Operations Management

Chapter 4 –
Forecasting Demand

PowerPoint presentation to accompany

Heizer/Render, Operations Management, 12 th Ed.

What is Forecasting?

Predict the next number in the pattern:

a) 3.7, 3.7, 3.7, 3.7, 3.7, ?

b) 2.5, 4.5, 6.5, 8.5, 10.5, ?

c) 5.0, 7.5, 6.0, 4.5, 7.0, 9.5, 8.0, 6.5, ?

What is Forecasting?

a) 3.7, 3.7, 3.7, 3.7, 3.7, **3.7**

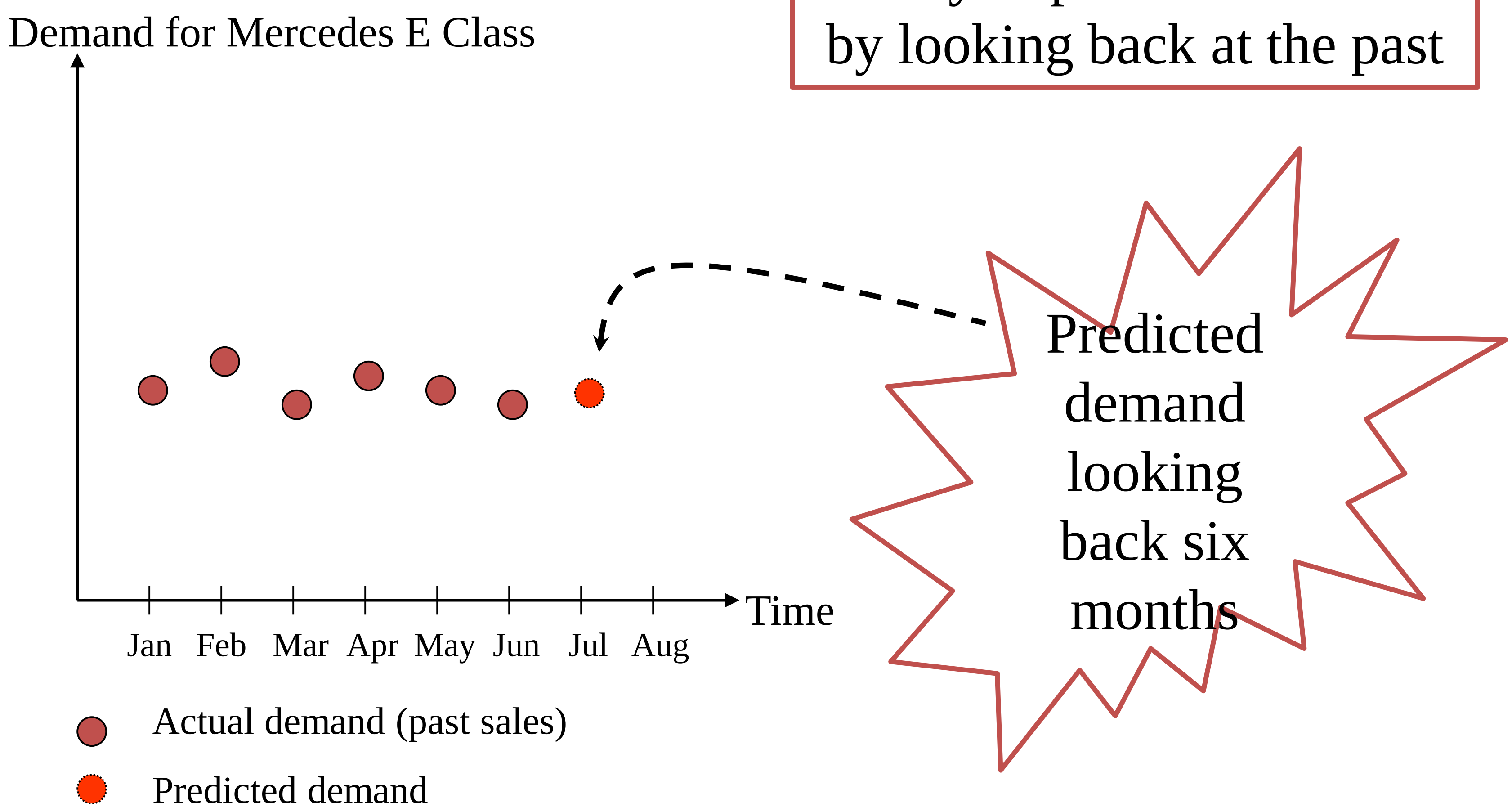
b) 2.5, 4.5, 6.5, 8.5, 10.5, **12.5**

c) 5.0, 7.5, 6.0, 4.5, 7.0, 9.5, 8.0, 6.5, **9**

- Demand forecasting is a Process of predicting a future event
- Underlying basis of all business decisions:
  Production, Inventory, Personnel, Facilities

What is Forecasting?

We try to predict the future by looking back at the past



Realities of Forecasting

- Most forecasting methods assume that there is some underlying stability in the system
- Forecasts are more accurate for shorter time periods
- Every forecast should include an error estimate
- Forecasts are seldom perfect……. unpredictable outside factors may impact the forecast

Realities of Forecasting

If Forecasts are seldom perfect(almost always wrong), why do we need to forecast?

↓

“Best" educated guesses about future are more valuable for purpose of Planning than no forecasts and hence no planning.

Importance of Forecasting in OM

- Departments throughout the organization depend on forecasts to formulate and execute their plans.

- Finance needs forecasts to project cash flows and capital requirements.

- Human resources need forecasts to anticipate hiring needs.

Production needs forecasts to plan production levels, workforce, material requirements, inventories, etc.

Forecasting Time Horizons

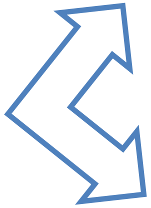
1. *Short-range forecast*
   - Up to 1 year, generally less than 3 months
   - Purchasing, job scheduling, workforce levels, job assignments, production levels
2. *Medium-range forecast*
   - 3 months to 3 years
   - Sales and production planning, budgeting
3. *Long-range forecast*
   - 3^+ years
   - New product planning, facility location, research and development

Types of forecasting methods

Qualitative methods	Quantitative methods
Rely on subjective opinions from one or more experts.	Rely on data and analytical techniques.

Qualitative Methods

- Used when situation is vague and little data exist



New products

New technology

- Involves intuition, experience

Types of forecasting methods

Qualitative Methods

Briefly, the qualitative methods are:

Qualitative Methods

- Executive Judgement
- Sales Force Composite
- Market Research/Survey
- Delphi Method

Executive Judgment: Opinion of a group of high level experts or managers is pooled

Types of forecasting methods

Qualitative Methods

Sales Force Composite

Each regional salesperson provides his/her sales estimates. Those estimates are then reviewed to make sure they are realistic. All regional estimates are then pooled at national levels to obtain an overall estimates.

Market Research/ Survey

Solicits input from customers pertaining to their future purchasing plans.

Delphi Method

Typically, the procedure consists of the following steps:

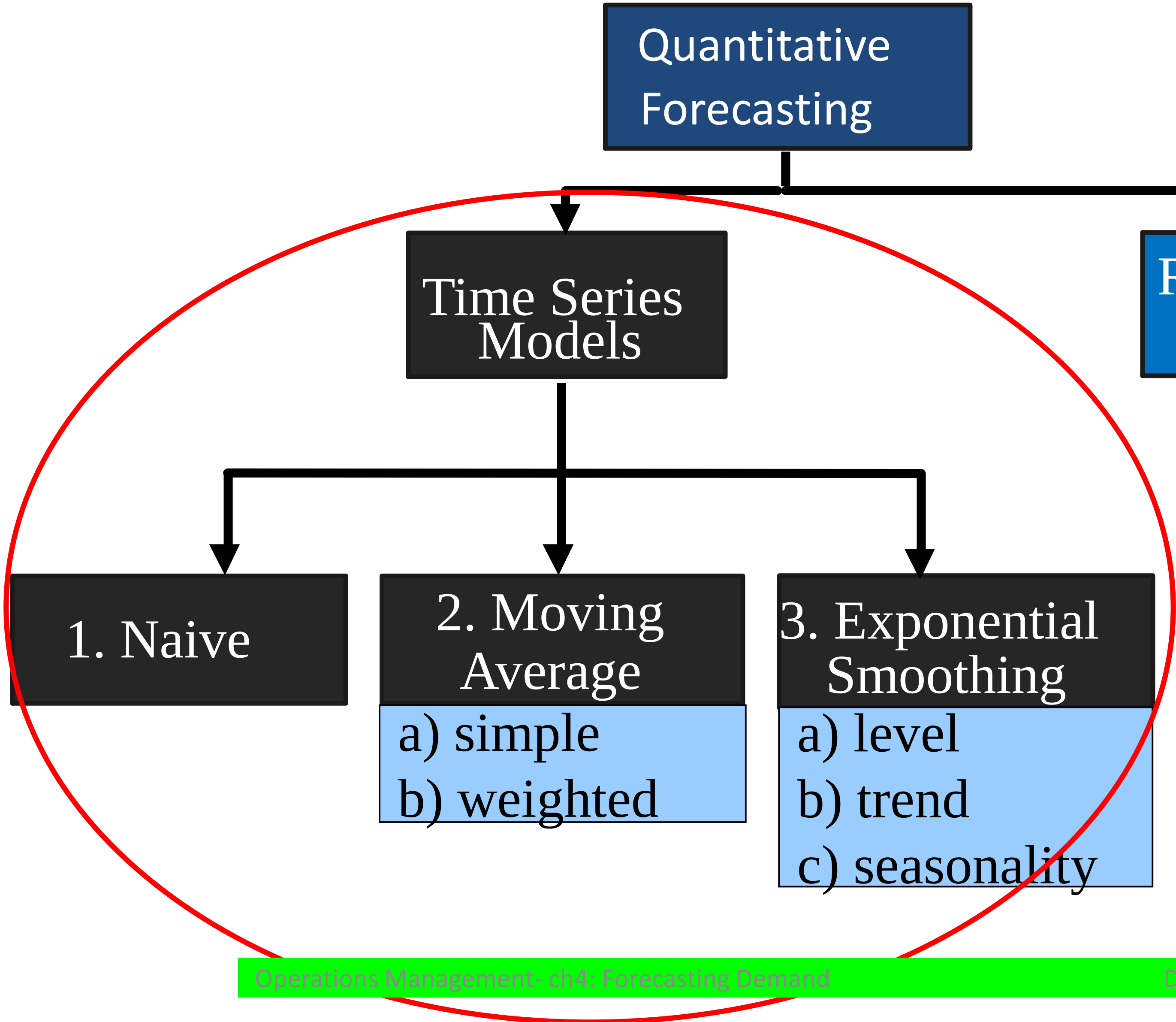
Types of forecasting methods

Qualitative Methods

- Each expert in the group makes his/her own forecasts in form of statements
- The coordinator collects all group statements and summarizes them
- The coordinator provides this summary and gives another set of questions to each group member including feedback as to the input of other experts
- The above steps are repeated until a consensus is reached.

Types of forecasting methods

Quantitative Methods



Types of forecasting methods

Quantitative Methods

- Used when situation is ‘stable’ and historical data exist

Existing products

Current technology

- Involves mathematical techniques

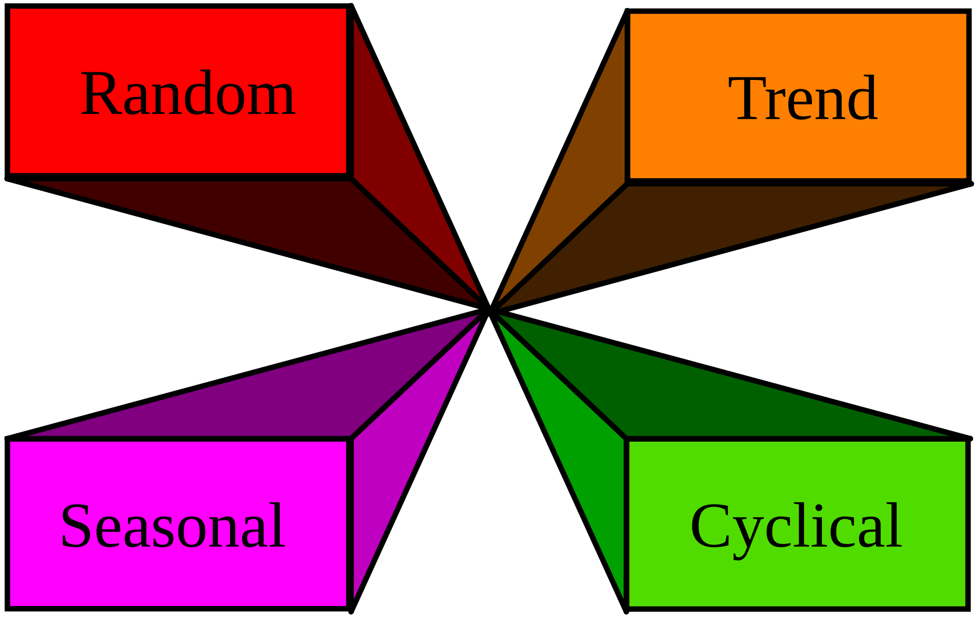
Types of forecasting methods

Quantitative Methods

Time Series Models

- Try to predict the future based on past data
- Assume that factors influencing the past will continue to influence the future

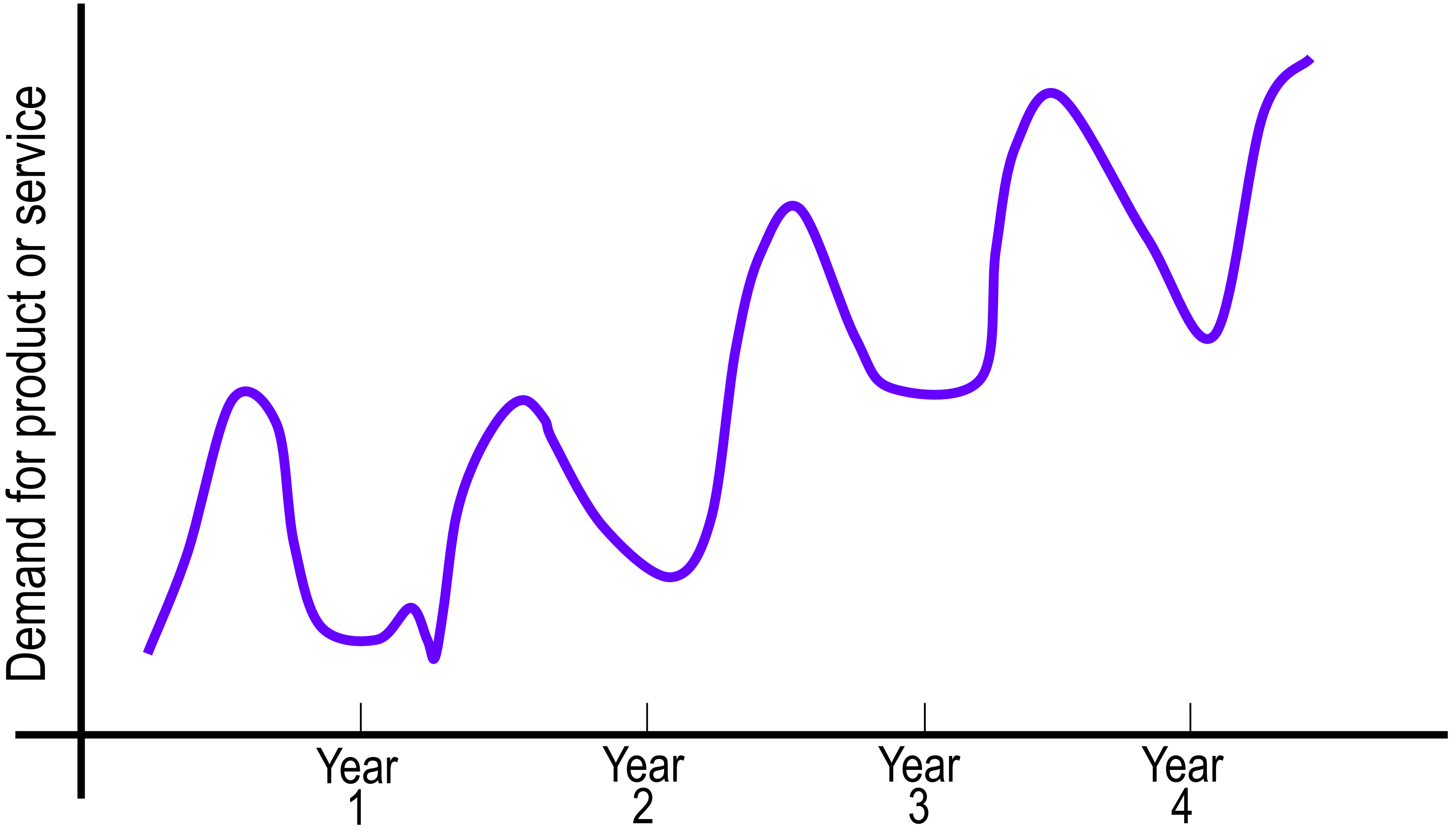
Time Series Models: Components



Types of forecasting methods

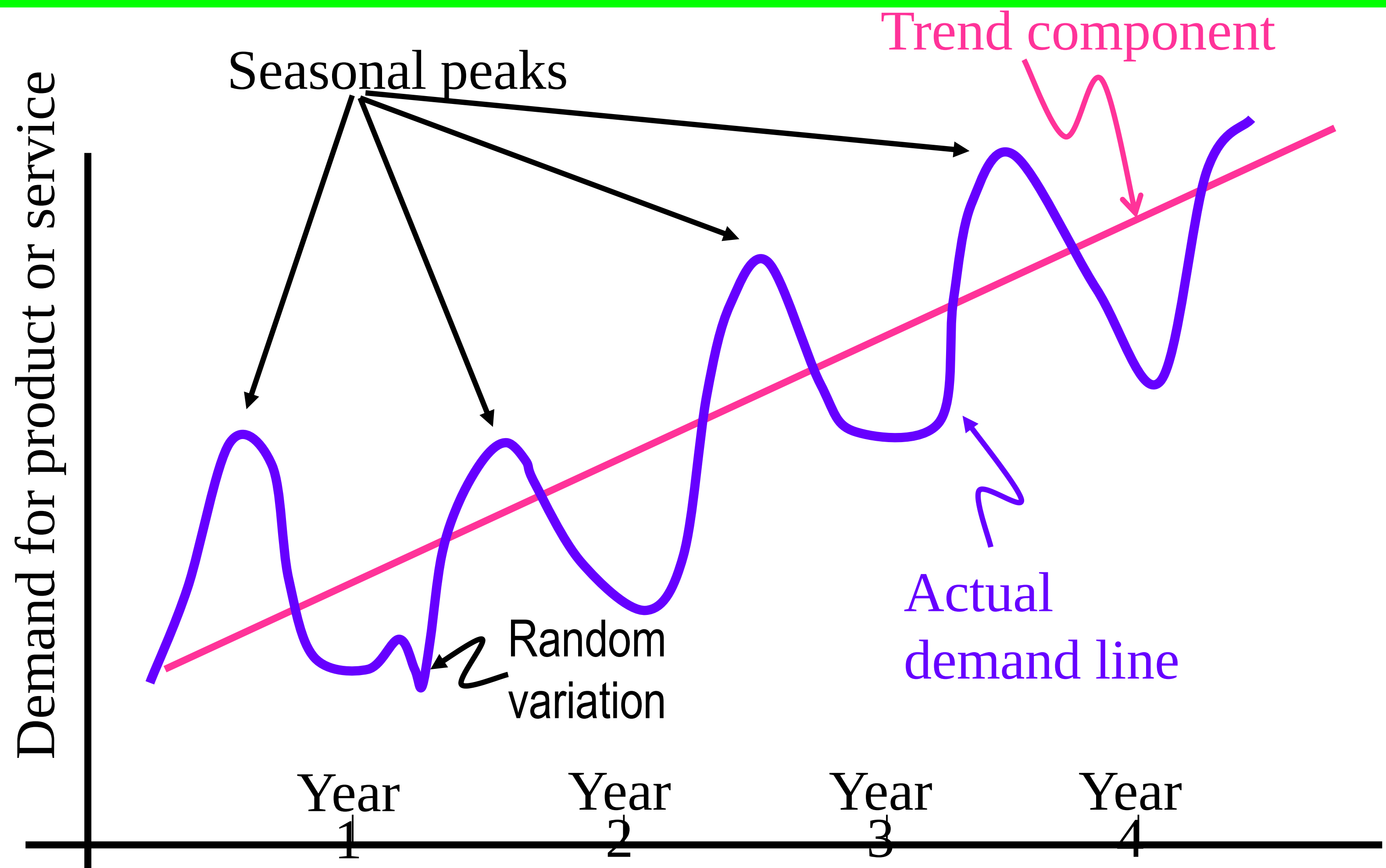
Quantitative Methods

Product Demand over Time



Types of forecasting methods

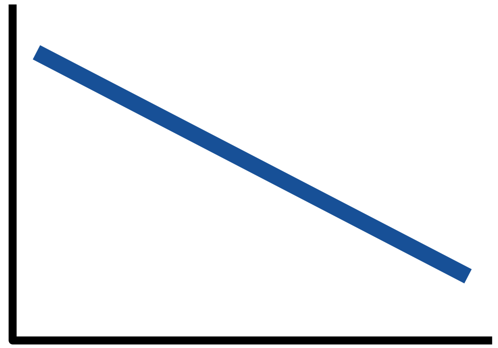
Quantitative Methods



Now let's look at some time series approaches to forecasting…

Trend Component

- Persistent, overall upward or downward pattern
- Changes due to population, technology, age, culture, etc.
- Typically several years duration



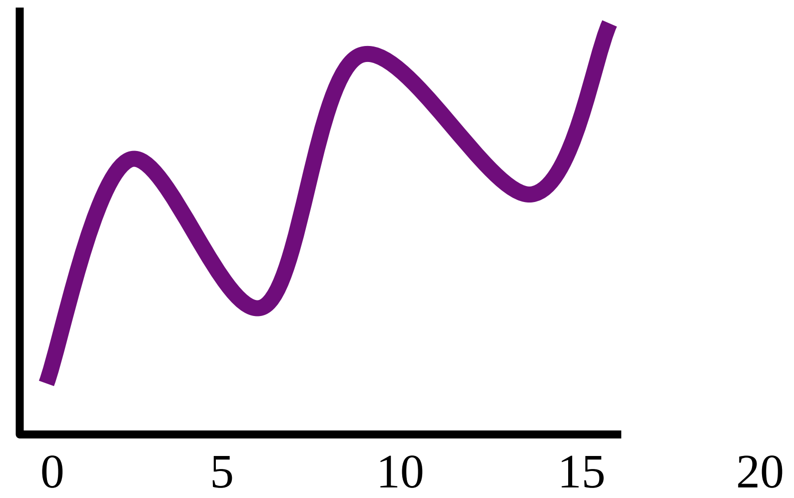
Seasonal Component

- Regular pattern of up and down fluctuations
- Due to weather, customs, etc.
- Occurs within a single year

PERIOD LENGTH	"SEASON" LENGTH	NUMBER OF "SEASONS" IN PATTERN
Week	Day	7
Month	Week	4 – 4.5
Month	Day	28 – 31
Year	Quarter	4
Year	Month	12
Year	Week	52

Cyclical Component

- Repeating up and down movements
- Affected by business cycle, political, and economic factors
- Multiple years duration

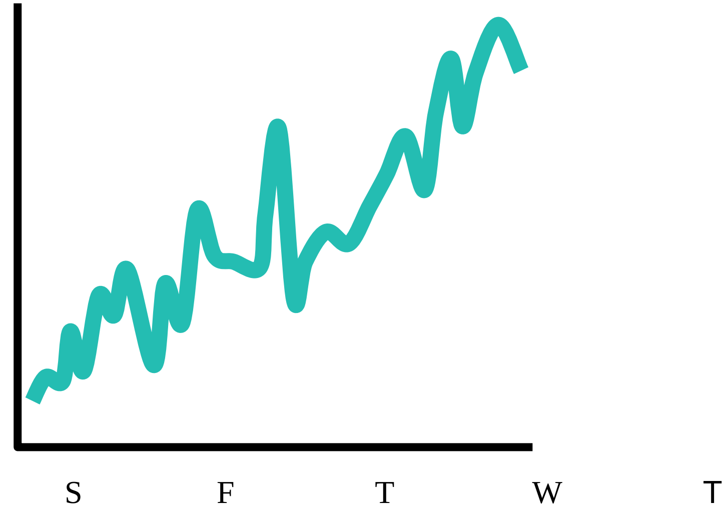


Random Component

- Erratic شاذ, unsystematic غير منتظم, ‘residual’ fluctuations

➢ Due to random variation or unforeseen events

➢ Short duration
and nonrepeating



Types of forecasting methods

Quantitative Methods

1. Naive Approach

- Demand in *next* period is the same as demand in *most recent* period

✓May sales = 48 → June forecast = 48

- Usually not good

Types of forecasting methods

Quantitative Methods

2a. Simple Moving Average

Assumes an average is a good estimator of future behavior

- Used if *little or no trend*
- Used for smoothing

The formula for the simple moving average is:

$$F_{t+1} = \frac{A_t + A_{t-1} + A_{t-2} + \dots + A_{t-n+1}}{n}$$

F_{t+1} = Forecast for the upcoming period, t+1
n = Number of periods to be averaged
A_t = Actual occurrence in period t

Types of forecasting methods

Quantitative Methods

Ex: 1

Week	Demand
1	650
2	678
3	720
4	785
5	859
6	920
7	850
8	758
9	892
10	920
11	789
12	844

- *Question: What are the 3-week and 6-week moving average forecasts for demand?*
- Assume you only have 3 weeks and 6 weeks of actual demand data for the respective forecasts

Types of forecasting methods

Quantitative Methods

Calculating the moving averages gives us:

Week	Demand	3	
1	650		
2	678		
3	720		
4	785	682.67	
5	859	727.67	
6	920	788.00	
7	850	854.67	768.67
8	758	876.33	802.00
9	892	842.67	815.33
10	920	833.33	844.00
11	789	856.67	866.50
12	844	867.00	854.83

$F_4=(650+678+720)/3$

$=682.67$

$F_7=(650+678+720+785+859+920)/6$

$=768.67$

Types of forecasting methods

Quantitative Methods

Ex: 2

Week	Demand
1	820
2	775
3	680
4	655
5	620
6	600
7	575

- Question: What is the 3 week moving average forecast for this data?
- Assume you only have 3 weeks and 5 weeks of actual demand data for the respective forecasts

Types of forecasting methods

Quantitative Methods

Solution

Week	Demand	3-Week	5-Week
1	820		
2	775		
3	680		
4	655	758.33	
5	620	703.33	
6	600	651.67	710.00
7	575	625.00	666.00

Types of forecasting methods

Quantitative Methods

Ex: 3

MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	
May	19	
June	23	
July	26	
August	30	
September	28	
October	18	
November	16	
December	14	

Types of forecasting methods

Quantitative Methods

Solution

MONTH	ACTUAL SHED SALES	3-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	$(10 + 12 + 13)/3 = 11\,{}^{2}/_{3}$
May	19	$(12 + 13 + 16)/3 = 13\,{}^{2}/_{3}$
June	23	$(13 + 16 + 19)/3 = 16$
July	26	$(16 + 19 + 23)/3 = 19\,{}^{1}/_{3}$
August	30	$(19 + 23 + 26)/3 = 22\,{}^{2}/_{3}$
September	28	$(23 + 26 + 30)/3 = 26\,{}^{1}/_{3}$
October	18	$(29 + 30 + 28)/3 = 28$
November	16	$(30 + 28 + 18)/3 = 25\,{}^{1}/_{3}$
December	14	$(28 + 18 + 16)/3 = 20\,{}^{2}/_{3}$

Types of forecasting methods

Quantitative Methods

Ex: 4

You're manager in Amazon's electronics department. You want to forecast ipod sales for months 4-6 using a 3-period moving average.

Month	Sales (000)
1	4
2	6
3	5
4	?
5	?
6	?

$$F_{t+1} = \frac{A_t + A_{t-1} + A_{t-2} + \ldots + A_{t-n+1}}{n}$$

Types of forecasting methods

Quantitative Methods

Month	Sales (000)	Moving Average (n=3)
1	4	NA
2	6	NA
3	5	NA
4	?	(4+6+5)/3=5
5	?	
6	?	

$$F_{t+1} = \frac{A_t + A_{t-1} + A_{t-2} + \ldots + A_{t-n+1}}{n}$$

Types of forecasting methods

Quantitative Methods

What if ipod sales were actually 3 in month 4?

Month	Sales (000)	Moving Average (n=3)
1	4	NA
2	6	NA
3	5	NA
4	? 3	5
5	?	
6	?	

Types of forecasting methods

Quantitative Methods

Forecast for Month 5?

Month	Sales (000)	Moving Average (n=3)
1	4	NA
2	6	NA
3	5	NA
4	3	5
5	?	(6+5+3)/3=4.667
6	?	

Types of forecasting methods

Quantitative Methods

2b. Weighted Moving Average

Simple moving average models weight all previous periods equally

Weighted Moving Average Gives more emphasis to recent data

The formula for the moving average is:

$$F_{t+1} = w_1 A_t + w_2 A_{t-1} + w_3 A_{t-2} + \ldots + w_n A_{t-n+1}$$

Weights decrease for older data

$$\sum_{i=1}^{n} w_i = 1$$

Types of forecasting methods

Quantitative Methods

Ex: 1

Given the weekly demand and weights, what is the forecast for the 4th period or Week 4?

Week	Demand
1	650
2	678
3	720
4	

Weights:

t-1	.5
t-2	.3
t-3	.2

Note that the weights place more emphasis on the most recent data, that is time period "t-1".

Types of forecasting methods

Quantitative Methods

Solution

Week	Demand	Forecast
1	650	
2	678	
3	720	
4		693.4

$F_4 = 0.5(720)+0.3(678)+0.2(650)=693.4$

Types of forecasting methods

Quantitative Methods

Ex: 2

Question: Given the weekly demand information and weights, what is the weighted moving average forecast of the 5th period or week?

Week	Demand
1	820
2	775
3	680
4	655

Weights:

t-1	.7
t-2	.2
t-3	.1

Types of forecasting methods

Quantitative Methods

Solution

Week	Demand	Forecast
1	820	
2	775	
3	680	
4	655	
5		672

$$F_5 = (0.1)(755)+(0.2)(680)+(0.7)(655)= 672$$

Types of forecasting methods

Quantitative Methods

Ex: 3

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	[(3 x 13) + (2 x 12) + (10)]/6 = 12 $^{1}/_{6}$
May		
June		
July		
August		
September		
October		
November		
December		

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago
6	Sum of the weights

Forecast for this month =

$$\frac{3 \times \text{Sales last mo.} + 2 \times \text{Sales 2 mos. ago} + 1 \times \text{Sales 3 mos. ago}}{\text{Sum of the weights}}$$

Weighted Moving Average

MONTH	ACTUAL SHED SALES	3-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	[(3 x 13) + (2 x 12) + (10)]/6 = 12 $^{1}/_{6}$
May	19	[(3 x 16) + (2 x 13) + (12)]/6 = 14 $^{1}/_{3}$
June	23	[(3 x 19) + (2 x 16) + (13)]/6 = 17
July	26	[(3 x 23) + (2 x 19) + (16)]/6 = 20 $^{1}/_{2}$
August	30	[(3 x 26) + (2 x 23) + (19)]/6 = 23 $^{5}/_{6}$
September	28	[(3 x 30) + (2 x 26) + (23)]/6 = 27 $^{1}/_{2}$
October	18	[(3 x 28) + (2 x 30) + (26)]/6 = 28 $^{1}/_{3}$
November	16	[(3 x 18) + (2 x 28) + (30)]/6 = 23 $^{1}/_{3}$
December	14	[(3 x 16) + (2 x 18) + (28)]/6 = 18 $^{2}/_{3}$

Types of forecasting methods

Quantitative Methods

Ex: 4 Weighted Moving Average: 3/6, 2/6, 1/6

Month	Sales (000)	Weighted Moving Average
1	4	NA
2	6	NA
3	5	NA
4	?	31/6 = 5.167
5	?	
6	?	

$$F_{t+1} = w_1 A_t + w_2 A_{t-1} + w_3 A_{t-2} + \ldots + w_n A_{t-n+1}$$

Types of forecasting methods

Quantitative Methods

$$F_{t+1} = w_1 A_t + w_2 A_{t-1} + w_3 A_{t-2} + \ldots + w_n A_{t-n+1}$$

Month	Sales (000)	Weighted Moving Average
1	4	NA
2	6	NA
3	5	NA
4	3	31/6 = 5.167
5	7	25/6 = 4.167
6		32/6 = 5.333

Types of forecasting methods

Quantitative Methods

3a. Exponential Smoothing

- Assumes the most recent observations have the highest predictive value

– gives more weight to recent time periods

$$F_{t+1} = F_t + \alpha\underbrace{(A_t - F_t)}_{e_t}$$

F_{t+1} = Forecast value for time $t+1$

A_t = Actual value at time t

α = Smoothing constant

Need initial forecast F_t to start.

Types of forecasting methods

Quantitative Methods

- Form of weighted moving average
  - Weights decline exponentially
  - Most recent data weighted most
- Requires smoothing constant (α)
  - Ranges from 0 to 1
  - Subjectively chosen
- Involves little record keeping of past data

Types of forecasting methods
Quantitative Methods

Ex: 4

Week	Demand
1	820
2	775
3	680
4	655
5	750
6	802
7	798
8	689
9	775
10	

- Question: Given the weekly demand data, what are the exponential smoothing forecasts for periods 2-10 using a=0.10 and a=0.60?
- Assume $F_1=D_1$

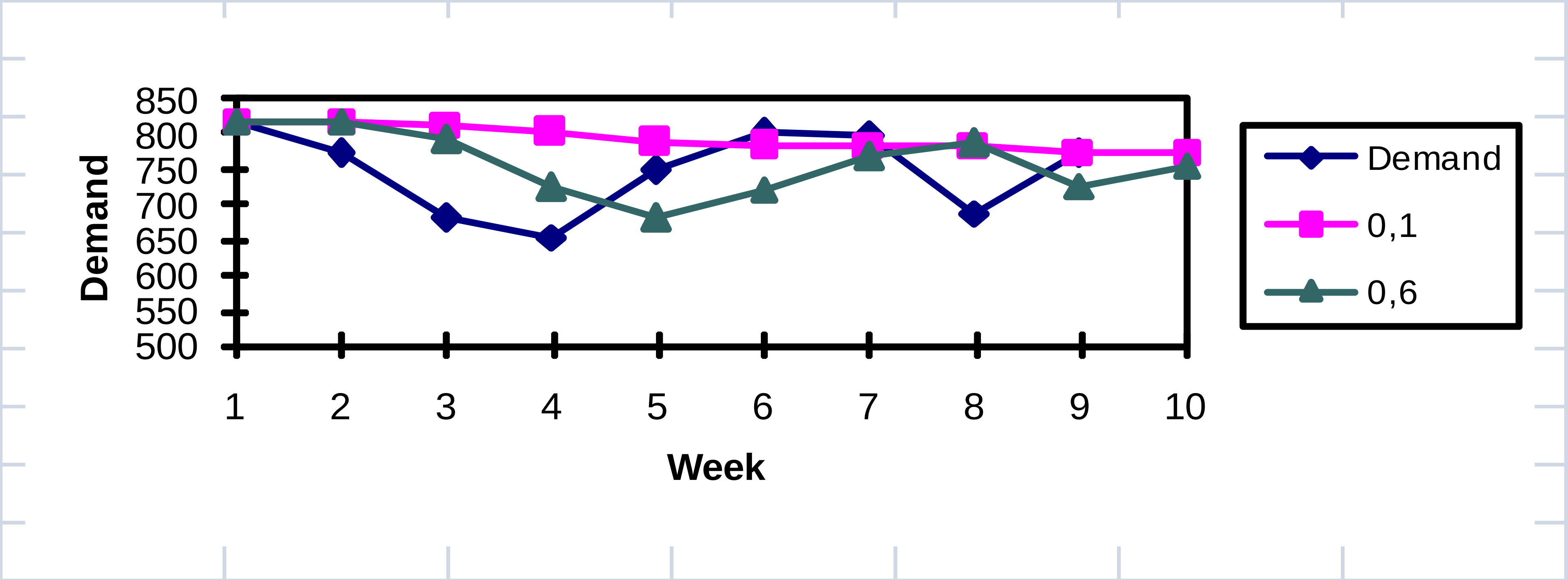
Solution

Week	Demand	*0.1*	*0.6*
1	820	820.00	820.00
2	775	820.00	820.00
3	680	815.50	820.00
4	655	801.95	817.30
5	750	787.26	808.09
6	802	783.53	795.59
7	798	785.38	788.35
8	689	786.64	786.57
9	775	776.88	786.61
10		776.69	780.77

Types of forecasting methods

Quantitative Methods

Exponential Smoothing Problem Plotting



Note how that the smaller alpha the smoother the line in this example.

Types of forecasting methods

Quantitative Methods

Ex: 4

Week	Demand
1	820
2	775
3	680
4	655
5	

Question: What are the exponential smoothing forecasts for periods 2-5 using a =0.5?

Assume $F_1=D_1$

Types of forecasting methods

Quantitative Methods

$F_1=820+(0.5)(820-820)=820$

$F_3=820+(0.5)(775-820)=797.75$

Week	Demand	*0.5*
1	820	820.00
2	775	820.00
3	680	797.50
4	655	738.75
5		696.88

Exponential Smoothing Example

Ex: 4

Predicted demand = 142 Ford Mustangs

Actual demand = 153

Smoothing constant a = .20

New forecast = 142 + .2(153 – 142)

= 142 + 2.2

= 144.2 ≈ 144 cars

Effect of Smoothing Constants

- Smoothing constant generally $.05 \leq \alpha \leq .50$
- As *a* increases, older values become less significant

	WEIGHT ASSIGNED TO				
SMOOTHING CONSTANT	MOST RECENT PERIOD (α)	2ND MOST RECENT PERIOD $\alpha(1-\alpha)$	3RD MOST RECENT PERIOD $\alpha(1-\alpha)^2$	4th MOST RECENT PERIOD $\alpha(1-\alpha)^3$	5th MOST RECENT PERIOD $\alpha(1-\alpha)^4$
$\alpha = .1$	.1	.09	.081	.073	.066
$\alpha = .5$	.5	.25	.125	.063	.031

Ex: 5

$$F_{t+1} = F_t + \alpha(A_t - F_t)$$

i **Ai**

Week	Demand
1	820
2	775
3	680
4	655
5	750
6	802
7	798
8	689
9	775
10	

Given the weekly demand data what are the exponential smoothing forecasts for periods 2-10 using α =0.10?

Assume $F_1=D_1$

Types of forecasting methods

Quantitative Methods

$$F_{t+1} = F_t + \alpha(A_t - F_t)$$

i	Ai	Fi
Week	Demand	α = *0.1*
1	820	820.00
2	775	
3	680	
4	655	
5	750	
6	802	
7	798	
8	689	
9	775	
10		

$F_2 = F_1 + \alpha(A_1 – F_1)$ $=820+.1(820–820)$

$=820$

Types of forecasting methods

Quantitative Methods

$$F_{t+1} = F_t + \alpha(A_t - F_t)$$

i	Ai	Fi
Week	Demand	α = *0.1*
1	820	820.00
2	775	820.00
3	680	
4	655	
5	750	
6	802	
7	798	
8	689	
9	775	
10		

$F_3 = F_2 + \alpha(A_2 – F_2)$ $=820+.1(775–820)$

$=815.5$

Types of forecasting methods

Quantitative Methods

$$F_{t+1} = F_t + \alpha(A_t - F_t)$$

i	Ai	Fi
Week	Demand	α = *0.1*
1	820	820.00
2	775	820.00
3	680	815.50
4	655	
5	750	
6	802	
7	798	
8	689	
9	775	
10		

This process continues through week 10

Types of forecasting methods

Quantitative Methods

$$F_{t+1} = F_t + \alpha(A_t - F_t)$$

i	Ai	Fi	
Week	Demand	α = *0.1*	α = *0.6*
1	820	820.00	820.00
2	775	820.00	820.00
3	680	815.50	793.00
4	655	801.95	725.20
5	750	787.26	683.08
6	802	783.53	723.23
7	798	785.38	770.49
8	689	786.64	787.00
9	775	776.88	728.20
10		776.69	756.28

What if the α constant equals 0.6

Types of forecasting methods

Quantitative Methods

$$F_{t+1} = F_t + \alpha(A_t - F_t)$$

i	Ai	Fi	
Month	Demand	$\alpha =$ *0.3*	$\alpha =$ *0.6*
January	120	100.00	100.00
February	90	106.00	112.00
March	101	101.20	98.80
April	91	101.14	100.12
May	115	98.10	94.65
June	83	103.17	106.86
July		97.12	92.54
August			
September			

What if the α constant equals 0.6

Ex: 6

Company A, a personal computer producer purchases generic parts and assembles them to final product. Even though most of the orders require customization, they have many common components. Thus, managers of Company A need a good forecast of demand so that they can purchase computer parts accordingly to minimize inventory cost while meeting acceptable service level. Demand data for its computers for the past 5 months is given in the following table.

Types of forecasting methods

Quantitative Methods

$$F_{t+1} = F_t + \alpha(A_t - F_t)$$

i	Ai	Fi	
Month	Demand	α = *0.3*	α = *0.5*
January	80	84.00	84.00
February	84	82.80	82.00
March	82	83.16	83.00
April	85	82.81	82.50
May	89	83.47	83.75
June		85.13	86.38
July		??	??

What if the α constant equals 0.5

Types of forecasting methods

Quantitative Methods

4. **Linear Regression Model**

$$\hat{y} = a + bx$$

Fitting a trend line to historical data points to project into the medium to long-range

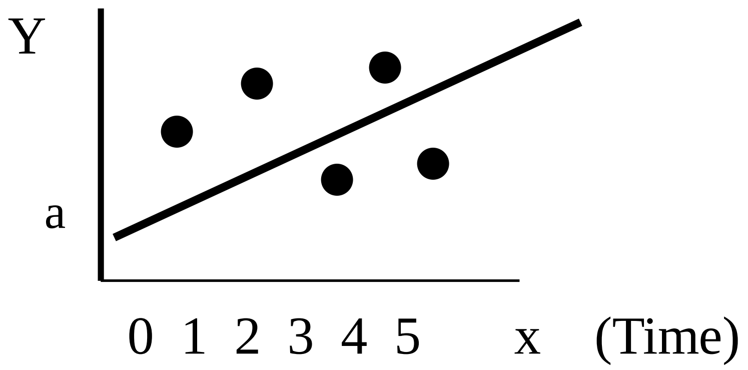
Linear trends can be found using the least squares technique

where y = computed value of the variable to be predicted (dependent variable)

a = y-axis intercept

b = slope of the regression line

x = the independent variable



- **Y** is the regressed forecast value or dependent variable in the model,
- **a** is the intercept value of the the regression line, and
- **b** is similar to the slope of the regression line. However, since it is calculated with the variability of the data in mind, its formulation is not as straight forward as our usual notion of slope.

$$a = \bar{y} - b\bar{x}$$

$$b = \frac{\sum xy - n(\bar{y})(\bar{x})}{\sum x^2 - n(\bar{x})^2}$$

Question: Given the data below, what is the simple linear regression model that can be used to predict sales?

Week	Sales
1	150
2	157
3	162
4	166
5	177

Answer: First, using the linear regression formulas, we can compute “a” and “b”.

Week	Week*Week	Sales	Week*Sales
1	1	150	150
2	4	157	314
3	9	162	486
4	16	166	664
5	25	177	885
3	*55*	*162.4*	*2499*
Average	Sum	Average	Sum

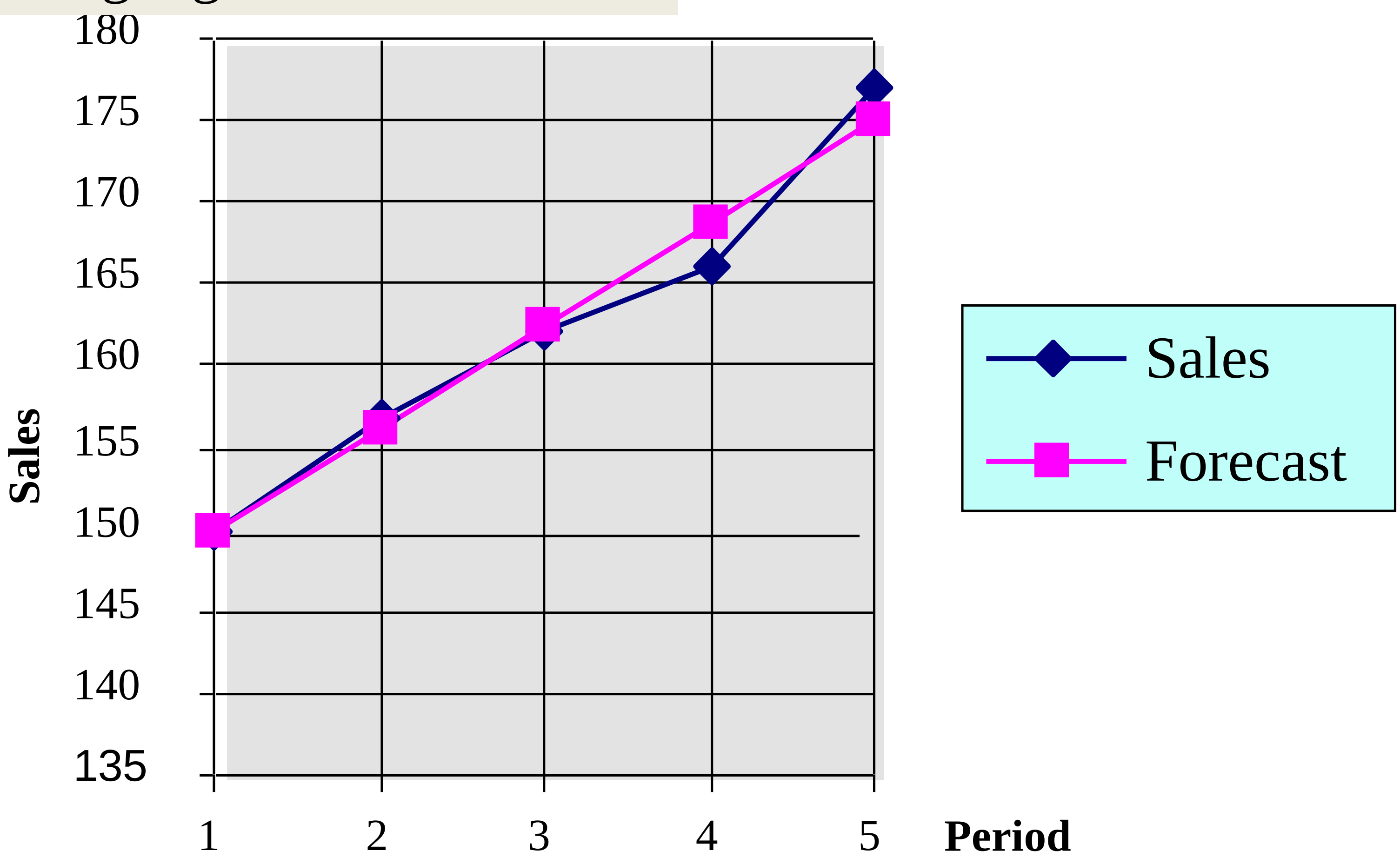
$$b = \frac{\sum xy - n(\bar{y})(\bar{x})}{\sum x^2 - n(\bar{x})^2} = \frac{2499 - 5(162.4)(3)}{55 - 5(9)} = \frac{63}{10} = \mathbf{6.3}$$

$$a = \bar{y} - b\bar{x} = 162.4 - (6.3)(3) = \mathbf{143.5}$$

The resulting regression model is:

$$Y_t = 143.5 + 6.3x$$

Now if we plot the regression generated forecasts against the actual sales we obtain the following chart:



EX 1

YEAR	ELECTRICAL POWER DEMAND	YEAR	ELECTRICAL POWER DEMAND
1	74	5	105
2	79	6	142
3	80	7	122
4	90		

YEAR (x)	ELECTRICAL POWER DEMAND (y)	x^2	xy
1	74	1	74
2	79	4	158
3	80	9	240
4	90	16	360
5	105	25	525
6	142	36	852
7	122	49	854
$\Sigma x = 28$	$\Sigma y = 692$	$\Sigma x^2 = 140$	$\Sigma xy = 3{,}063$

$$\bar{x} = \frac{\sum x}{n} = \frac{28}{7} = 4 \qquad \bar{y} = \frac{\sum y}{n} = \frac{692}{7} = 98.86$$

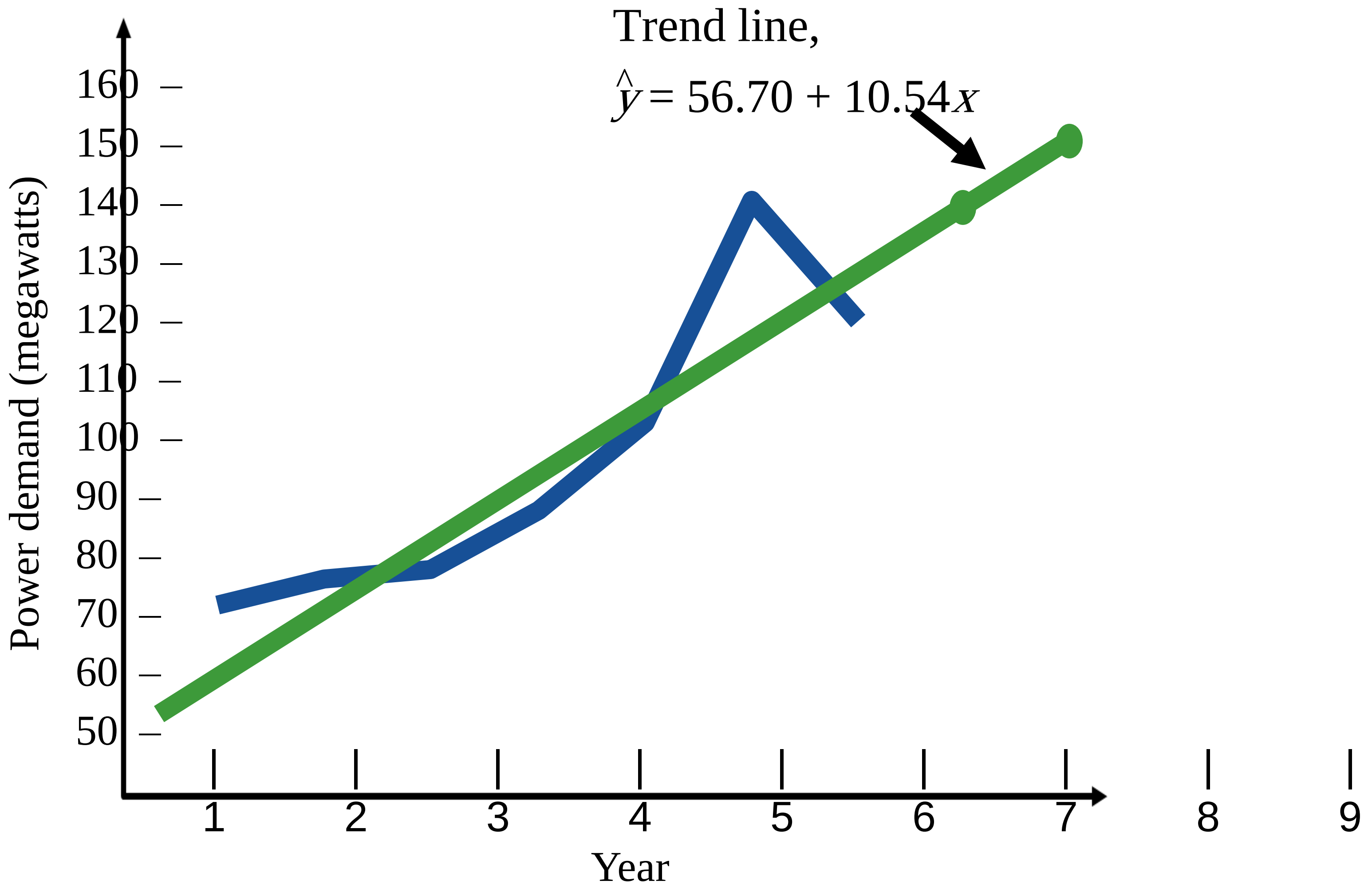
YEAR (x)	ELECTRICAL POWER DEMAND (y)	x^2	xy
7	122	49	854
Σx = 28	Σy = 692	Σx^2 = 140	Σxy = 3,063

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{3,063 - (7)(4)(98.86)}{140 - (7)(4^2)} = \frac{295}{28} = 10.54$$

$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

Thus, $\hat{y} = 56.70 + 10.54x$

Demand in year 8 = 56.70 + 10.54(8)
= 141.02, or 141 megawatts



Trend line,
$\hat{y} = 56.70 + 10.54x$
Power demand (megawatts)
160
150
140
130
120
110
100
90
80
70
60
50
1
2
3
4
5
6
7
8
9
Year

To Use a Forecasting Method

- Collect historical data
- Select a model
  - Moving average methods
    - Select *n* (number of periods)
    - For weighted moving average: select weights
  - Exponential smoothing
    - Select α

- Selections should produce a good forecast

…but what is a good forecast?

A Good Forecast

- Has a small error
  - Error = Demand - Forecast

The objective is to obtain the most accurate forecast no matter the technique

We generally do this by selecting the model that gives us the lowest forecast error

Ideal values =0 (i.e., no forecasting error)

Measures of Forecast Error

a. MAD = Mean Absolute Deviation

$$MAD = \frac{\sum_{t=1}^{n} \overbrace{|A_t - F_t|}^{e_t}}{n}$$

b. MSE = Mean Squared Error

$$MSE = \frac{\sum_{t=1}^{n} (A_t - F_t)^2}{n}$$

c. RMSE = Root Mean Squared Error

$$RMSE = \sqrt{MSE}$$

Common Measures of Error

Given the quarters demand data, what are the exponential smoothing forecasts for periods 2-9 using α =0.10? and α =0.50?
Assume $F_1 = D_1$

QUARTER	ACTUAL TONNAGE UNLOADED
1	180
2	168
3	159
4	175
5	190
6	205
7	180
8	182
9	?

Common Measures of Error

$$F_{t+1} = F_t + a(A_t - F_t)$$

Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum |\text{Actual - Forecast}|}{n}$$

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = .10$	FORECAST WITH $\alpha = .50$
1	180	175	175
2	168	175.50 = 175.00 + .10(180 – 175)	177.50
3	159	174.75 = 175.50 + .10(168 – 175.50)	172.75
4	175	173.18 = 174.75 + .10(159 – 174.75)	165.88
5	190	173.36 = 173.18 + .10(175 – 173.18)	170.44
6	205	175.02 = 173.36 + .10(190 – 173.36)	180.22
7	180	178.02 = 175.02 + .10(205 – 175.02)	192.61
8	182	178.22 = 178.02 + .10(180 – 178.02)	186.30
9	?	178.59 = 178.22 + .10(182 – 178.22)	184.15

Determining the MAD

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = .10$	ABSOLUTE DEVIATION FOR a = .10	FORECAST WITH $\alpha = .50$	ABSOLUTE DEVIATION FOR a = .50
1	180	175	5.00	175	5.00
2	168	175.50	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
Sum of absolute deviations:			82.45		[illegible]
	MAD = $\frac{\Sigma\lvert\text{Deviations}\rvert}{n}$		10.31		12.33

Determining the MSE

Mean Squared Error (MSE)

$$MSE = \frac{\sum_{t=1}^{n}(A_t - F_t)^2}{n}$$

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR $\alpha = .10$	$(ERROR)^2$
1	180	175	$5^2 = 25$
2	168	175.50	$(-7.5)^2 = 56.25$
3	159	174.75	$(-15.75)^2 = 248.06$
4	175	173.18	$(1.82)^2 = 3.31$
5	190	173.36	$(16.64)^2 = 276.89$
6	205	175.02	$(29.98)^2 = 898.80$
7	180	178.02	$(1.98)^2 = 3.92$
8	182	178.22	$(3.78)^2 = 14.29$
			Sum=1526.52

$$MSE = \frac{\sum(\text{Forecast errors})^2}{n} = 1{,}526.52/8 = 190.8$$

Determining the MAPE

Mean Absolute Percent Error

$$\text{MAPE} = \frac{\sum_{i=1}^{n} 100\left|\text{Actual}_i - \text{Forecast}_i\right| / \text{Actual}_i}{n}$$

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST FOR α = .10	ABSOLUTE PERCENT ERROR 100(ERROR/ACTUAL)
1	180	175.00	100(5/180) = 2.78%
2	168	175.50	100(7.5/168) = 4.46%
3	159	174.75	100(15.75/159) = 9.90%
4	175	173.18	100(1.82/175) = 1.05%
5	190	173.36	100(16.64/190) = 8.76%
6	205	175.02	100(29.98/205) = 14.62%
7	180	178.02	100(1.98/180) = 1.10%
8	182	178.22	100(3.78/182) = 2.08%
			Sum of % errors = 44.75%

$$\text{MAPE} = \frac{\sum \text{absolute percent error}}{n} = \frac{44.75\%}{8} = 5.59\%$$

Comparison of Forecast Error

Quarter	Actual Tonnage Unloaded	Rounded Forecast with $\alpha = .10$	Absolute Deviation for $\alpha = .10$	Rounded Forecast with $\alpha = .50$	Absolute Deviation for $\alpha = .50$
1	180	175	5.00	175	5.00
2	168	175.5	7.50	177.50	9.50
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.30
			82.45		98.62

	$\alpha = .10$	$\alpha = .50$
MAD	10.31	12.33
MSE	190.82	195.24
MAPE	5.59%	6.76%