Assignment N° 3
Boolean Algebra

Exercise 1: Indicate the laws (axioms and theorems) used in the demonstrations below:

Algebraic transformations	Used laws
$(A \oplus B).B + A.B =$ $(\bar{A}.B + A.\bar{B}).B + A.B$	
$= B.(B.\bar{A} + \bar{B}.A) + A.B$	
$= B.(B.\bar{A}) + B.(\bar{B}.A) + A.B$	
$= (B.B).\bar{A} + (B.\bar{B}).A + A.B$	
$= B.\bar{A} + (B.\bar{B}).A + A.B$	
$= B.\bar{A} + 0.A + A.B$	
$= B.\bar{A} + 0 + A.B$	
$= B.\bar{A} + A.B$	
$= B.(\bar{A} + A)$	
$= B.1$	
$= B$	

Exercise 02:

- Apply the law of Morgan on 3 variables.
- Give the Dual function of the following logical function: $(x + \bar{x}.y) + z = x + y + z$
- Prove that the NOR operator is not associative.

Exercise 03: Consider the following logical function: $f(x, y, z) = xy + y\bar{z}$

- Express this function based only on the NAND operator.
- Express this function based only on the NOR operator.
- Write this function in its canonical disjunctive form.

Exercise 04: Prove the following equalities algebraically:

- $A + \bar{A}B = A + B$
- $A(\bar{A} + B) = A.B$
- $A + AB = A$
- $\overline{A + B + B.C} = \bar{A}.\bar{B}$

Exercise 05 : Give the simplest expressions of the following functions (using the properties of Boolean Algebra):

- $F1 = (x\bar{y} + z).(x + \bar{y}).z$
- $F2 = (x + y + z).(\bar{x} + y + z) + xy + yz$
- $F3 = \bar{a}.b.c + a.c + (a + b).\bar{c}$

Exercise 06 : Show the following expressions using a truth table:

- $A \oplus B = A.\bar{B} + \bar{A}.B = (A + B).(\bar{A} + \bar{B})$
- $\overline{A \oplus B} = A.B + \bar{A}.\bar{B} = (\bar{A} + B).(A + \bar{B})$

Exercise 07 : Give the truth tables of the following functions:

- F1 (a,b) =1 : if « a » is less than or equal to « b » ($a \leq b$)
- F2(x,y,z)= 1 : if the number of variables at 1 is even.
- F3(e_0,e_1,e_2) = 1 : if at least two variables are equal to 0.
- F4(A,B,C) = 1 : if the number $(ABC)_2$ is odd.
- $F5(x, y, z) = xy + x\bar{y} + \overline{yz}$

Exercise 08: Draw the logic diagram of the following functions:

- $(a+b).(a+\bar{b})$
- $\bar{a}.\bar{b}+\overline{a+b+c+d}$
- $\overline{(\overline{x+\bar{y}})+(\overline{x\oplus z})}$

Exercise 09: Give the complements of the following functions expressed in their simplified forms:

- $a(bca+\bar{a}c(b+\bar{c})+abc)bc$
- $\bar{a}\bar{b}\bar{c}+\bar{a}\bar{b}c$
- $\overline{ab+\bar{c}a}+\overline{bc}$
- $\bar{A}B\oplus\bar{A}\bar{B}$

Exercise 10: Let the functions be defined by their truth tables below:

A	B	C	F_1(A,B,C)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

A	B	C	D	F_2(A,B,C,D)
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

1. Find all minterms for which F1=1
2. Give the 1st canonical form (sum of products) of this function
3. Find all maxterms for which F1=0
4. Give the 2nd canonical form (product of sums) of the function F1
5. Simplify the two expressions of F1 using the rules of Boolean algebra
6. Construct the Karnaugh map and determine the simplified expression of F1
7. Draw the logic diagram (circuit) of F1
8. Deduce the complement of F1 from the Karnaugh map
9. Same questions for F2

Exercise 11: Consider the following logical functions:

- *F1(A,B,D,C)=* $\bar{B}DA+\bar{D}\bar{B}C+\bar{A}\bar{B}D+\bar{B}CD+A\bar{B}C$
- *F2(x,y,z,t)=* $\bar{x}\bar{y}t+zt+\bar{x}\bar{y}z+\bar{x}t+x\bar{y}z+\bar{x}\bar{y}\bar{t}$
- *F3(A,B,D,C)=* $AB+\bar{C}\bar{D}+\bar{A}\bar{B}C\bar{D}+\bar{A}BC\bar{D}$
- *F4(A,B,C,D)=* $D(\bar{A}+C)+\bar{B}(\bar{C}+AC)+\bar{B}C+\bar{C}D$

For each of them:

1. Construct the truth table.
2. Simplify the function using the rules of Boolean algebra.
3. Construct the Karnaugh map and determine the simplified expression of the function.
4. Derive the complement of the function from the Karnaugh map.

Exercise 12: Let the logical function F(a,b,c) of three logical variables a, b, c be defined by its logic diagram below

1. Find the logical expression of this function
2. Simplify the logical expression using Karnaugh map
3. Draw the circuit with the minimum number of logic gates