad X_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad X_3 = \begin{pmatrix} 0 \\ -0,984 \\ 0,179 \end{pmatrix}$

1,5

2/ $C_{ij} = \left(\frac{\sigma_i + \sigma_j}{2}, 0\right), \ R_{ij} = \frac{\sigma_i - \sigma_j}{2}$

$C_{12} = (51,45 \ ; 0) \quad R_{12} = 26,45$

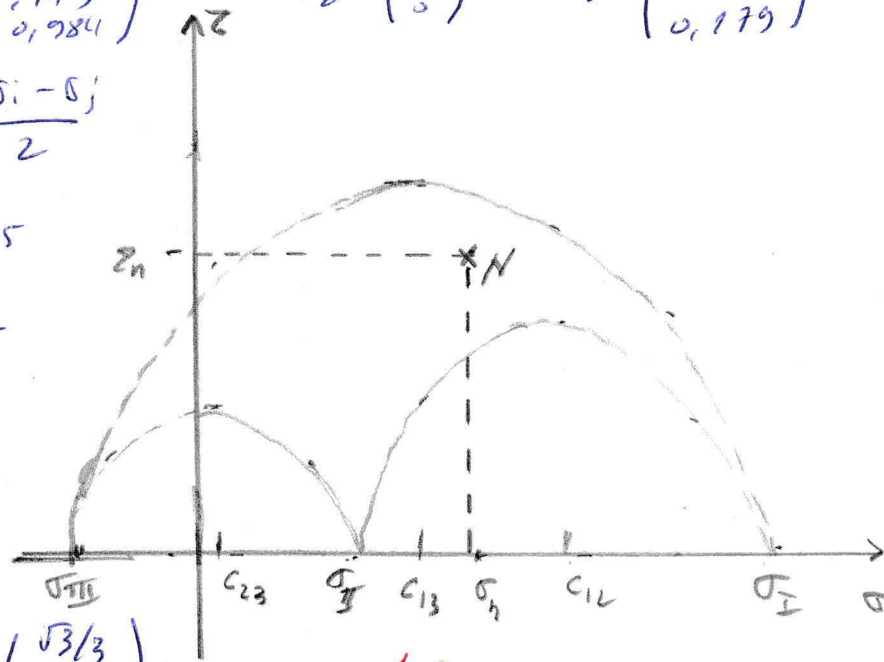
0,5

$C_{23} = (6,05 \ ; 0) \quad R_{23} = 18,95$

0,5

$C_{13} = (32,5 \ ; 0) \quad R_{13} = 45,4$

0,5



1,0

3/ $n_1 = n_2 = n_3 \Rightarrow \quad n = \begin{pmatrix} \sqrt{3}/3 \\ \sqrt{3}/3 \\ \sqrt{3}/3 \end{pmatrix}$

1,0

$\sigma = n^t \ \Sigma . n$

$\sigma_n = \begin{pmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{pmatrix} \begin{pmatrix} 25 & 0 & 0 \\ 0 & -10 & 16 \\ 0 & 16 & 75 \end{pmatrix} \begin{pmatrix} \sqrt{3}/3 \\ \sqrt{3}/3 \\ \sqrt{3}/3 \end{pmatrix} = \frac{\sqrt{3}}{3} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 25 \\ 6 \\ 91 \end{pmatrix} \frac{\sqrt{3}}{3}$

0,5

$= \frac{1}{3}(25 + 6 + 91) = \frac{122}{3} = 40,67 \ daN/mm^2$

1,0

$\tau_n = \sqrt{|\Sigma . n|^2 - \sigma^2}$

$= \sqrt{\left(\frac{25\sqrt{3}}{3}\right)^2 + \left(\frac{6\sqrt{3}}{3}\right)^2 + \left(\frac{91\sqrt{3}}{3}\right)^2 - 40,67^2}$

1,0

$= \sqrt{\frac{1}{3}(25^2 + 6^2 + 91^2) - 40,67^2} = 36,42 \ daN/mm^2$

0,5 point representatif : $N(\sigma_n, \tau_n)$