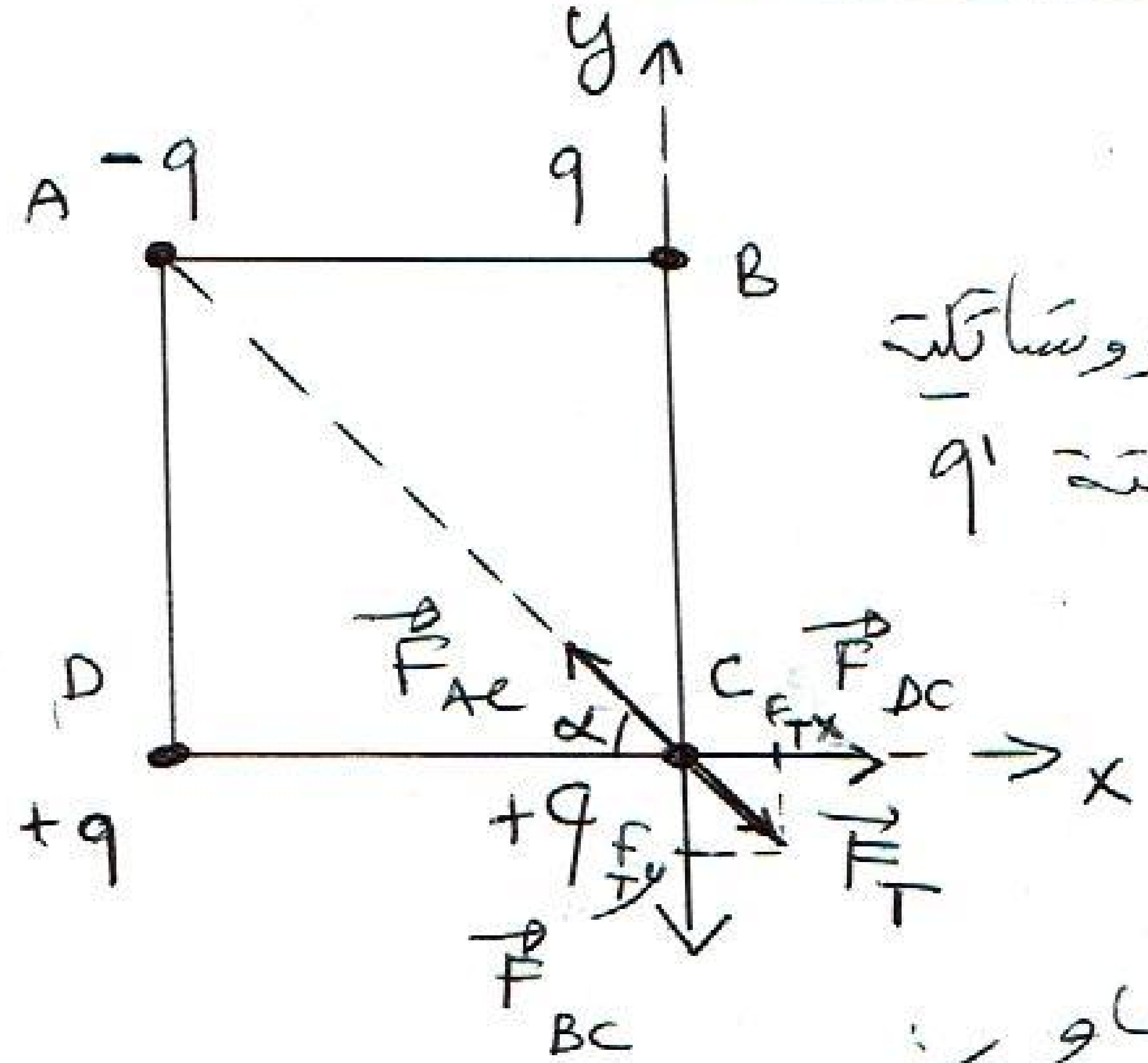
حل السلسلة (1)

حل ت (1):



- حساب القوة الكهروستاتيكية المؤثرة على الشحنة q' في النقطة C:

ط1/

$$\vec{F}_T = \vec{F}_{AC} + \vec{F}_{BC} + \vec{F}_{DC}$$

بالإسقاط على المحاور:

$$\begin{cases} F_{Tx} = -F_{AC}\cos\alpha + F_{DC} \\ F_{Ty} = F_{AC}\sin\alpha - F_{BC} \end{cases}$$

بتطبيق قانون كولون:

$$F_{AC} = \frac{k|q_A||q_C|}{AC^2} = \frac{kqq'}{2a^2}$$

$$F_{BC} = \frac{k|q_B||q_C|}{BC^2} = \frac{kqq'}{a^2} = F_{DC}$$

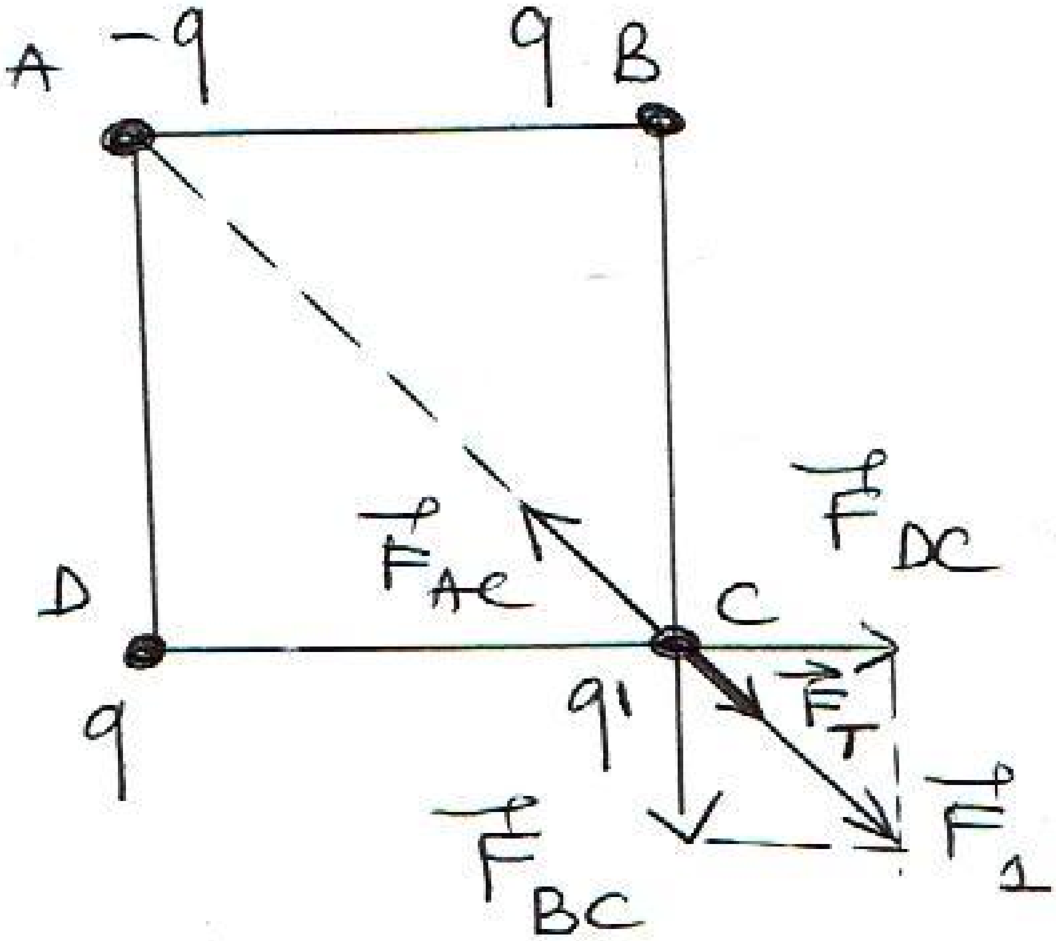
$$\cos\alpha = \sin\alpha = \frac{\sqrt{2}}{2} \quad / \quad \alpha = 45°$$

$$\begin{cases} F_{Tx} = \frac{kqq'}{a^2}\left(-\frac{\sqrt{2}}{4} + 1\right) \\ F_{Ty} = \frac{kqq'}{a^2}\left(\frac{\sqrt{2}}{4} - 1\right) = -F_{Tx} \end{cases} \Rightarrow \begin{cases} F_{Tx} = 0{,}65\frac{kqq'}{a^2} \\ F_{Ty} = -0{,}65\frac{kqq'}{a^2} \end{cases}$$

$$\boxed{\vec{F}_T = F_{Tx}\vec{i} + F_y\vec{j}}$$

$$F_T = \sqrt{F_{Tx}^2 + F_{Ty}^2} = F_{Tx}\sqrt{2} = \frac{Kqq'}{a^2}\left(-\frac{\sqrt{2}}{4} + 1\right)\sqrt{2}$$

$$\boxed{F_T = 0{,}91\ \frac{Kqq'}{a^2}}$$



2/ب حساب $\vec{F}_T$:

$$\vec{F}_T = \vec{F}_{AC} + \vec{F}_{BC} + \vec{F}_{DC}$$

نطبق قانون كولون :

$$\boxed{F_{AC} = \frac{Kqq'}{2a^2}}$$

$$\boxed{F_{BC} = F_{DC} = \frac{Kqq'}{a^2}}$$

$$\vec{F}_T = \vec{F}_{AC} + \underbrace{\vec{F}_{BC} + \vec{F}_{DC}}_{=\vec{F}_1}$$

$$\vec{F}_1 = \vec{F}_{BC} + \vec{F}_{DC} \Rightarrow \|\vec{F}_1\| = \sqrt{F_{BC}^2 + F_{DC}^2 + 2F_{BC}F_{DC}\underbrace{\cos\frac{\pi}{2}}_{=0}}$$

$$\Rightarrow F_1 = \sqrt{F_{BC}^2 + F_{DC}^2} \longrightarrow \vec{F}_{BC} \perp \vec{F}_{DC}$$

$$F_1 = F_{BC}\sqrt{2} \Rightarrow \boxed{F_1 = \sqrt{2}\,\frac{Kqq'}{a^2}}$$

$$\vec{F}_T = \vec{F}_{AC} + \vec{F}_1$$

$$\Rightarrow F_T = \sqrt{F_{AC}^2 + F_1^2 + 2F_{AC}F_1 \underbrace{\cos\pi}_{=-1}}$$

$$F_T = \sqrt{\left(F_{AC} - F_1\right)^2} \Rightarrow F_T = \left|F_{AC} - F_1\right|$$

$$F_T = \left|\frac{Kqq'}{2a^2} - \frac{\sqrt{2}\,Kqq'}{a^2}\right|$$

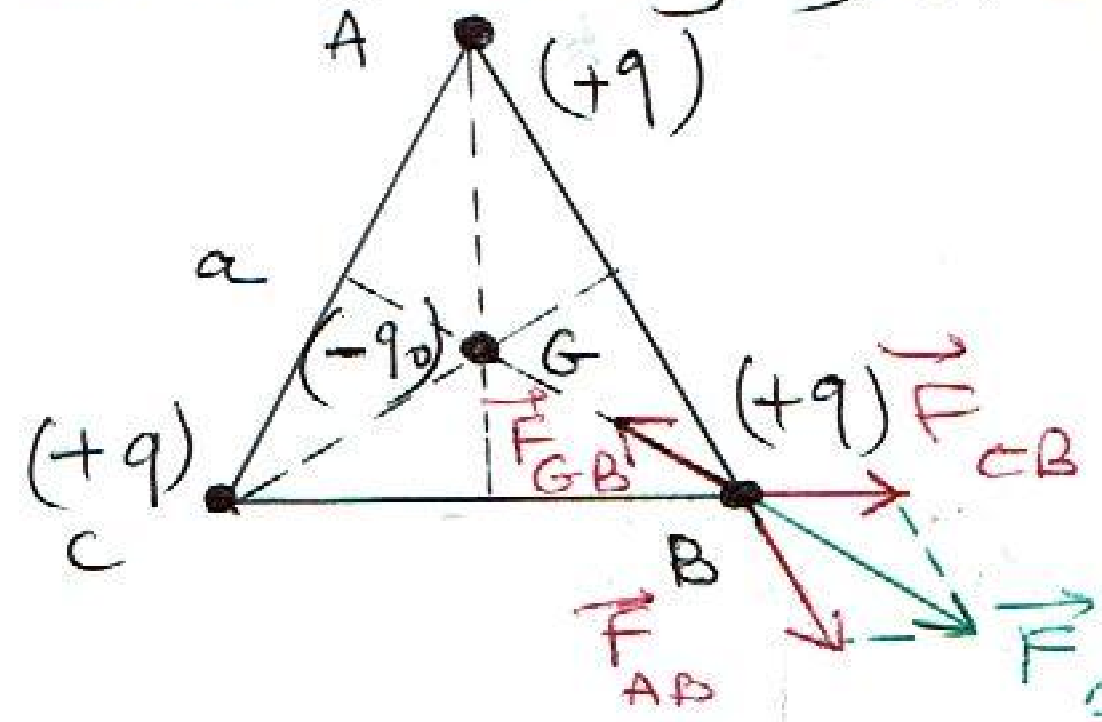
$$\boxed{F_T = \left(\sqrt{2} - \frac{1}{2}\right)\frac{Kqq'}{a^2}}$$

$$\boxed{F_T = 0{,}91\,\frac{Kqq'}{a^2}}$$

اتجاه $\vec{F}_T$ هو اتجاه F_1 لأن : $F_{AC} < F_1$.

حل (2):

- تعيين قيمة الشحنة q_0 لكي تنعدم محصلة القوى المؤثرة على كل الشحنات:



- نعتبر الشحنات q في الرؤوس A، B، C موجبة وعليه ⇐ $q_0 < 0$ (سالبة)
- نختار الرأس B: القوى المؤثرة على q هي: $\vec{F}_{AB}$ (قوة تنافر)، $\vec{F}_{CB}$ (قوة تنافر) و $\vec{F}_{GB}$ (قوة تجاذب)

مجموع القوى المؤثرة على q هي:

$$\underbrace{\vec{F}_{AB}+\vec{F}_{CB}+\vec{F}_{GB}}_{\text{محصلة القوى}} = \vec{0}$$

نعتبر: $\vec{F}_{AB}+\vec{F}_{CB}=\vec{F}_1$

$$\Rightarrow \|\vec{F}_1\| = \sqrt{F_{AB}^2+F_{CB}^2+2F_{AB}F_{CB}\cos 60°}$$

$$F_1 = \sqrt{F_{AB}^2 + F_{CB}^2 + 2F_{AB}F_{CB}\left(\frac{1}{2}\right)}$$

$$F_1 = \sqrt{F_{AB}^2 + F_{CB}^2 + F_{AB}F_{CB}}$$

نطبق قانون كولون ونحسب شدة القوى:

$$\boxed{F_{AB} = \frac{Kq^2}{a^2}} \quad \boxed{F_{CB} = \frac{Kq^2}{a^2}} \Rightarrow F_{AB} = F_{CB}$$

$$F_{GB} = \frac{K\,|q_0|\,q}{GB^2}$$

1ب) في مثلث متساوي الأضلاع

$$\boxed{GB = \frac{2h}{3}} \qquad (GB = GA = GC)$$

$$h^2 + \left(\frac{a}{2}\right)^2 = a^2 \Rightarrow \boxed{h = \frac{\sqrt{3}\,a}{2}}$$

$$GB = \frac{\cancel{2}}{3}\,\frac{\sqrt{3}}{\cancel{2}}\,a \Rightarrow \boxed{GB = \frac{a}{\sqrt{3}}}$$

2ب)

$$\cos 30° = \frac{a/2}{GB}$$

$$\Rightarrow GB = \frac{a/2}{\cos 30°} = \frac{a/2}{\sqrt{3}/2} = \frac{a}{\sqrt{3}}$$

$$\boxed{GB = \frac{a}{\sqrt{3}}}$$

$$\boxed{GB = GC = GA = \frac{a}{\sqrt{3}}}$$

$$F_{GB} = \frac{k|q_0|q}{\left(\frac{a}{\sqrt{3}}\right)^2} \Rightarrow \boxed{F_{GB} = \frac{3k|q_0|q}{a^2}}$$

$$F_1 = \sqrt{F_{AB}^2 + F_{CB}^2 + F_{AB}F_{CB}} \quad \Big|\; (F_{AB} = F_{CB})$$

$$F_1 = F_{AB}\sqrt{3} = \frac{\sqrt{3}\,kq^2}{a^2} \Rightarrow \boxed{F_1 = \frac{\sqrt{3}\,kq^2}{a^2}}$$

$$\vec{F_1} + \vec{F_{GB}} = \vec{0} \Rightarrow \begin{cases} \|\vec{F_{GB}}\| = \|\vec{F_1}\| \\ \vec{F_{GB}} \text{ و } \vec{F_1} \text{ متعاكسان} \\ \text{في الاتجاه.} \end{cases}$$

$$\Rightarrow \frac{3\cancel{k}|q_0|\cancel{q}}{\cancel{a^2}} = \frac{\sqrt{3}\,\cancel{k}q^{\cancel{2}}}{\cancel{a^2}}$$

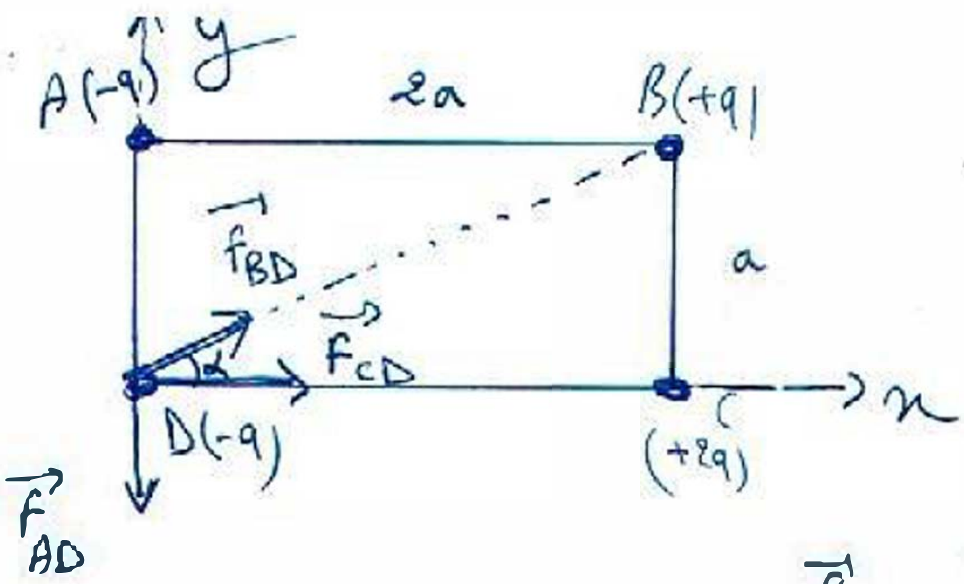
$$|q_0| = \frac{\sqrt{3}\,q}{3} = \frac{q}{\sqrt{3}}$$

$$\boxed{|q_0| = \frac{q}{\sqrt{3}}}$$

و منه،

$$\boxed{q_0 = -\frac{q}{\sqrt{3}}}$$

حل ت 3 :



- حساب القوة الكهروستاتيكية المؤثرة على الشحنة الموجودة في النقطة "D"

$$\vec{F}_D = \vec{F}_{CD} + \vec{F}_{AD} + \vec{F}_{BD}$$

بالإسقاط على المحاور نجد :

$$\begin{cases} F_{Dx} = F_{CD} + F_{BD} \cos\alpha \\ F_{Dy} = F_{BD} \sin\alpha - F_{AD} \end{cases}$$

$$\cos\alpha = \frac{DC}{BD} = \frac{2a}{\sqrt{a^2+4a^2}} = \frac{2a}{a\sqrt{5}}$$

$$\cos\alpha = \frac{2}{\sqrt{5}} \quad ; \quad \sin\alpha = \frac{a}{a\sqrt{5}}$$

$$\sin\alpha = \frac{1}{\sqrt{5}}$$

$$F_{CD} = \frac{K|q_C||q_D|}{CD^2} = \frac{2Kq^2}{4a^2} = \frac{Kq^2}{2a^2}$$

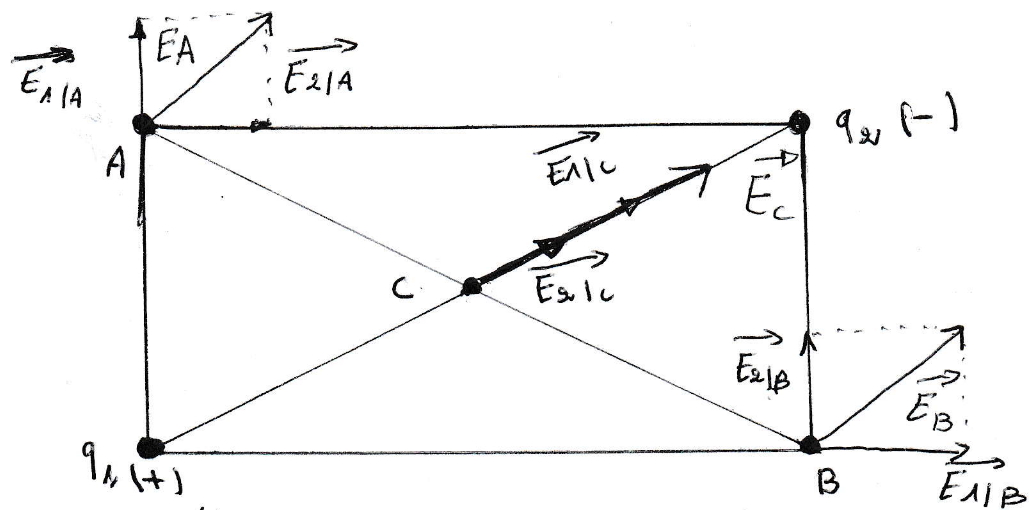
$$F_{BD} = \frac{K|q_B||q_D|}{BD^2} = \frac{Kq^2}{5a^2} \quad , \quad F_{AD} = K\frac{|q_A||q_D|}{AD^2} = \frac{Kq^2}{a^2}$$

$$\begin{cases} F_{Dx} = \frac{Kq^2}{2a^2} + \frac{Kq^2}{5a^2}\cdot\frac{2}{\sqrt{5}} = \frac{Kq^2}{a^2}\left(\frac{1}{2} + \frac{2}{5\sqrt{5}}\right) = \frac{Kq^2}{a^2}(0{,}68) \\ F_{Dy} = \frac{Kq^2}{5a^2}\cdot\frac{1}{\sqrt{5}} - \frac{Kq^2}{a^2} = \frac{Kq^2}{a^2}\left(-1 + \frac{1}{5\sqrt{5}}\right) = \frac{Kq^2}{a^2}(-0{,}91) \end{cases}$$

$$F_D = \sqrt{F_{Dx}^2 + F_{Dy}^2} = \frac{Kq^2}{a^2}(1.14)$$

$$\boxed{F_D = (1.14)\,Kq^2/a^2}$$

حل تمرين 4



حساب شدة الحقل عند النقاط A ، B و C

• الحقل عند النقطة A:

$$\vec{E}_A = \vec{E}_{1/A} + \vec{E}_{2/A}$$

$$E_A = \sqrt{E_{1/A}^2 + E_{2/A}^2 + 2E_{1/A}E_{2/A}\cos 90}$$

$$= \sqrt{E_{1/A}^2 + E_{2/A}^2}$$

$$E_{1/A} = \frac{K|q_1|}{r_{1/A}^2} = \frac{Kq_1}{r_{1/A}^2} = \frac{9\times10^9\times(3,2\times10^{-11})}{(6\times10^{-2})^2}$$

$$E_{1/A} = 80 \text{ V/m}$$

$$E_{2/A} = \frac{K|q_2|}{r_{2/A}^2} = \frac{-Kq_2}{r_{2/A}^2}$$

$$= \frac{-9\times10^9\times(-4,26\times10^{-11})}{(8\times10^{-2})^2}$$

$$E_{2/A} = 59,90 \text{ V/m}$$

$$E_A = \sqrt{80^2 + 59,90^2}$$

$$\boxed{E_A = 99,94 \text{ V/m}}$$

• الحقل عند النقطة B:

$$\vec{E}_B = \vec{E}_{1/B} + \vec{E}_{2/B} \Leftrightarrow E_B = \sqrt{E_{1/B}^2 + E_{2/B}^2}$$

لأن $\vec{E}_{1/B} \perp \vec{E}_{2/B}$

$$E_{1/B} = \frac{K|q_1|}{r_{1/B}^2} = \frac{Kq_1}{r_{1/B}^2}$$

$$E_{1/B} = \frac{9\times10^9\times3,2\times10^{-11}}{(8\times10^{-2})^2} = 45 \text{ V/m}$$

$$E_{2/B} = \frac{K|q_2|}{r_{2/B}^2} = \frac{-Kq_2}{r_{2/B}^2}$$

$$E_{2/B} = \frac{-9\times10^9\times(-4,26\times10^{-11})}{(6\times10^{-2})^2} = 106,5 \text{ V/m}$$

$$E_B = \sqrt{45^2 + 106,5^2} \Leftrightarrow \boxed{E_B = 115,6 \text{ V/m}}$$

• حساب الحقل عند النقطة C:

$$\vec{E}_C = \vec{E}_{1/C} + \vec{E}_{2/C}$$

$$E_C = \sqrt{E_{1/C}^2 + E_{2/C}^2 + 2E_{1/C}E_{2/C}\cos 0}$$

$$= \sqrt{(E_{1/C} + E_{2/C})^2} \Leftrightarrow E_C = E_{1/C} + E_{2/C}$$

$$E_{1/C} = \frac{K|q_1|}{r_{1/C}^2} = \frac{Kq_1}{r_{1/C}^2} \quad \Big| \quad E_{2/C} = \frac{K|q_2|}{r_{2/C}^2} = \frac{-Kq_2}{r_{2/C}^2}$$

$$r_{1/C} = r_{2/C} = \frac{1}{2}AB = \frac{1}{2}\sqrt{b^2 + a^2}$$

$$= \frac{1}{2}\sqrt{6^2 + } = \frac{1}{2}10 = 5 \text{ cm}$$

ت ع

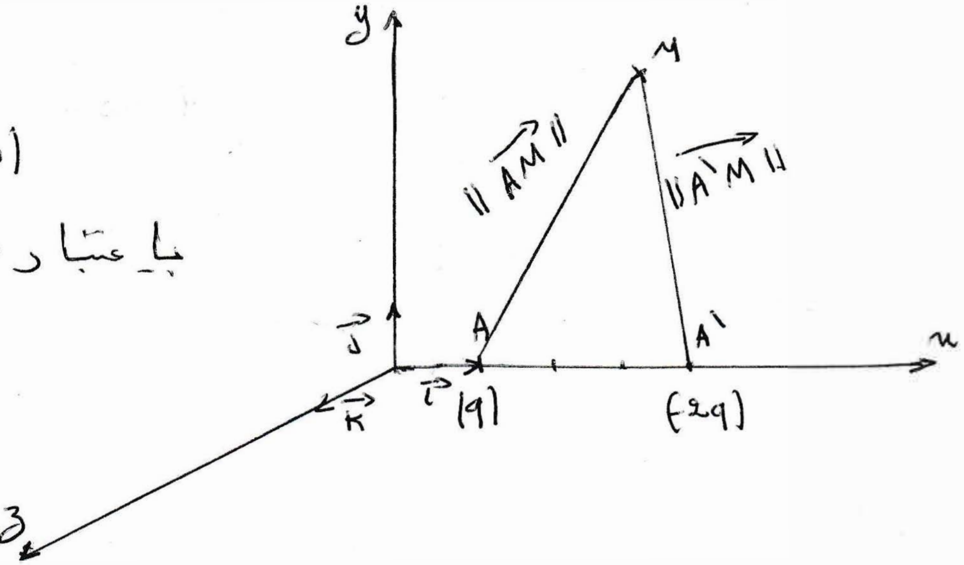
$$E_{1/C} = \frac{9\times10^9\times3,2\times10^{-11}}{(5\times10^{-2})^2} = 115,2 \text{ V/m}$$

$$E_{2/C} = \frac{9\times10^9\times(-4,26\times10^{-11})}{(5\times10^{-2})^2} = 153,4 \text{ V/m}$$

$$E_C = 153,4 + 115,2 \Leftrightarrow$$

$$\boxed{E_C = 168,6 \text{ V/m}}$$

حل تمرين 5



$A(a,0,0)$

$A'(4a,0,0)$

باعتبار $a=1$

① حساب الكمون في نقطة كيفية $M(x,y,z)$.

$$V_M = V_{A/M} + V_{A'/M}$$

$$= \frac{Kq}{\|\overrightarrow{AM}\|} + \frac{K(-2q)}{\|\overrightarrow{A'M}\|}$$

$$V_M = Kq\left(\frac{1}{\|\overrightarrow{AM}\|} - \frac{2}{\|\overrightarrow{A'M}\|}\right)$$

$$\|\overrightarrow{AM}\| = \sqrt{(x_M-x_A)^2+(y_M-y_A)^2+(z_M-z_A)^2}$$
$$= \sqrt{(x-a)^2+(y-0)^2+(z-0)^2}$$

$$\|\overrightarrow{AM}\| = \sqrt{(x-a)^2+y^2+z^2}$$

$$\|\overrightarrow{A'M}\| = \sqrt{(x_M-x_{A'})^2+(y_M-y_{A'})^2+(z_M-z_{A'})^2}$$
$$= \sqrt{(x-4a)^2+(y-0)^2+(z-0)^2}$$

$$\|\overrightarrow{A'M}\| = \sqrt{(x-4a)^2+y^2+z^2}$$

$$\boxed{V_M = Kq\left(\frac{1}{\sqrt{(x-a)^2+y^2+z^2}} - \frac{2}{\sqrt{(x-4a)^2+y^2+z^2}}\right)}$$

② حساب المساحة المتساوية الكمون

$V_M = 0 \iff V = 0$

$$V_M = 0 \Rightarrow \begin{cases} Kq = 0 & \text{مستحيل} \\ \text{و} \\ \dfrac{1}{\sqrt{(a-x)^2+y^2+z^2}} - \dfrac{2}{\sqrt{(x-4a)^2+y^2+z^2}} = 0 \end{cases}$$

$$\frac{1}{\sqrt{(a-x)^2+y^2+z^2}} = \frac{2}{\sqrt{(x-4a)^2+y^2+z^2}}$$

$$4\left[(a-x)^2+y^2+z^2\right] = \left[(x-4a)^2+y^2+z^2\right]$$

$$4a^2 - \cancel{8ax} + 4x^2 + 4y^2 + 4z^2 = x^2 - \cancel{8ax} + 16a^2 + y^2 + z^2$$

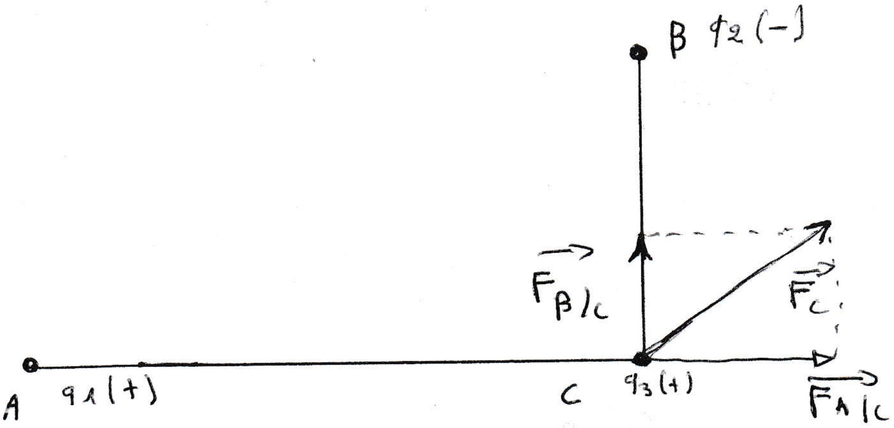
$$4x^2+4y^2+4z^2-x^2-y^2-z^2 = 16a^2-4a^2$$

$$\left(3x^2+3y^2+3z^2 = 12a^2\right] \times \frac{1}{3}$$

$$\boxed{x^2+y^2+z^2 = (2a)^2}$$

المساحة المتساوية الكمون عبارة عن سطح كرة نصف قطرها $r=2a$ و مركزها $w(0,0,0)$.

حل تمرين 6



① حساب القوة المؤثرة على q_3:

$$\vec{F_C} = \vec{F_{A/C}} + \vec{F_{B/C}}$$

$$F_C = \sqrt{F_{A/C}^2 + F_{B/C}^2 + \underbrace{\cancel{2F_{A/C}F_{B/C}\cos 90°}}_{0}}$$

$$\Leftrightarrow F_C = \sqrt{F_{A/C}^2 + F_{B/C}^2}$$

• $F_{A/C} = \frac{K|q_A q_C|}{r_{AC}^2} = \frac{K q_A q_C}{r_{AC}^2}$

ت ع: $F_{A/C} = \frac{9\times10^9(1,5\times10^{-3})(0,2\times10^{-3})}{(1,2)^2}$

$= 1875\ N$

• $F_{B/C} = \frac{K|q_B q_C|}{r_{B/C}^2} = \frac{-K q_B q_C}{r_{B/C}^2}$

ت ع: $F_{B/C} = \frac{-9\times10^9(-0,5\times10^{-3})(0,2\times10^{-3})}{(0,5)^2}$

$F_{B/C} = 3600\ N$

$F_C = \sqrt{1875^2 + 3600^2}$

$\boxed{F_C = 4059,02\ N}$

② حساب الحقل و الكمون الكهربائيين الناتجين عن q_1 و q_2 عند النقطة C

• الحقل:

$$\vec{F_C} = q_C \vec{E_C} \Leftrightarrow \vec{E_C} = \frac{\vec{F_C}}{q_C}$$

$$E_C = \frac{F_C}{|q_C|}$$

ت ع: $E_C = \frac{4059,02}{0,2\times10^{-3}}$

$\boxed{E_C = 20,29\times10^6\ V/m}$

• الكمون

$$V_C = V_{A/C} + V_{B/C} = \frac{Kq_A}{AC} + \frac{Kq_B}{BC}$$

ت ع: $V_C = \frac{9\times10^9\times1,5\times10^{-3}}{1,2} + \frac{9\times10^9(-0,5\times10^{-3})}{0,5}$

$\boxed{V_C = 2,25\times10^6 V}$

③ حساب الطاقة الكامنة للشحنة q_3

$$E_{p(q_3)} = q_3 V_C$$

ت ع: $= 0,2\times10^{-3}\times2,25\times10^6$

$\boxed{E_{p(q_3)} = 450\ jol}$