Dynamics

Prerequisites: Statics

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The study of how ***rigid*** bodies react to forces acting on them.

Statics: The study of bodies in equilibrium.

Newton's first law

$$F_R = 0$$

Dynamics: The study of motion.

1. **Kinematics** – concerned with the geometric aspects of motion, *s*, *v*, *a* and *t*.

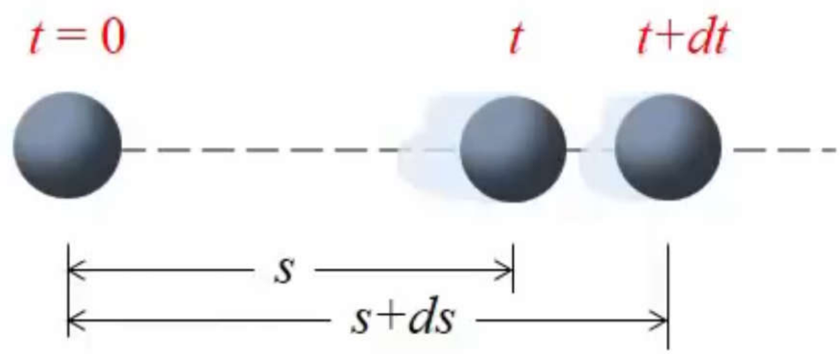
S **displacement**

V **velocity**

a **acceleration**

t **time**

Kinematics: concerned with the geometric aspects of motion ,*s*,*v* ,*a* and *t*



$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} = \frac{d^2s}{dt}$$

The study of how ***rigid*** bodies react to forces acting on them.

Statics: The study of bodies in equilibrium.

Dynamics: The study of motion.

1. **Kinematics** – concerned with the geometric aspects of motion, s, v, a and t.
2. **Kinetics** - concerned with how forces causing the motion.

Newton's second law $$\boldsymbol{F_R = ma}$$

Course structure:

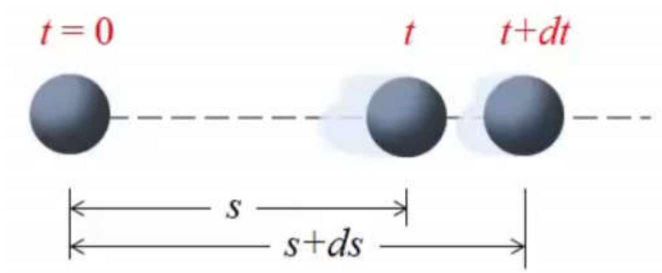
1. **Particle kinematics**
2. ***Particle kinetics:***
   a- force and acceleration
   b-work and energy
   c-impulse and momentum
3. **Rigid body planar (2D) kinematics**
4. **Rigid body planar (2D) kinetics*:***
   a- force and acceleration
   b-work and energy
   c-impulse and momentum

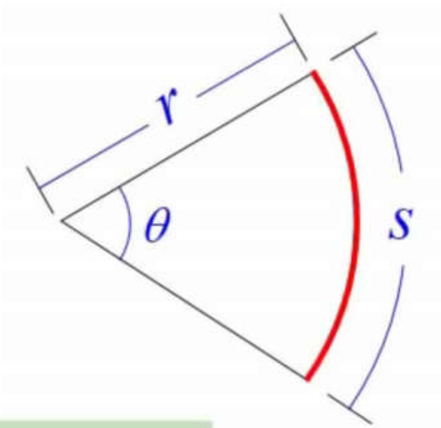
$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$F = ma$$

$$s = r\theta$$





Note: only scalar equations are shown from here on for simplification purpose.

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad dt = \frac{ds}{v} = \frac{dv}{a} \quad \Rightarrow \quad ads = vdv$$

$$F = ma$$

$$s = r\theta$$

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$ads = vdv \longrightarrow Fds = mads = mvdv$$

$$F = ma$$

$$s = r\theta$$

$$\int_{s_1}^{s_2} Fds = m \int_{s_1}^{s_2} vdv$$

$$\int_{s_1}^{s_2} Fds = \frac{1}{2}mv_2^2 \quad \frac{1}{2}mv_1^2$$

Principle of work and energy

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$F = ma$$

$$F = m\frac{dv}{dt}$$

$$\int_{t_1}^{t_2} F dt = m \int_{t_1}^{t_2} dv$$

$$s = r\theta$$

$$\int_{t_1}^{t_2} \boldsymbol{F dt = mv_2 \quad mv_1}$$

Principle of linear impulse and momentum

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$F = ma$$

$$s = r\theta$$

$$\omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d\omega}{dt} = \frac{a_t}{r}$$

$$\boldsymbol{\alpha d\theta = \omega d\omega}$$

$$\boldsymbol{M_G = I_G\alpha}$$

$$\boldsymbol{U_M = \int_{\theta_1}^{\theta_2} M d\theta}$$ **Work of moment**

$$\boldsymbol{I_G\omega_1 + I_G\omega_1 \sum_{1}^{n} \int_{t_1}^{t_2} M_G\, dt = I_G\omega_2}$$ **Angular impulse and momentum**

Particle kinematic

Rectilinear, continuous motion

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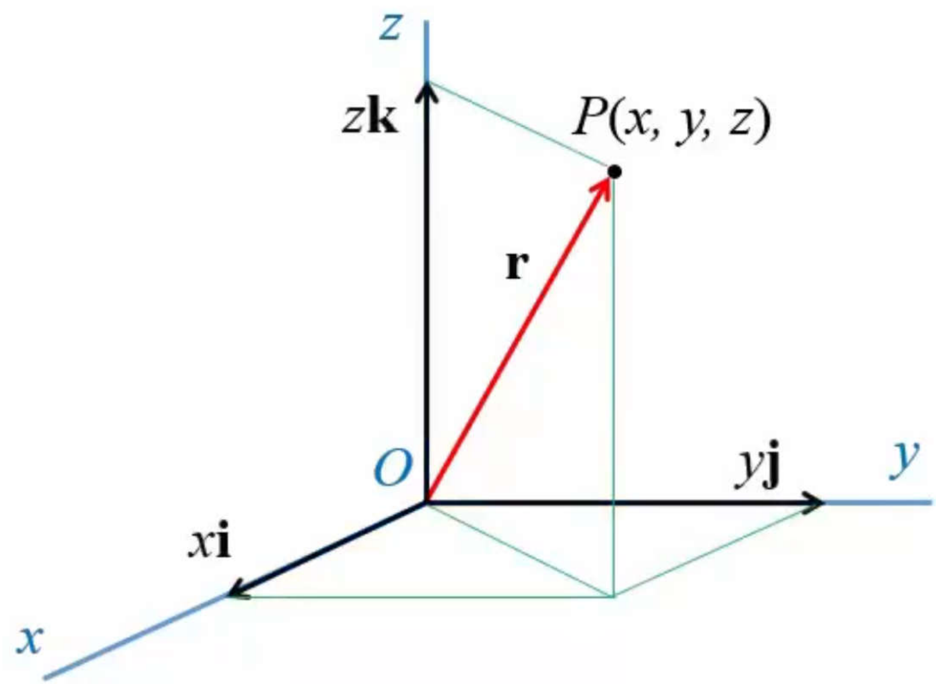
- To revisit position vector as learned in Statics.
- To define the important vector quantities: displacement, velocity and acceleration.
- To distinguish between displacement and distance travelled, velocity and speed, instantaneous velocity and average velocity.
- To study particle motion along a straight line and the representation of motion using position-time graph and velocity-time graph.

We have seen:

Position vector:

$$r = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

$\boldsymbol{r}(\boldsymbol{t})$ is a function of time



$$r = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

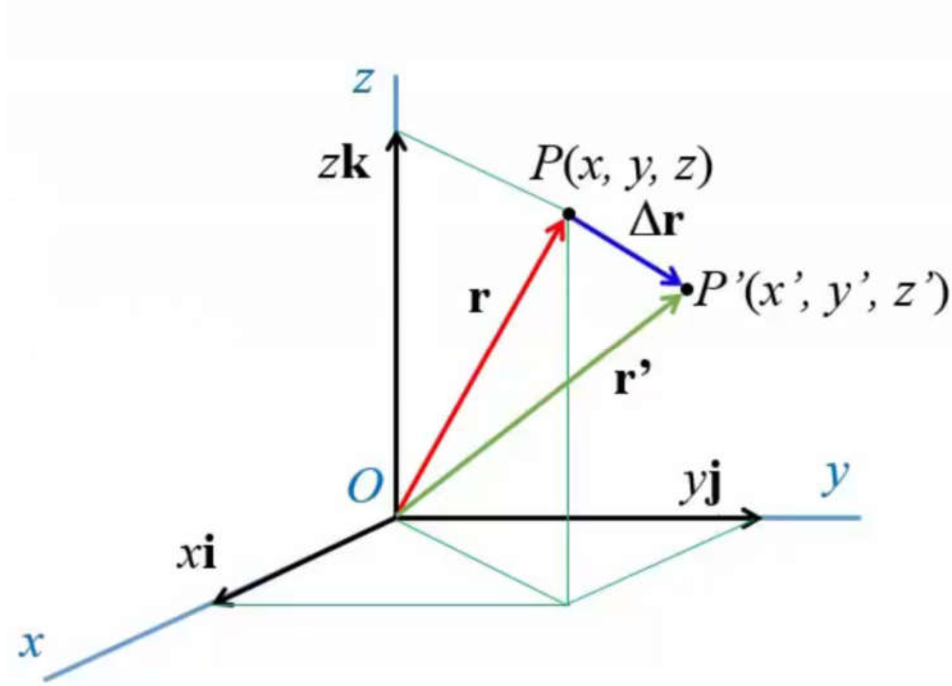
$$r' = x'\boldsymbol{i} + y'\boldsymbol{j} + z'\boldsymbol{k}$$

$$\Delta \boldsymbol{r} = \boldsymbol{r}' \quad \boldsymbol{r}$$

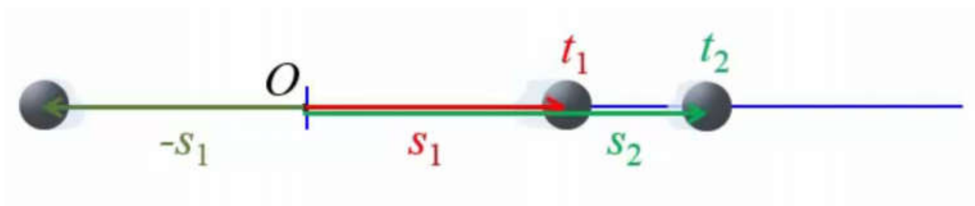
$\Delta \boldsymbol{r}$ Displacement

$$\boldsymbol{v} = \frac{\boldsymbol{dr}}{\boldsymbol{dt}}$$ velocity

$$\boldsymbol{a} = \frac{\boldsymbol{dv}}{\boldsymbol{dt}}$$ acceleration



Rectilinear, Continuous motion



$s(t)$ is a function of time

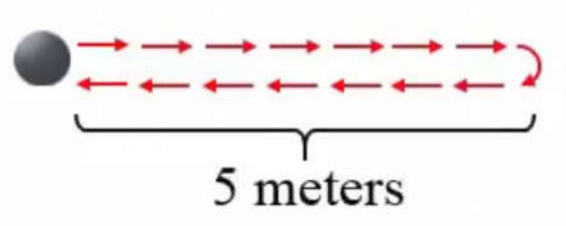
Continuous, means that $s(t)$ consists only of one equation.

Average velocity: $$v_{avg} = \frac{\triangle s}{\triangle t} = \frac{s_2 \quad s_1}{t_2 \quad t_1}$$

Instantaneous velocity: $$v = \frac{ds}{dt}$$

Rectilinear, Continuous motion

Distance travelled: S_T



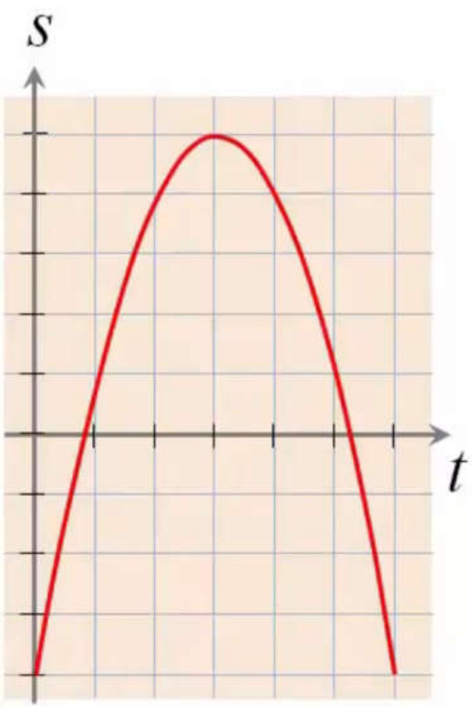
$$\triangle s = 0$$

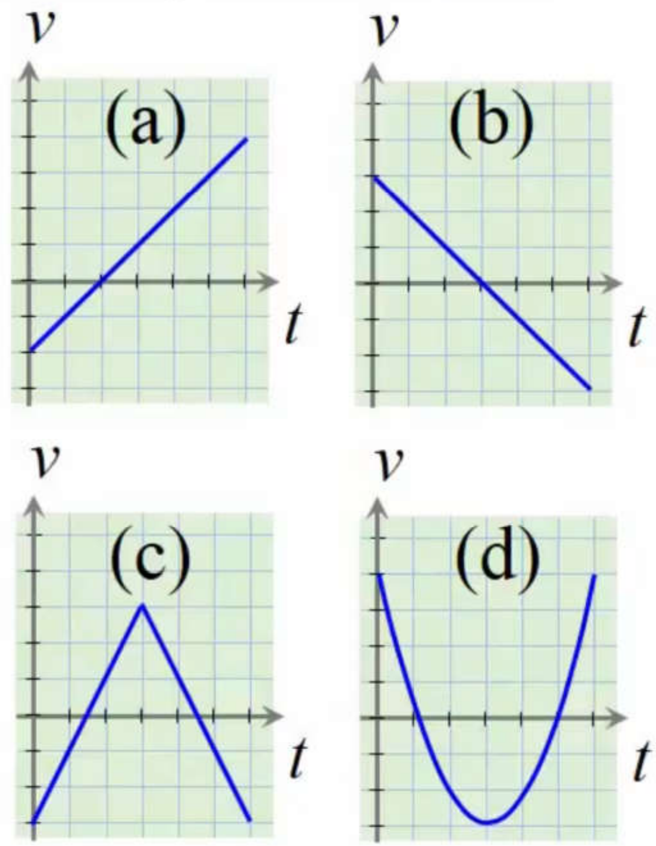
$$v_{avg} = \frac{\triangle s}{st} = 0$$

$$S_T = 10m$$

$$(v_{sp})_{avg} = \frac{S_T}{\triangle t} = \frac{10m}{5s} = 2\, m/s$$

Question 3: For the given position-time function, which graph could represent its velocity-time function?





Particle kinematic

Instantaneous acceleration, average acceleration and kinematic equations

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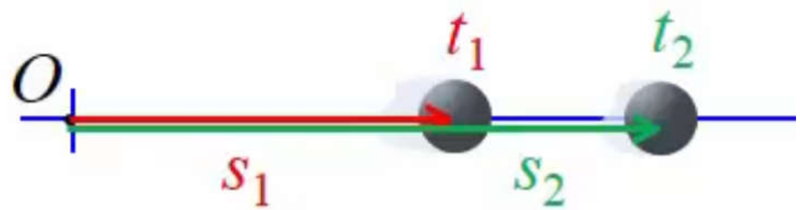
Rectilinear, Continuous motion



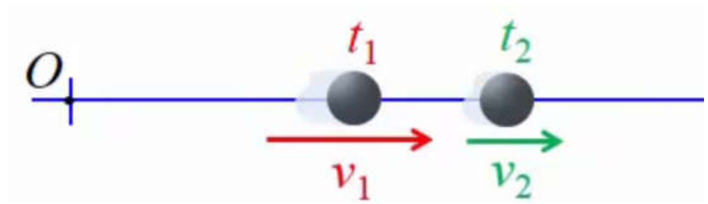
Average velocity

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{s_2 \quad s_1}{t_2 \quad t_1}$$

$\Delta t \rightarrow 0$ **Instantaneous velocity**

$$v = \frac{ds}{dt}$$

Rectilinear, Continuous motion



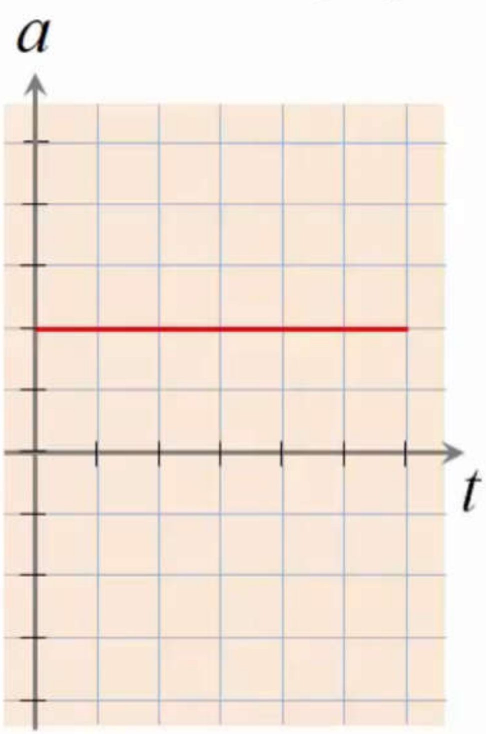
Average acceleration

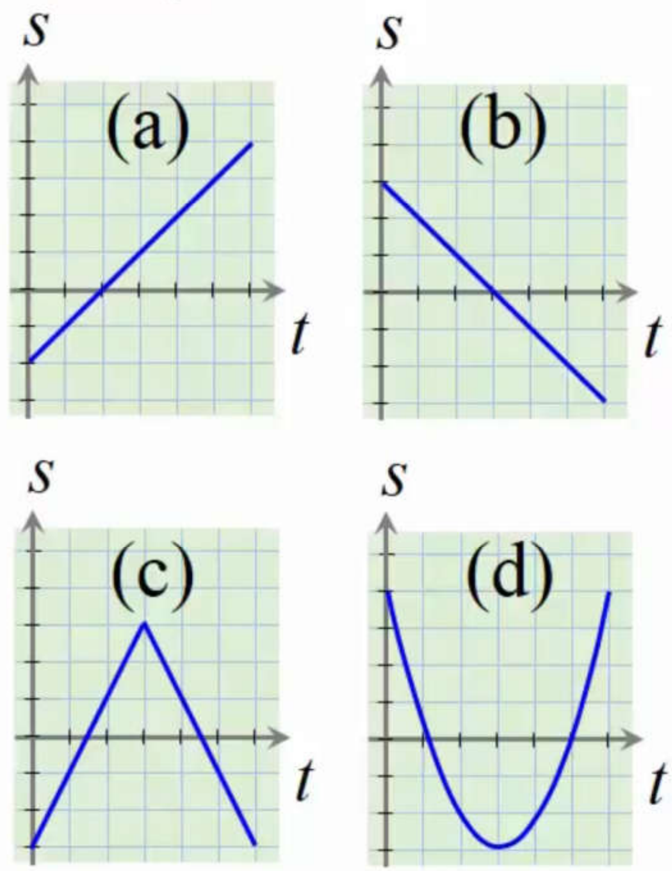
$$a_{avg} = \frac{\Delta\, v}{\Delta\, t} = \frac{v_2 \quad\quad v_1}{t_2 \quad\quad t_1}$$

$\Delta\, t \rightarrow 0$ **Instantaneous acceleration**

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt}$$

Question 1: For the given acceleration-time function, which graph could represent its position-time function?





Rectilinear, Continuous motion

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad dt = \frac{ds}{v} = \frac{dv}{a} \quad \Rightarrow \quad ads = vdv$$

The Three kinematic equations

Example 1: The velocity of a particle moving along a straight line is $v = (t^2+2t)$ m/s, where t is in seconds. If its position $s = 0$ when $t = 0$, determine its acceleration and position when $t = 4$ s.

$$v = \frac{ds}{dt} \quad a = \frac{dv}{dt}$$

$$ads = vdv$$

$$v = \frac{ds}{dt} \Rightarrow (t^2 + 2)dt = ds$$

$$\Rightarrow \int_{t=0}^{t} (t^2 + 2)dt = \int_{s=0}^{s} ds$$

$$\Rightarrow \frac{1}{3}t^3 + 2t = s = s(t)$$

$$s(4) = 37.3m$$

Example 2: The acceleration of a particle moving along a straight line is $a = \sqrt{s}\ \text{m/s}^2$, where s is in meters. If its position $s = 0$ and velocity $v = 0$ when $t = 0$, determine its **velocity** when $s = 16$ m. What time is that?

$$v = \frac{ds}{dt} \quad a = \frac{dv}{dt}$$

$$ads = vdv$$

$$ads = vdv \Rightarrow \sqrt{s}ds = vdv$$

$$\Rightarrow \int_{s=0}^{s} \sqrt{s}ds = \int_{v=0}^{v} vdv \Rightarrow \frac{2}{3}s^{3/2} = \frac{1}{2}v^2$$

$$\Rightarrow v = \frac{2\sqrt{3}}{3}s^{3/4}$$

$$\boldsymbol{v(16) = 9.24\, m/s}$$

Particle kinematic

Rectilinear motion with constant acceleration

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Objectives:

- To derive the three equations for rectilinear motion with constant acceleration from the three basic kinematic equations.
- To explain the problem-solving strategy for problems involving rectilinear motion with constant acceleration.

The three basic kinetic equations:

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$ads = vdv$$

Constant acceleration:

$$a = a_c, v_0, s_0$$

$$a = \frac{dv}{dt} \rightarrow adt = dv \rightarrow a_c dt = dv$$

$$\rightarrow \int_0^t a_c dt = \int_{v_0}^{v} dv$$

$$\rightarrow a_c t = v \quad v_0$$

$$v = v_0 + a_c t$$

The three basic kinetic equations:

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$ads = vdv$$

Constant acceleration:

$$a = a_c, v_0, s_0$$

$$v = \frac{ds}{dt} \rightarrow vdt = ds \rightarrow (v_0 + a_c)dt = ds$$

$$\rightarrow \int_0^t (v_0 + a_c t)dt = \int_{s_0}^{s} ds$$

$$\rightarrow v_0 t + \frac{1}{2} a_c t^2 = s \quad s_0$$

$$\rightarrow s = \frac{1}{2} a_c t^2 + v_0 t + s_0$$

The three basic kinetic equations:

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$ads = vdv$$

Constant acceleration:

$$a = a_c, v_0, s_0$$

$$ads = vdv \rightarrow a_c ds = vdv$$

$$\rightarrow \int_{s_0}^{s} a_c ds = \int_{v_0}^{v} vdv$$

$$\rightarrow a_c(s \quad s_0) = \frac{1}{2}\left(v^2 \quad v_0^2\right)$$

$$\rightarrow v^2 = v_0^2 + 2a_c(s \quad s_0)$$

Particle kinematic

Rectilinear, erratic motion with motion graph

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The three basic kinetic equations:

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$ads = vdv$$

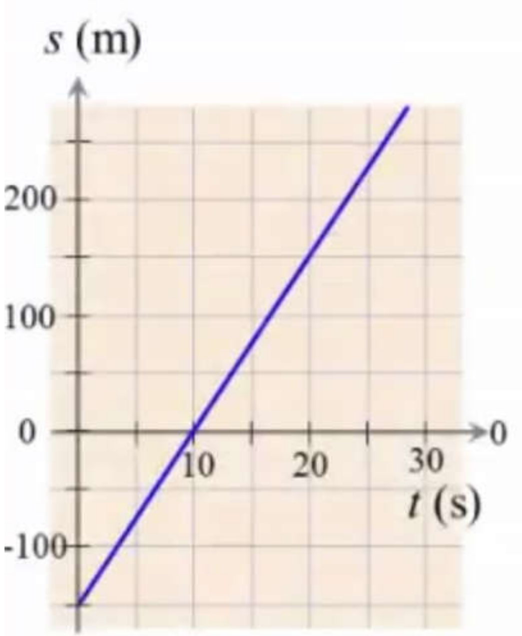
$$s = f(t)$$

$$v = f'(t)$$

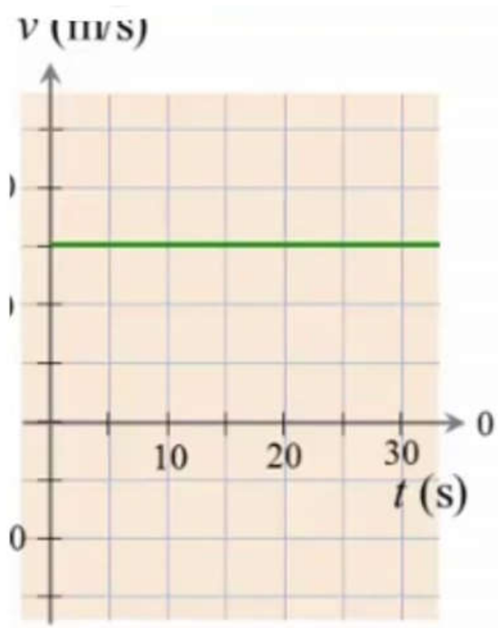
$$a = f''(t)$$

Continuous motion: s consists of only one equation

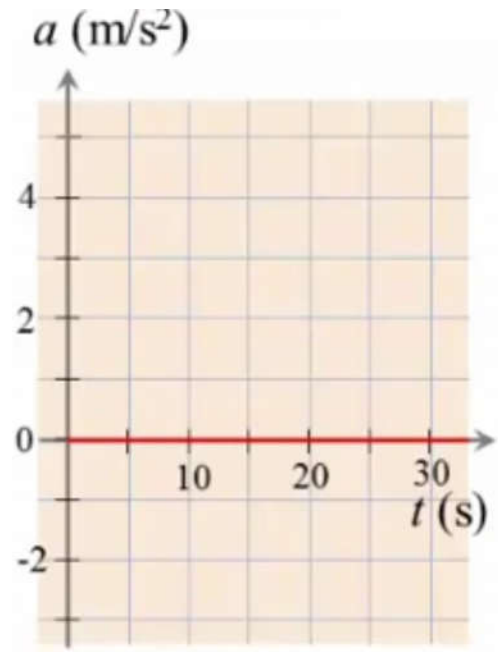
$s = 15t \quad 150(m)$ $\qquad v = \dfrac{ds}{dt} = 15(m/s)$ $\qquad a = \dfrac{dv}{dt} = 0$



s-t graph



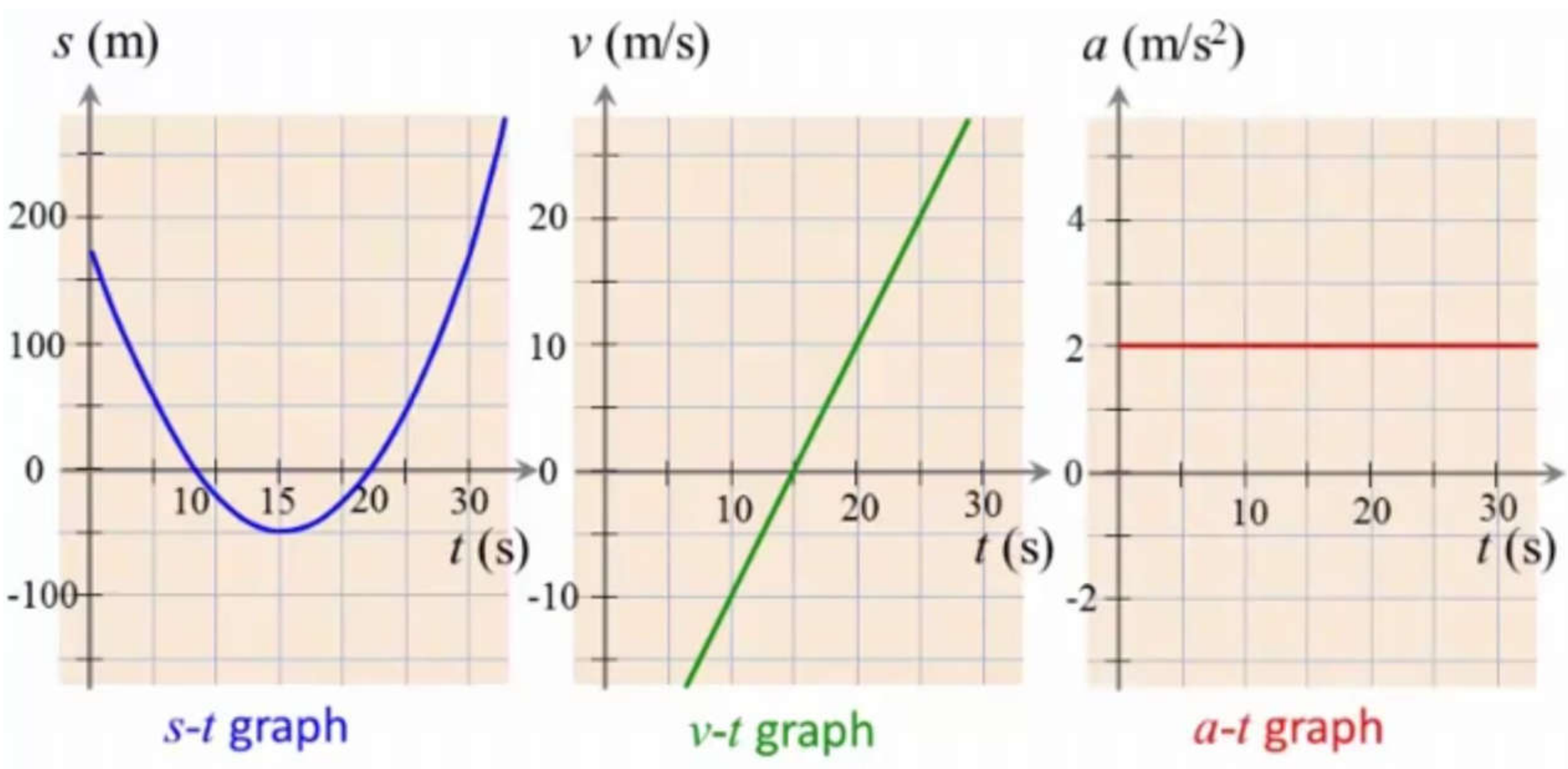
v-t graph



a-t graph

Continuous motion: s consists of only one equation

$s = t^2 \quad 30t + 175(m)$ $\qquad$ $v = 2t \quad 30(m/s)$ $\qquad$ $a = 2(m/s^2)$



Erratic motion: non-continuous motion

$$s = \begin{cases} \text{Equation ①} \\ \text{Equation ②} \\ \text{Equation ③} \\ \cdots \end{cases} \quad v = \begin{cases} \text{Equation ①} \\ \text{Equation ②} \\ \text{Equation ③} \\ \cdots \end{cases} \quad a = \begin{cases} \text{Equation ①} \\ \text{Equation ②} \\ \text{Equation ③} \\ \cdots \end{cases}$$

Attention: Watch out for the beginning and the end of each time period

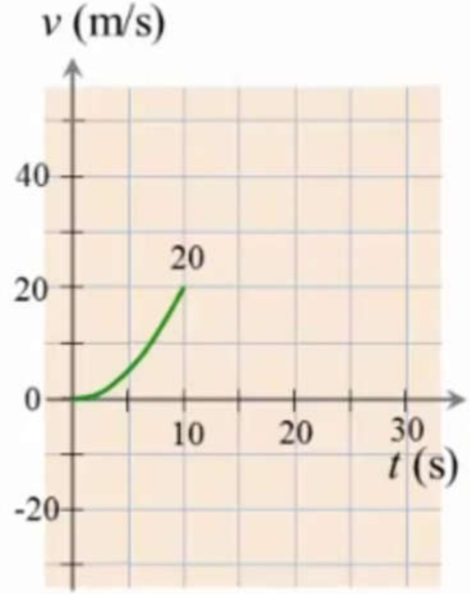
Example 1: An object travels along a straight path. $s_0 = 0$ and $v_0 = 0$. Its acceleration (in m/s^2) as a function of time is given. Construct its *s-t*, *v-t* and *a-t* motion graphs.

$$a = \begin{cases} 0.4t & 0 \le t < 10\text{ s} \quad ① \\ 2.4 & 10 \le t < 20\text{ s} \quad ② \\ 0 & 20 \le t < 30\text{ s} \quad ③ \end{cases}$$

① $0 \le t < 10\text{ s}:$

$$adt = \int_0^t 0.4t\,dt = \int_0^v dv$$

$$\rightarrow v = 0.2t^2$$



v-t graph

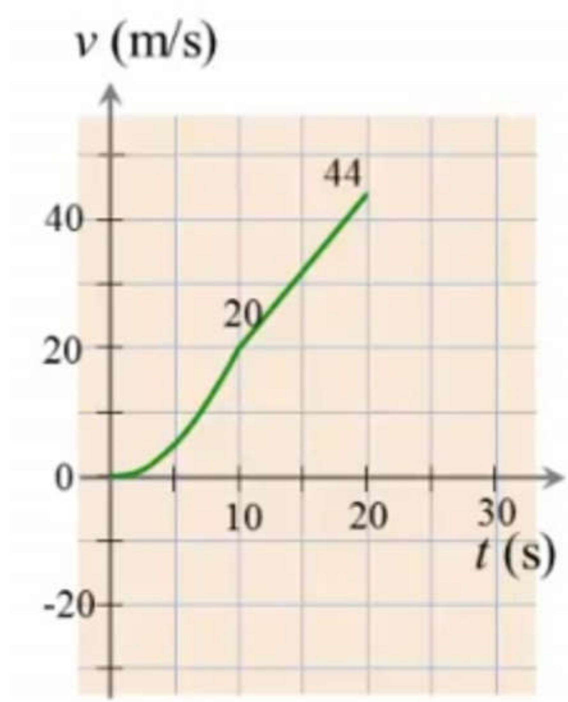
$$a = \begin{cases} 0.4t & 0 \le t < 10\text{ s} \quad ① \\ 2.4 & 10 \le t < 20\text{ s} \quad ② \\ 0 & 20 \le t < 30\text{ s} \quad ③ \end{cases}$$

② $10 \le t < 20\text{ s}:$

$$adt = \int_{10}^{t} 2.4dt = \int_{20}^{v} dv$$

$$\rightarrow v \quad 20 = 2.4t \quad 24$$

$$\rightarrow v = 2.4t \quad 4$$



v-t graph

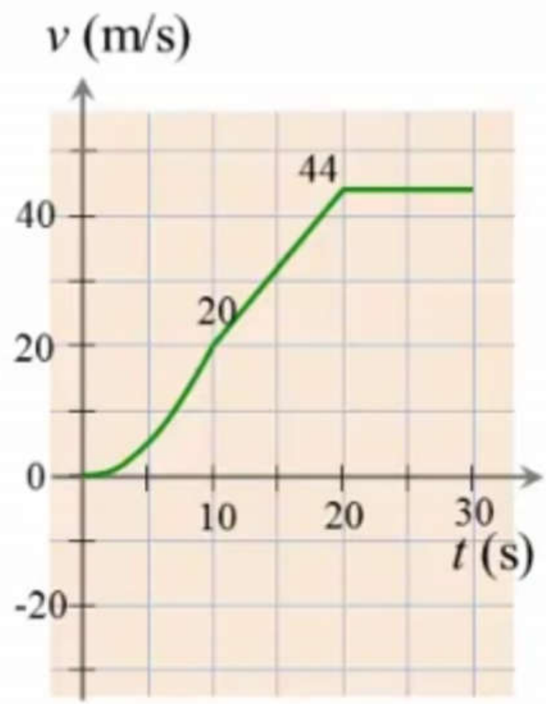
$$a = \begin{cases} 0.4t & 0 \le t < 10\text{ s} & ① \\ 2.4 & 10 \le t < 20\text{ s} & ② \\ 0 & 20 \le t < 30\text{ s} & ③ \end{cases}$$

③ $20 \le t < 30\text{ s}:$

$$adt = \int_{20}^{t} 0dt = \int_{44}^{v} dv$$

$$\rightarrow v \quad 44 = 0$$

$$\rightarrow v = 44$$



v-t graph

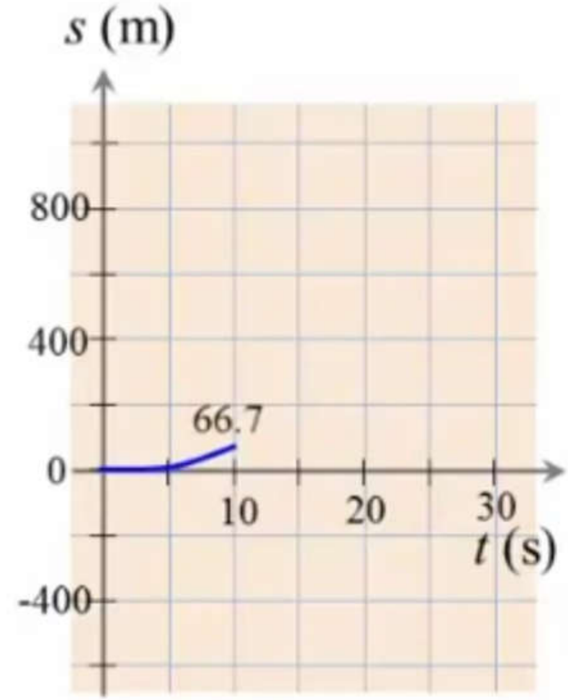
$$v = \begin{cases} 0.2t^2 & 0 \le t < 10\text{ s} \quad ① \\ 2.4t - 4 & 10 \le t < 20\text{ s} \quad ② \\ 44 & 20 \le t < 30\text{ s} \quad ③ \end{cases}$$

$$v = \frac{ds}{dt}$$

① $0 \le t < 10\text{ s}$:

$$vdt = ds \Rightarrow \int_0^t 0.2t^2 dt = \int_0^s ds$$

$$\Rightarrow s = 0.067t^3$$



s-t graph

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$$v = \begin{cases} 0.2t^2 & 0 \le t < 10\text{ s} \quad ① \\ 2.4t - 4 & 10 \le t < 20\text{ s} \quad ② \\ 48 & 20 \le t < 30\text{ s} \quad ③ \end{cases}$$

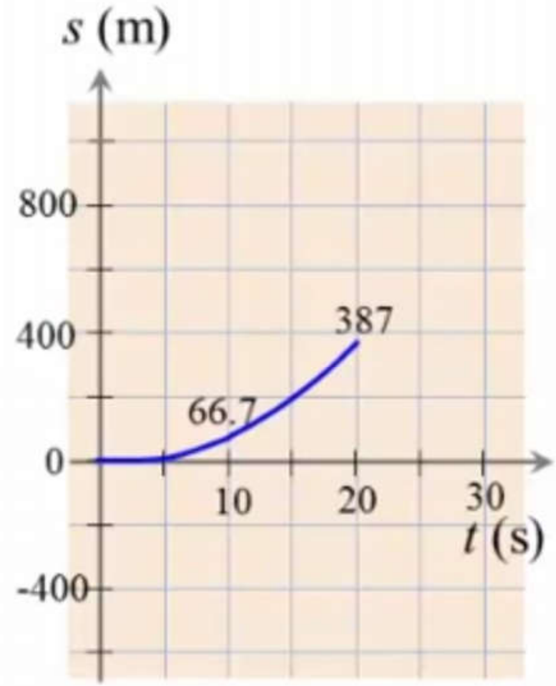
$$v = \frac{ds}{dt}$$

② $10 \le t < 20\text{ s}:$

$$vdt = ds \Rightarrow \int_{10}^{t} (2.4t - 4)dt = \int_{66.7}^{s} ds$$

$$\Rightarrow \left(1.2t^2 - 4t\right)\Big|_{10}^{t} = s - 66.7$$

$$\Rightarrow s = 1.2t^2 - 4t - 13.3$$



s-t graph

$$v = \begin{cases} 0.2t^2 & 0 \le t < 10\text{ s} \quad ① \\ 2.4t - 4 & 10 \le t < 20\text{ s} \quad ② \\ 44 & 20 \le t < 30\text{ s} \quad ③ \end{cases}$$

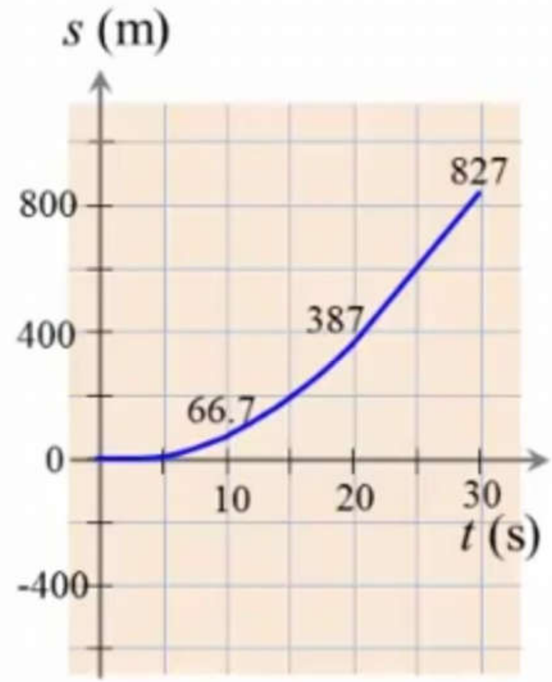
$$v = \frac{ds}{dt}$$

③ $20 \le t < 30\text{ s}$:

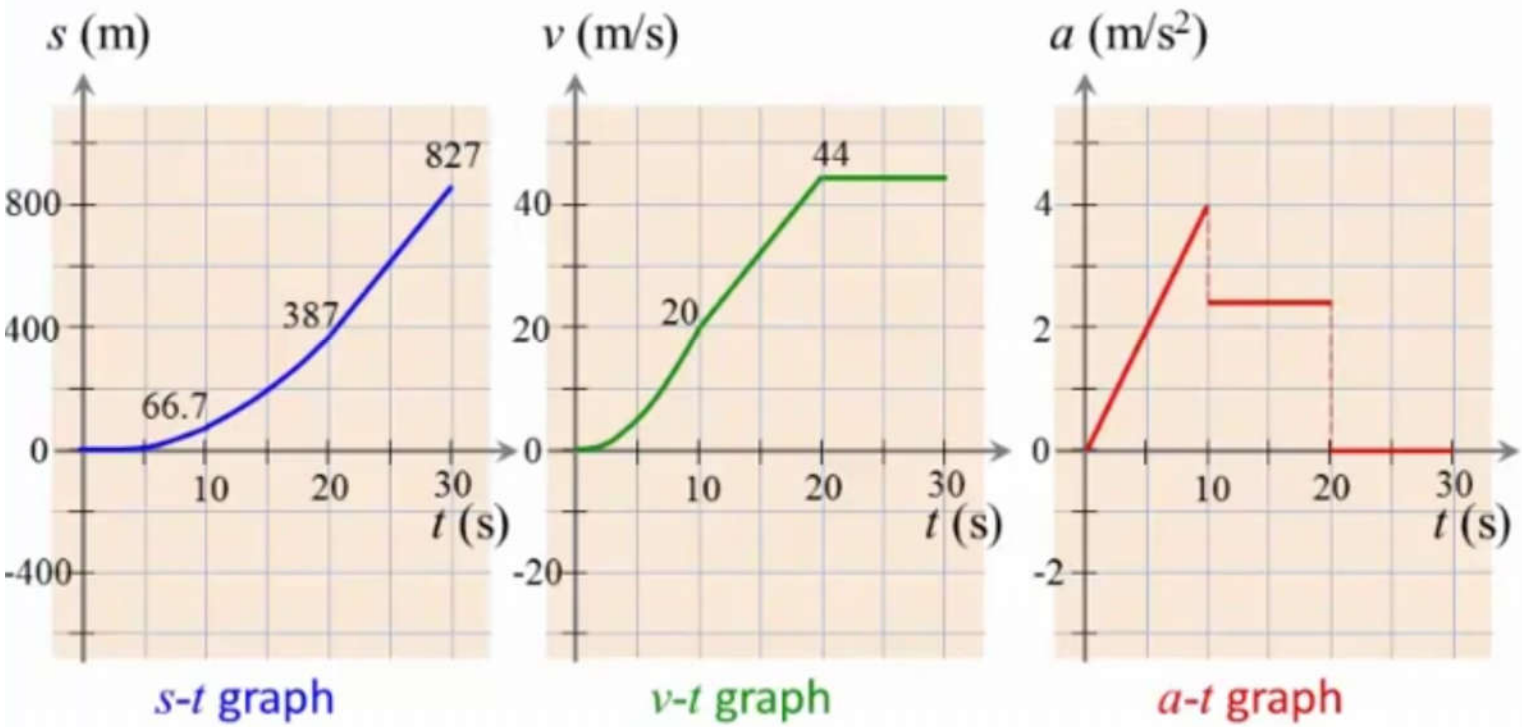
$$vdt = ds \Rightarrow \int_{20}^{t} 44dt = \int_{387}^{s} ds$$

$$\Rightarrow 44(t-20) = s - 387$$

$$\Rightarrow s = 44t - 493$$



s-t graph


s (m)
800
400
0
-400
66.7
387
827
10
20
30
t (s)
s-t graph
v (m/s)
40
20
0
-20
20
44
10
20
30
t (s)
v-t graph
a (m/s²)
4
2
0
-2
10
20
30
t (s)
a-t graph

Particle kinematic

Curvilinear motion

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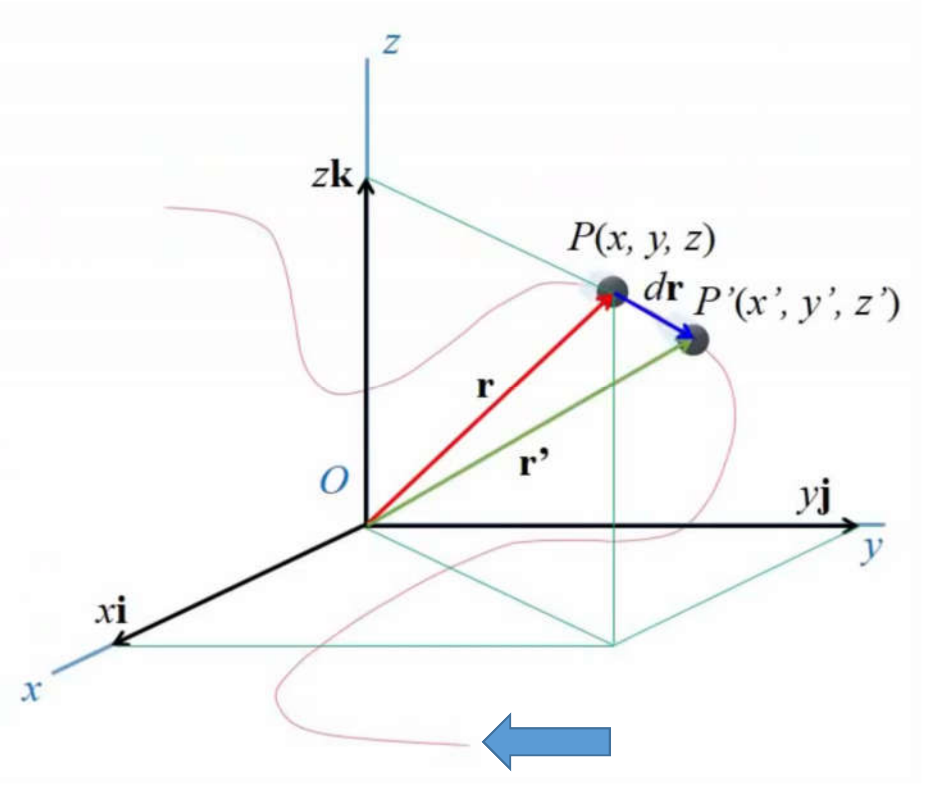
$$r = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

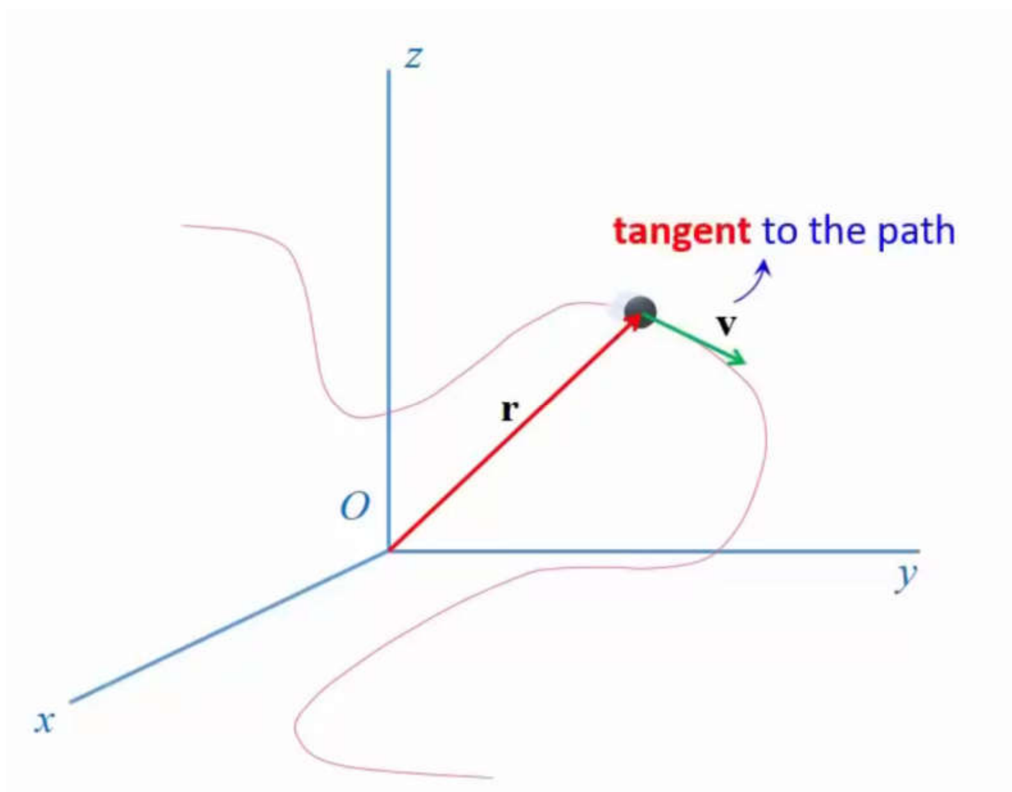
$$\boldsymbol{v} = \frac{\boldsymbol{dr}}{\boldsymbol{dt}}$$ velocity

$$= \boldsymbol{xi} + \boldsymbol{yj} + \boldsymbol{zk}$$

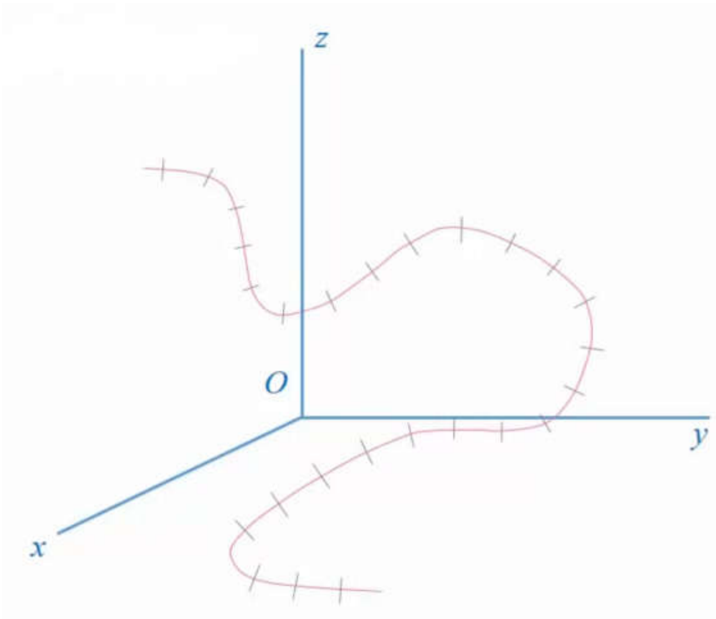
$$\boldsymbol{a} = \frac{\boldsymbol{dv}}{\boldsymbol{dt}} = \frac{\boldsymbol{d^2r}}{\boldsymbol{dt}}$$

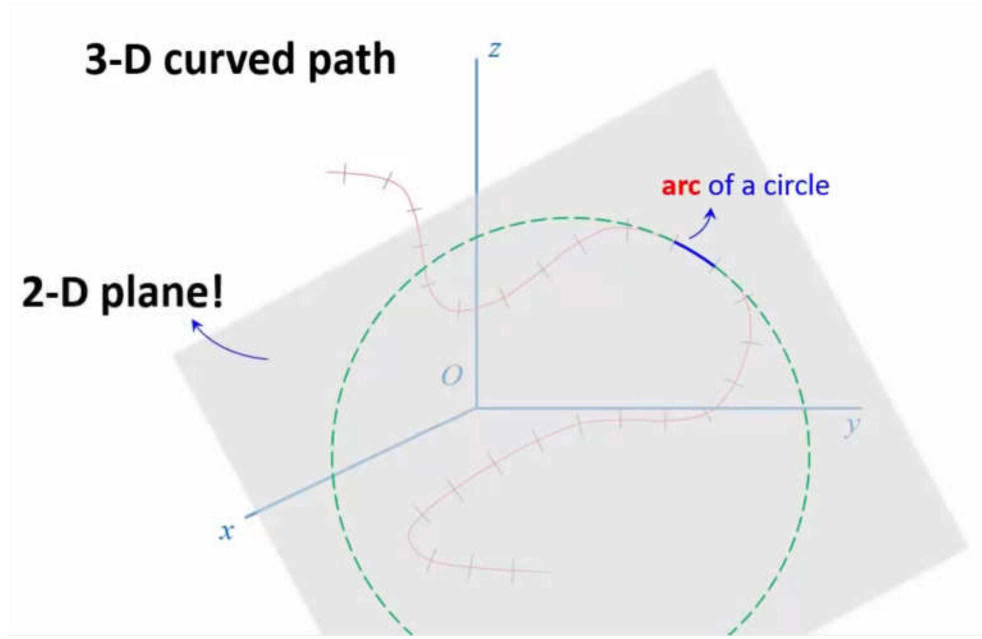
$$= \boldsymbol{xi} + \boldsymbol{yj} + \boldsymbol{zk}$$



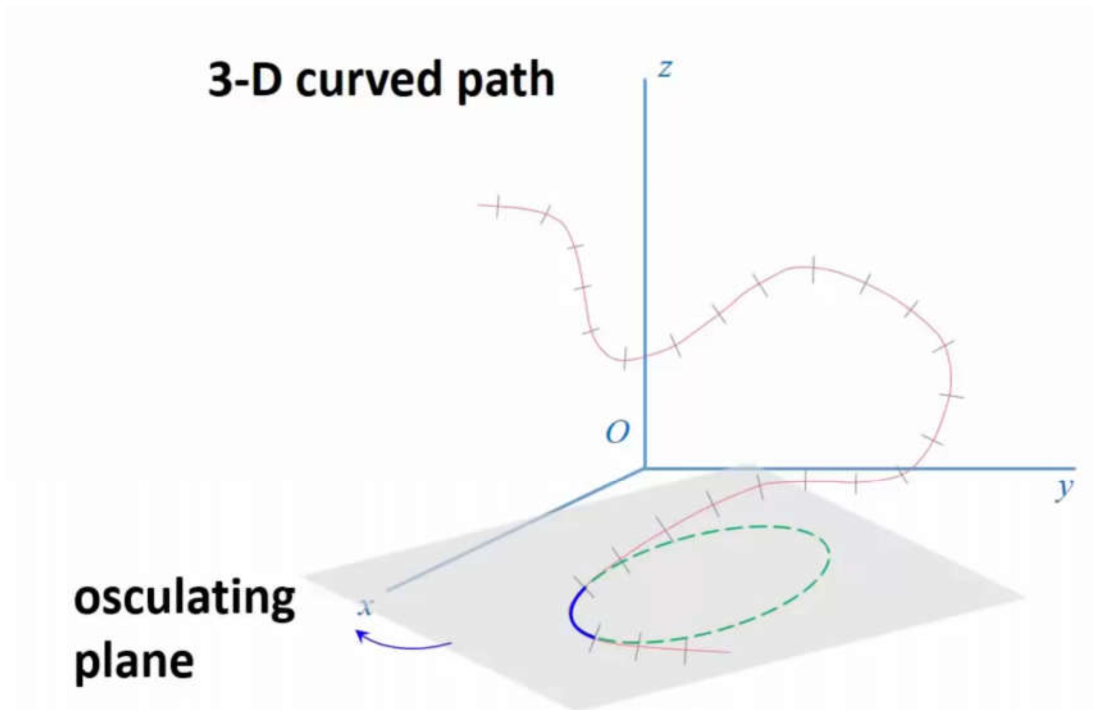

z
tangent to the path
v
r
O
y
x

3D-curved path

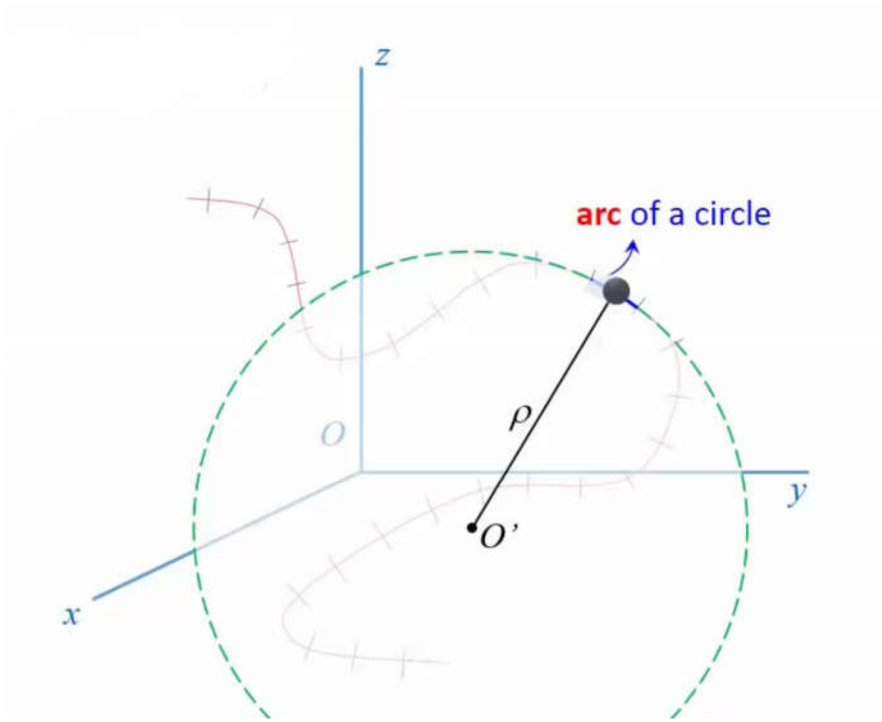


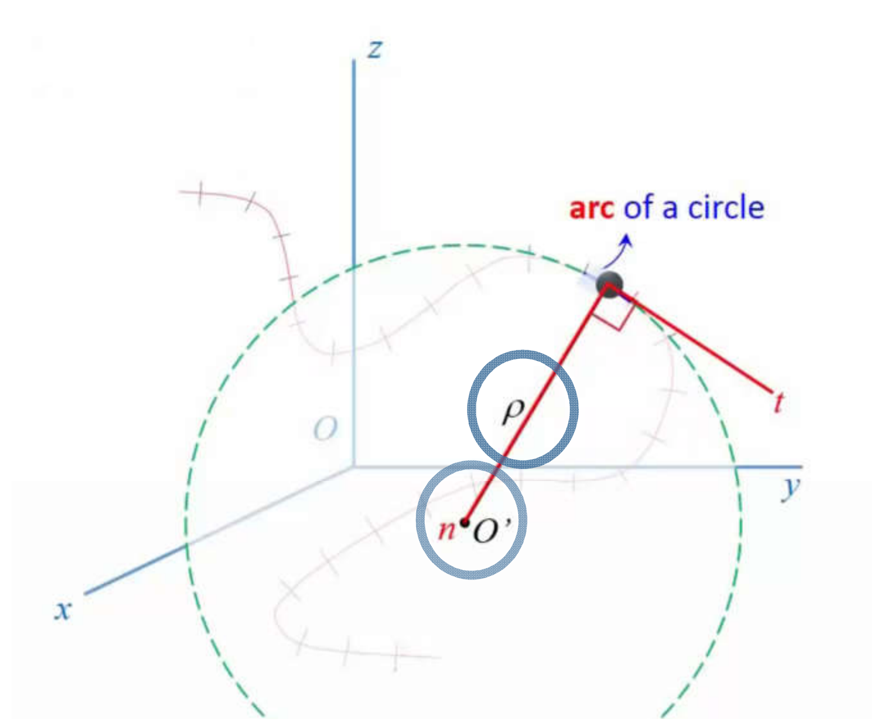


3D-curved path



The 3D motion is now considered as a sequence of 2D motions


z
arc of a circle
ρ
O
y
O'
x


z
arc of a circle
ρ
t
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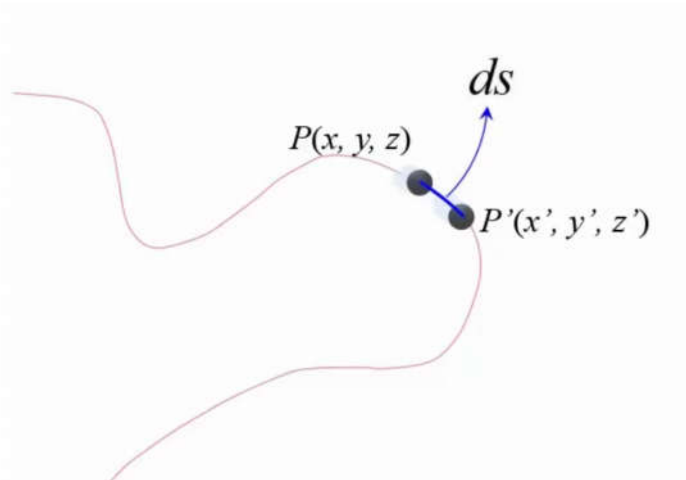
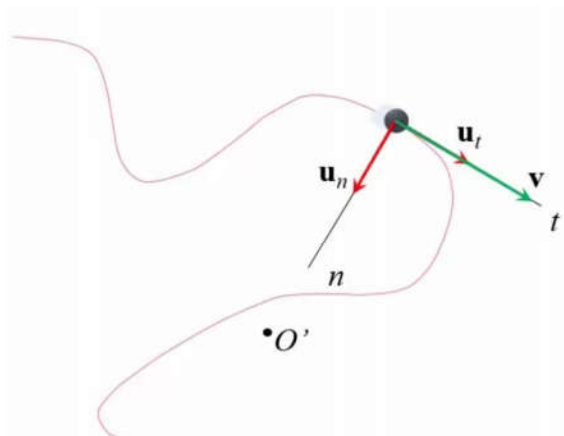
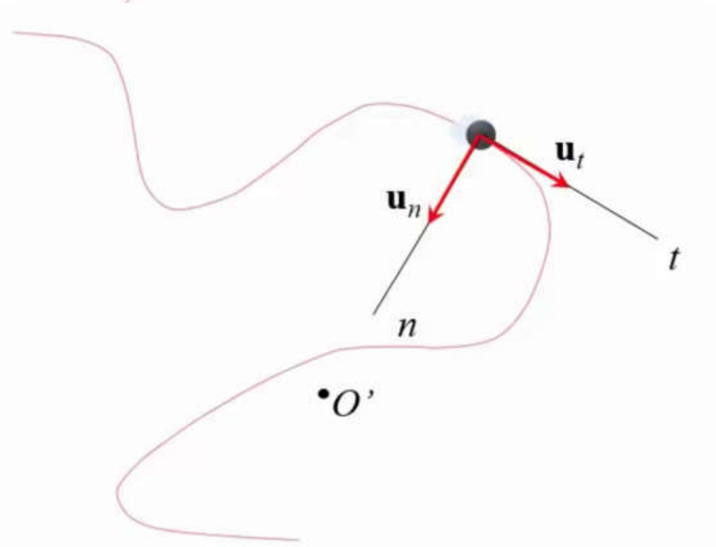
y
n
O'
x

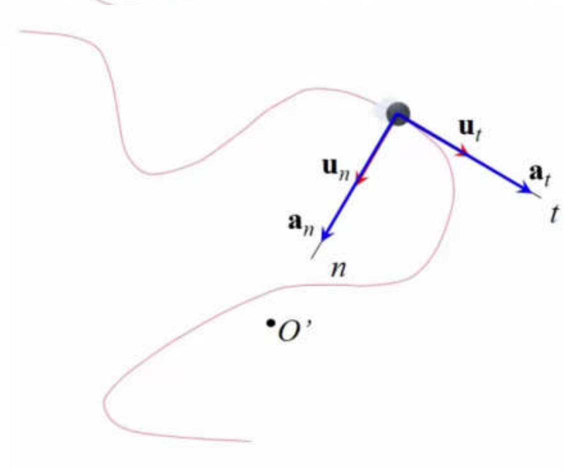




$$v = v u_t$$

$$v = \frac{ds}{dt}$$





$$a = a_t u_t + a_n u_n$$

$$a_t = \frac{dv}{dt}$$

$$a_n = \frac{v^2}{\rho}$$

$$a = \sqrt{a_t^2 + a_n^2}$$

a_n Is the centripetal acceleration

$$s = s_0 + v_0 t + \frac{1}{2}(a_t)_c t^2$$

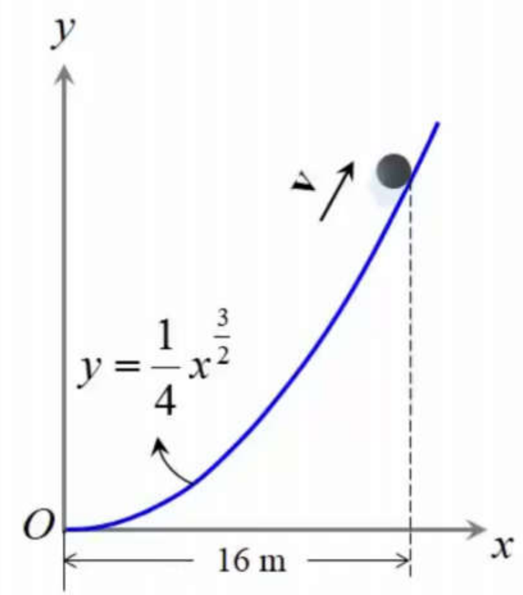
$$v = v_0 + (a_t)_c t^2$$

$$v^2 = v_0^2 + 2(a_t)_c(s \quad s_0)$$

If the path is known as y=f(x) then the curvature radius is :

$$\rho = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

Example: An object travels along a curved path as shown. If at the point shown its speed is 28.8 m/s and the speed is increasing at 8 m/s^2, determine the direction of its velocity, and the magnitude and direction of its acceleration at this point.



$$\text{slope} = \frac{dy}{dx} = \frac{3}{8}x^{\frac{1}{2}}$$

@ $x = 16$ m

$$\text{slope} = \frac{3}{8}\cdot 16^{\frac{1}{2}} = 1.5 = \tan\theta$$

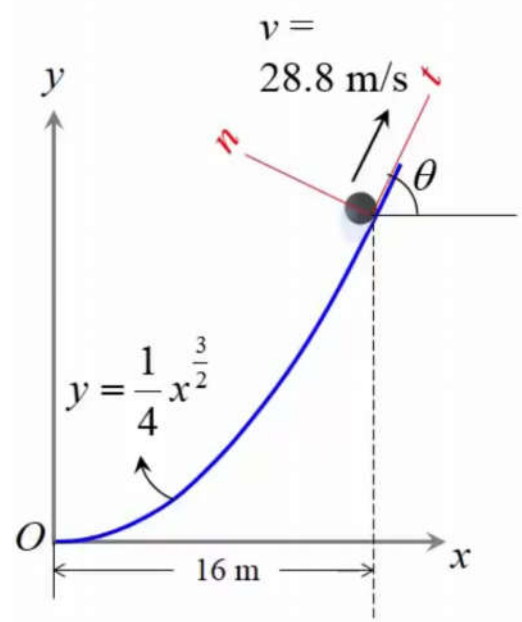
$$\therefore \quad \theta = \tan^{-1}1.5 = 56.3°$$

$$\mathbf{a} = a_t\mathbf{u}_t + a_n\mathbf{u}_n$$

$$a_t = \frac{dv}{dt} = 8 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{\left[1+(dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|}$$



$$\rho = \frac{\left[1+(dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|}$$

$$\frac{dy}{dx} = \frac{3}{8}x^{\frac{1}{2}} \qquad \frac{d^2y}{dx^2} = \frac{3}{16}x^{-\frac{1}{2}}$$

$$@\,x = 16\text{ m} \qquad \frac{dy}{dx} = 1.5 \qquad \frac{d^2y}{dx^2} = \frac{3}{64}$$

$$\therefore \quad \rho = 125\text{ m}$$

$$a_t = \frac{dv}{dt} = 8\text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{28.8^2}{125} = 6.64\ (\text{m/s}^2)$$

$$a = \sqrt{a_t^2 + a_n^2} = 10.4\text{ m/s}^2$$

$$\phi = 56.3° + \tan^{-1}\frac{6.64}{8} = 96.0°$$