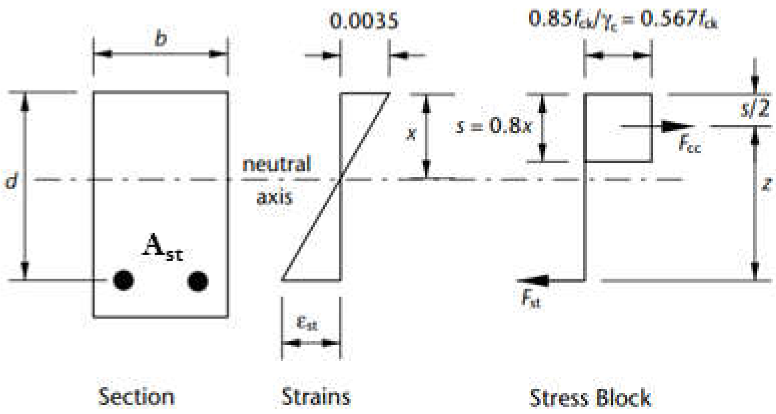
Design and Analysis of Reinforced Concrete (RC) Section of Beam in Bending at Ultimate Limit State (ULS) according to

Eurocode 2 (EN 1992)

1. Singly Reinforced Concrete Rectangular Section



Design:

- Given: dimensions, f_{ck}, f_{yk}, M_{uR}
- Required: A_{st}

Steps:

- Calculate $K = M_{uR} / f_{ck}bd^2$

  $K \leq 0.167$ (compression steel is NOT required)

- Calculate $z = d\left[0.5 + \sqrt{0.25 - \frac{K}{1.134}}\right]$

- Calculate $A_{st} = M_{uR} / 0.87f_{yk}z$

- Chose the number and diameter of bars.

Analysis:

- Given: dimensions, f_{ck}, f_{yk}, A_{st}
- Required: M_{uR}

Steps:

- Assume steel yields ($f_{st} = 0.87\ f_{yk}$) $\longrightarrow$ find s & x:

  Equilibrium: $F_{cc} = F_{st}$

  $\longrightarrow$ $s = (0.87\ f_{yk}\ A_{st} / 0.567\ f_{ck}\ b)$

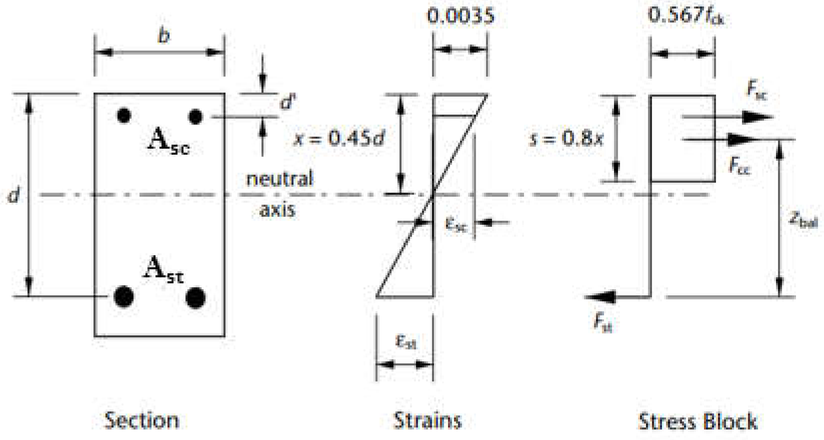
  $\longrightarrow$ $x = s / 0.8$

- Check if steel yields (check if $f_{st} = 0.87\ f_{yk}$)

  $x \leq 0.617\ d$

- If steel yields calculate $M_{uR} = F_{st}\ z = 0.87\ f_{yk}\ A_{st}\ (d - s/2)$

2. Doubly Reinforced Concrete Rectangular Section



<u>Design :</u>

- Given: dimensions, f_{ck}, f_{yk}, M_{uR}
- Required: A_{st}, A_{sc}

Steps:

- Calculate $K = M_{uR} / f_{ck}bd^2$

  $K > 0.167$ (compression steel is required)

- Check if compression steel yields (check if $f_{sc} = 0.87\ f_{yk}$)

  If $d'/d \leq 0.171\ d$

- Calculate A_{st} & A_{sc}:

  Compression steel:

  $A_{sc} = (K - K_{bal})\ f_{ck}bd^2 / 0.87\ f_{yk}\ (d\text{-}d')$ where $K_{bal} = 0.167$

  Tension steel:

  $A_{st} = [\ K_{bal}\ f_{ck}bd^2 / 0.87\ f_{yk}\ z_{bal}\ ] + A_{sc}$ where $z_{bal} = 0.82d$

- Chose the number and diameter of bars.

Analysis:

- Check if compression steel yields (check if $f_{sc} = 0.87\ f_{yk}$)

  If $d'/d \leq 0.171\ d$

- Assume tension steel yields

  Equilibrium: $F_{st} = F_{cc} . F_{sc}$

  $\longrightarrow s = [\ 0.87\ f_{yk}\ (A_{st} - A_{sc})\ /\ 0.567\ f_{ck}\ b\ ]$

  $\longrightarrow x = s\ /\ 0.8$

- Check if tension steel yields (check if $f_{st} = 0.87\ f_{yk}$)

  If $x/d \leq 0.617\ d \longrightarrow f_{st} = 0.87\ f_{yk}$

- If tension steel yields calculate $M_{uR} = F_{cc}\ (d - s/2) + F_{sc}\ (d - d')$

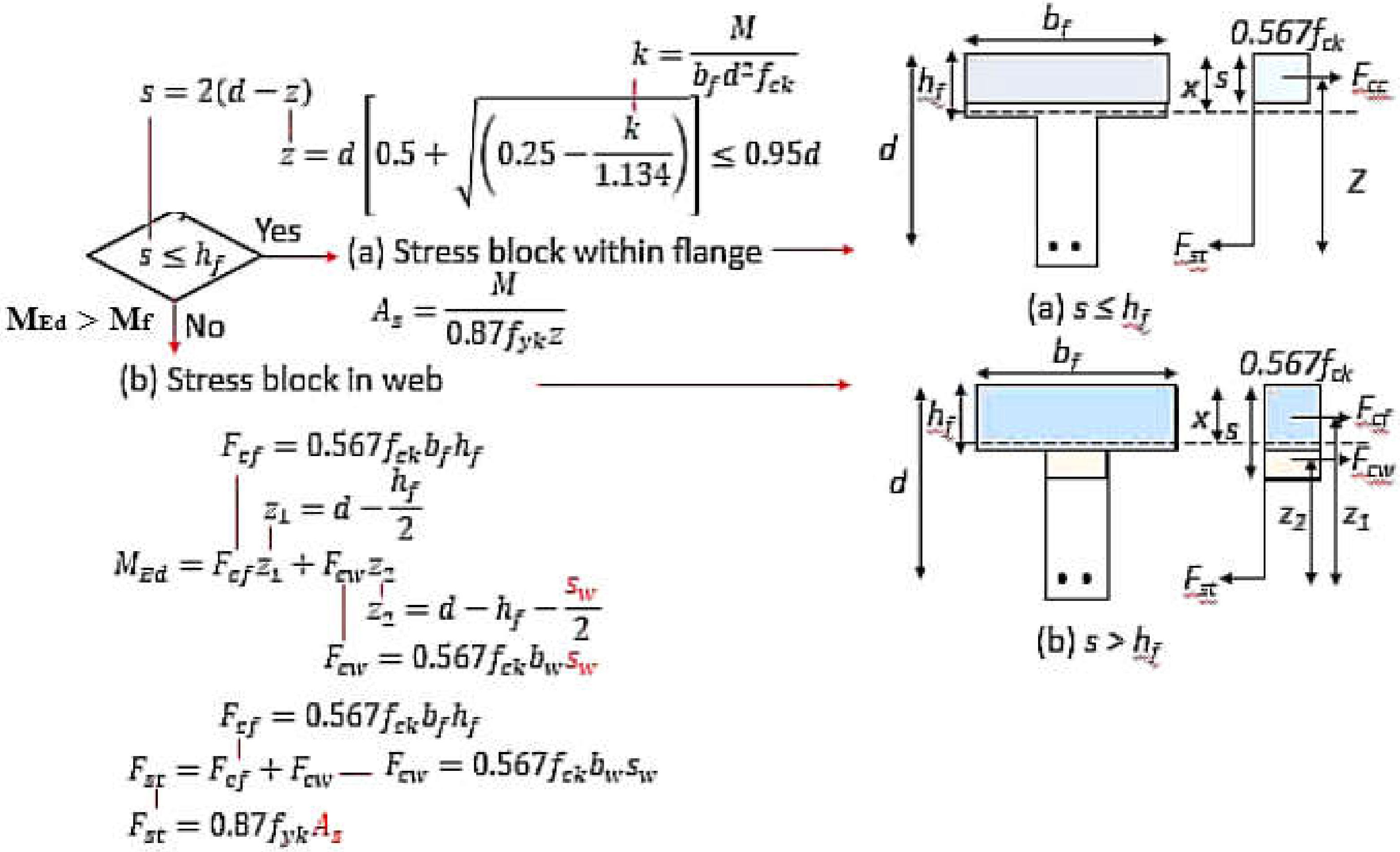
  $$M_{uR} = 0.567\ f_{ck}\ b\ s\ (d - s/2) + 0.87\ f_{yk}\ A_{sc}\ (d - d')$$

Table A.1 Sectional areas of groups of bars (mm^2)

Bar size (mm)	*Number of bars*									
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
6	28.3	56.6	84.9	113	142	170	198	226	255	283
8	50.3	101	151	201	252	302	352	402	453	503
10	78.5	157	236	314	393	471	550	628	707	785
12	113	226	339	452	566	679	792	905	1020	1130
16	201	402	603	804	1010	1210	1410	1610	1810	2010
20	314	628	943	1260	1570	1890	2200	2510	2830	3140
25	491	982	1470	1960	2450	2950	3440	3930	4420	4910
32	804	1610	2410	3220	4020	4830	5630	6430	7240	8040
40	1260	2510	3770	5030	6280	7540	8800	10100	11300	12600

3. Reinforced Concrete Flanged Section

Design:



Analysis:

Case $s \le h_f$:

Steps:

- Assume steel yields ($f_{st} = 0.87\ f_{yk}$) $\longrightarrow$ find s:

  Equilibrium: $F_{cc} = F_{st}$

  $\longrightarrow$ $s = (0.87\ f_{yk}\ A_{st} / 0.567\ f_{ck}\ b_f) \le h_f$ $\longrightarrow$ The stress block does lie within the flange

- Calculate Lever arm z:

  $z = d - s/2$

- Calculate $M_{uR} = F_{cc}\ z = 0.567\ f_{ck}\ b_f\ s\ z$

Case $s > h_f$:

Steps:

- Calculate $F_{cf} = 0.567\ f_{ck}\ b_f\ \ h_f$
- Calculate $F_{st} = 0.87\ f_{yk}\ A_{st}$
- If $F_{st} > F_{cf}$ $\longrightarrow$ $\mathbf{s > h_f}$
- $F_{cw} = 0.567\ f_{ck}\ b_w\ (s - h_f)$

  For equilibrium:

  $F_{cw} = F_{st} - F_{cf}$ $\longrightarrow$ s
- Calculate $M_{uR} = F_{cf}\ z1 + F_{cw}\ z2 = F_{cf}(d - h_f /2) + F_{cw}\ (\ d - h_f /2 - \ s/2)$

References:

Mosley,B. Bunjey, J. Hulse, R. 2012. Reinforced concrete design to Eurocode 2. SEVENTH EDITION. PALGRAVE MACMILLAN

Reinforced concrete design I: https://www.youtube.com/@eng-aim