Mohamed Khider University of Biskra

Faculty of FSESNV
Department of SM
University Year 2023/2024

Module: Series and Diff. Eq
Level: 2^{nd} Year LMD
Specialty: Physics

Dirigated Work N°2

(IMPROPER INTEGRALS)

Exercise 1 *Calculate the following integrals:*

$$\int_0^{+\infty} \frac{1}{x^2+4x+9}dx, \quad \int_0^1 \ln x dx, \quad \int_1^{+\infty} \frac{\ln x}{x^2}dx, \quad \int_0^{+\infty} e^{-2x}\sin x dx, \quad \int_0^1 \frac{\ln x}{\sqrt{1-x}}dx,$$

$$\int_0^{+\infty} \frac{\ln x}{\sqrt{x}\,(1-x)^{3/2}}dx, \quad \int_0^{+\infty} \frac{x\ln x}{(1+x^2)^2}dx, \quad \int_0^{+\infty} \frac{\arctan x}{(1+x^2)^{3/2}}dx, \quad \int_0^a \frac{x^2}{\sqrt{a^2-x^2}}dx,$$

Exercise 2 *Study the nature of the following integrals:*

$$\int_1^{+\infty} \frac{dx}{x^\alpha}, \quad \int_0^1 \frac{dx}{x^\alpha}, \quad \int_0^{+\infty} \frac{\sqrt{x}}{(1+x)^\alpha}dx, \quad \int_0^{\pi} \frac{dx}{(1-\cos x)^\alpha},$$

$$\int_1^{+\infty} \frac{\ln x}{x+e^{-x}}dx, \quad \int_0^{+\infty} \frac{e^{\sin x}}{\sqrt{x}}dx. \quad \int_0^{+\infty} \frac{\arctan x}{x^\alpha}dx.$$

Exercise 3 *Using the variable change, calculate the following integrals:*

$$\int_0^{\frac{\pi}{2}} \sqrt{\tan x}dx, \quad \int_0^1 \ln^p\left(\frac{1}{x}\right)dx, \quad \int_0^{+\infty} \cos\left(e^x\right)dx, \quad \int_0^{+\infty} \sin\left(x^2\right)dx,$$

Exercise 4 *Study the absolute convergence and the semi-convergence of the following integrals:*

$$\int_1^{+\infty} \frac{\sin x}{x^2}dx, \quad \int_0^1 \frac{\sqrt{x}\sin\left(\frac{1}{x^2}\right)}{\ln(1+x)}dx, \quad \int_1^{+\infty} \frac{\sin x}{x}dx, \quad \int_0^{+\infty} \frac{\sqrt{x}\sin x}{x+1}dx.$$

Exercise 5 *Determine the set of pairs (α,β) for which the generalized integral is convergent:*

$$\int_1^{+\infty} \frac{dx}{x^\alpha(1+x^\beta)}$$

Charged of courses

Dr. OUAAR, F