

Tutorial N°03 (TD N°03)

Of Chapter III

1. Singly Reinforced Concrete Rectangular Section

Exercise N°01:

The ultimate design moment to be resisted by the section in the bellow figure is 185 kNm. Determine the area of tension reinforcement (A_{st}) required given the characteristic material strengths are $f_{yk} = 500 \text{ N/mm}^2$ and $f_{ck} = 25 \text{ N/mm}^2$

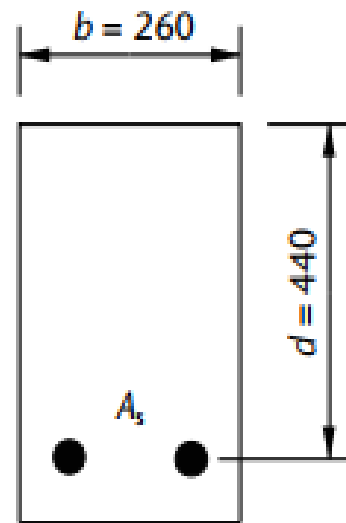


Fig.1

Solution:

Design:

$$\begin{aligned} K &= \frac{M}{bd^2f_{ck}} \\ &= \frac{185 \times 10^6}{260 \times 440^2 \times 25} \\ &= 0.147 < 0.167 \end{aligned}$$

therefore compression steel is not required.

Lever arm:

$$\begin{aligned} z &= d \left\{ 0.5 + \sqrt{\left(0.25 - \frac{K}{1.134} \right)} \right\} \\ &= 440 \left\{ 0.5 + \sqrt{\left(0.25 - \frac{0.147}{1.134} \right)} \right\} \\ &= 373 \text{ mm} \end{aligned}$$

$$\begin{aligned} A_s &= \frac{M}{0.87f_{yk}z} \\ &= \frac{185 \times 10^6}{0.87 \times 500 \times 373} \\ &= 1140 \text{ mm}^2 \end{aligned}$$

Then chose the number and diameter of bar from table A.1

Exercise N°02:

Determine the ultimate moment of resistance of the cross-section shown in the bellow figure given that the characteristic strengths are $f_{yk} = 500 \text{ N/mm}^2$ for the reinforcement and $f_{ck} = 25 \text{ N/mm}^2$ for the concrete.

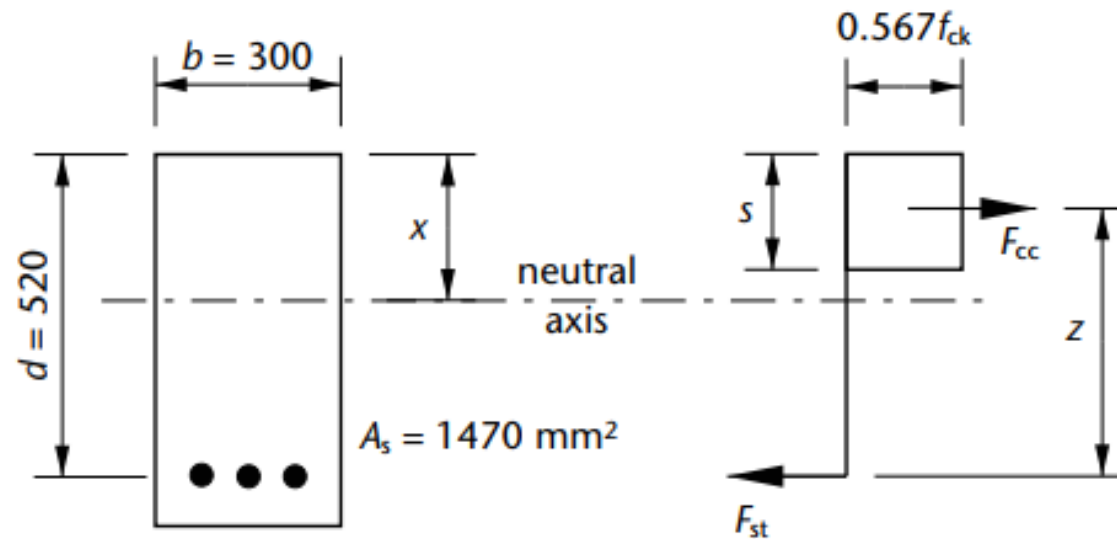


Fig.2

Solution:

Analysis:

For equilibrium of the compressive and tensile forces on the section

$$F_{cc} = F_{st}$$

therefore

$$0.567f_{ck}bs = 0.87f_{yk}A_s$$

$$0.567 \times 25 \times 300 \times s = 0.87 \times 500 \times 1470$$

therefore

$$s = 150 \text{ mm}$$

and

$$\begin{aligned} x &= s/0.8 = 150/0.8 \\ &= 188 \text{ mm} \end{aligned}$$

This value of x is less than the value of $0.617d$, therefore the steel has yielded and $f_{st} = 0.87f_{yk}$ as assumed

Moment of resistance of the section is

$$\begin{aligned} M &= F_{st} \times z \\ &= 0.87f_{yk}A_s(d - s/2) \\ &= 0.87 \times 500 \times 1470(520 - 150/2) \times 10^{-6} = 284 \text{ kNm} \end{aligned}$$

2. Doubly Reinforced Concrete Rectangular Section

Exercise N°03:

The ultimate design moment to be resisted by the section in the bellow figure is 285 kNm. Determine the area of tension reinforcement (A_s) required given the characteristic material strengths are $f_{yk} = 500 \text{ N/mm}^2$ and $f_{ck} = 25 \text{ N/mm}^2$

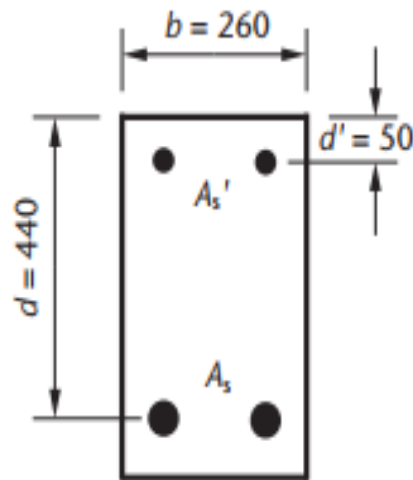


Fig.1

Solution:

Design:

$$\begin{aligned} K &= \frac{M}{bd^2f_{ck}} \\ &= \frac{285 \times 10^6}{260 \times 440^2 \times 25} = 0.226 \\ &> 0.167 \end{aligned}$$

therefore compression steel is required

$$d'/d = 50/440 = 0.11 < 0.171 \quad \text{the compression steel will have yielded}$$

Compression steel:

$$\begin{aligned} A'_s &= \frac{(K - K_{bal})f_{ck}bd^2}{0.87f_{yk}(d - d')} \\ &= \frac{(0.226 - 0.167)25 \times 260 \times 440^2}{0.87 \times 500(440 - 50)} \\ &= 438 \text{ mm}^2 \end{aligned}$$

Tension steel:

$$\begin{aligned} A_s &= \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z_{bal}} + A'_s \\ &= \frac{0.167 \times 25 \times 260 \times 440^2}{0.87 \times 500(0.82 \times 440)} + 438 \\ &= 1339 + 438 \\ &= 1777 \text{ mm}^2 \end{aligned}$$

Then chose the number and diameter of bar from table A.1

Exercise N°04:

Determine the ultimate moment of resistance of the cross-section shown in the bellow figure given that the characteristic strengths are $f_{yk} = 500 \text{ N/mm}^2$ for the reinforcement and $f_{ck} = 25 \text{ N/mm}^2$ for the concrete.

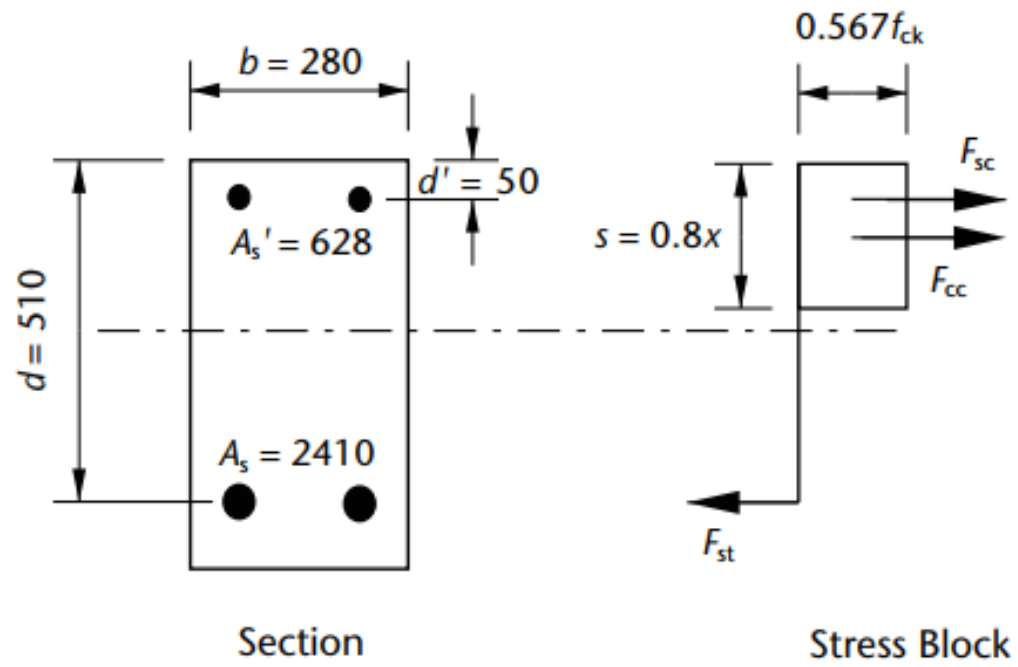


Fig.2

Solution:

Analysis:

$$\begin{aligned}s &= \frac{0.87f_{yk}(A_s - A'_s)}{0.567f_{ck}b} \\ &= \frac{0.87 \times 500(2410 - 628)}{0.567 \times 25 \times 280} \\ &= 195 \text{ mm}\end{aligned}$$

$$x = s/0.8 = 244 \text{ mm}$$

$$x/d = 244/510 = 0.48 < 0.617$$

so the tension steel will have yielded. Also

$$d'/x = 50/225 = 0.22 < 0.38$$

so the compression steel will also have yielded, as assumed.

$$\begin{aligned}M &= F_{cc}(d - s/2) + F_{sc}(d - d') \\ &= 0.567f_{ck}bs(d - s/2) + 0.87f_{yk}A'_s(d - d') \\ &= [0.567 \times 25 \times 280 \times 195(510 - 195/2) + 0.87 \times 500 \times 620(510 - 50)] \times 10^{-6} \\ &= 319 + 124 = 443 \text{ kNm}\end{aligned}$$

3. Reinforced Concrete Flanged Section

3.1. Case: $s \leq h_f$

Exercise N°05:

The ultimate moment of resistance of the T-section shown in the bellow figure is 250 kNm. Calculate the area of reinforcement required (A_{st}). The characteristic material strengths are $f_{yk} = 500 \text{ N/mm}^2$ and $f_{ck} = 25 \text{ N/mm}^2$. Assume the reinforcement is strained to the yield, so that $f_{st} = 0.87 f_{yk}$.

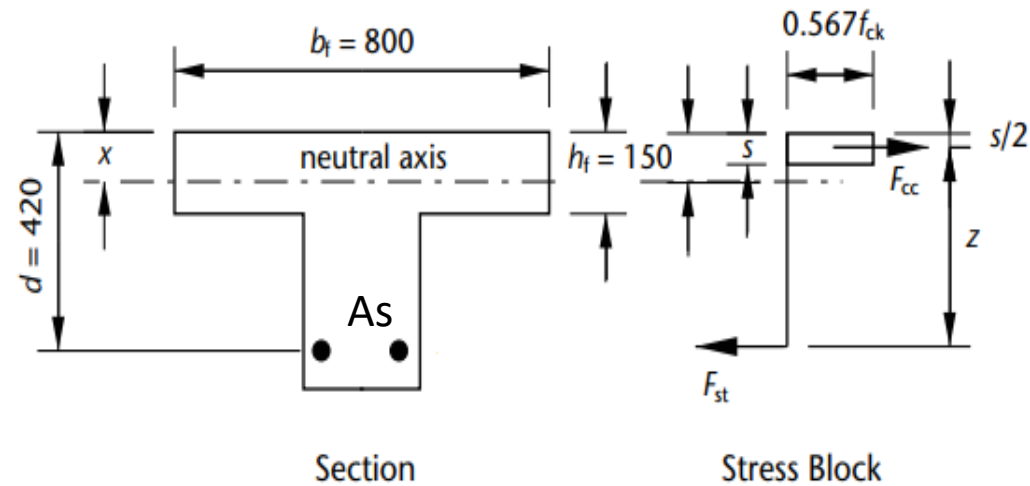


Fig.1

Solution:

Design:

$$k = \frac{M}{b_f d^2 f_{ck}}$$

$$K = 0.071$$

Lever arm:

$$z = d \left\{ 0.5 + \sqrt{\left(0.25 - \frac{K}{1.134} \right)} \right\} \leq 0.95d$$

$$z = 391.312 \text{ mm}$$

$$s = 2(d - z)$$

$$s = 57.376 \text{ mm} < h_f = 150 \text{ mm}$$

s is less than the flange thickness (h_f), the stress block does lie within the flange as assumed and the area of reinforcement is given by

$$A_s = \frac{M}{0.87 f_{yk} z}$$

$$A_s = 1469 \text{ mm}^2$$

Then chose the number and diameter of bar from table A.1

Exercise N°06:

Determine the ultimate moment of resistance of the T-section shown in the bellow figure. The characteristic material strengths are $f_{yk} = 500 \text{ N/mm}^2$ and $f_{ck} = 25 \text{ N/mm}^2$. Assume the reinforcement is strained to the yield, so that $f_{st} = 0.87 f_{yk}$.

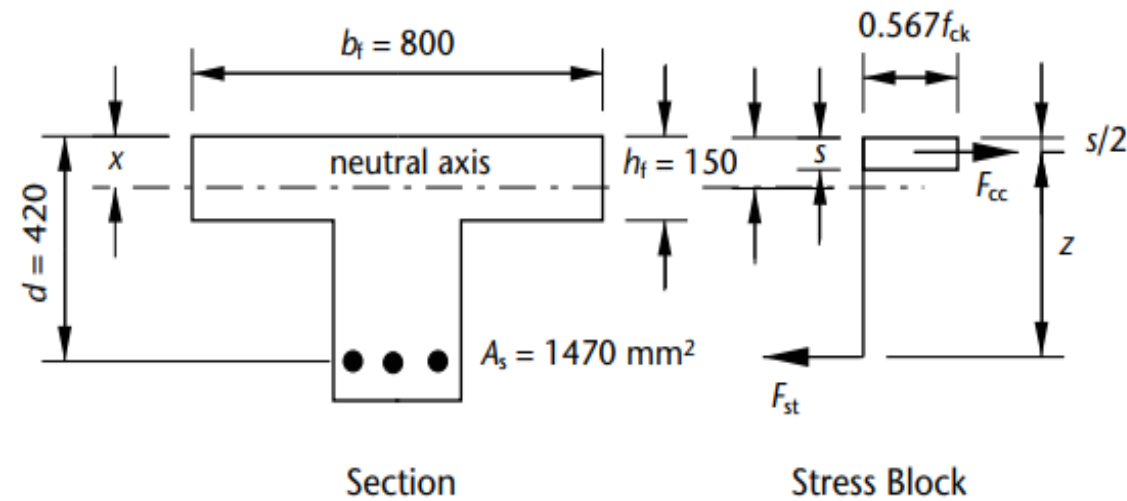


Fig.2

Solution:

Analysis:

$$\begin{aligned}s &= \frac{0.87 \times 500 \times 1470}{0.567 \times 25 \times 800} \\ &= 56 \text{ mm} < h_f = 150 \text{ mm} \\ x &= s/0.8 \\ &= 70 \text{ mm}\end{aligned}$$

Hence the stress block does lie within the flange and with this depth of neutral axis the steel will have yielded as assumed.

Lever arm:

$$\begin{aligned}z &= d - s/2 \\ &= 420 - 56/2 \\ &= 392 \text{ mm}\end{aligned}$$

Taking moments about the centroid of the reinforcement the moment of resistance is

$$\begin{aligned}M &= F_{cc} \times z \\ &= 0.567 f_{ck} b_f s z \\ &= 0.567 \times 25 \times 800 \times 56 \times 392 \times 10^{-6} \\ &= 249 \text{ kNm}\end{aligned}$$

3.2. Case: $s > h_f$

Exercise N°07:

The ultimate moment of resistance of the T-section shown in the bellow figure is 180 kNm. Calculate the area of reinforcement required (A_s). The characteristic material strengths are $f_{yk} = 500 \text{ N/mm}^2$ and $f_{ck} = 25 \text{ N/mm}^2$. Assume the reinforcement is strained to the yield, so that $f_{st} = 0.87 f_{yk}$.

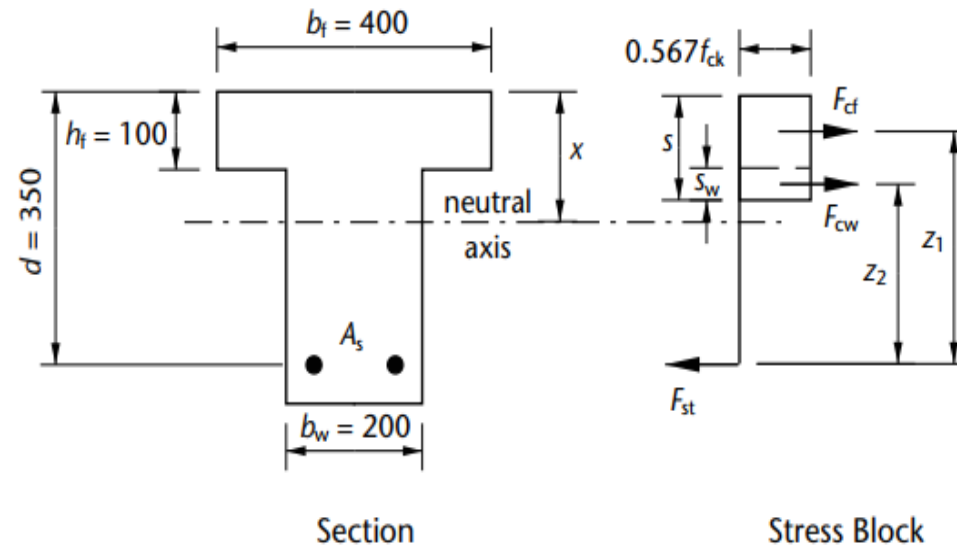


Fig.1

Solution:

Design:

F_{cf} is the force developed in the flange

F_{cw} is the force developed in the area of web in compression

Moment of resistance, M_f , of the flange is

$$M_f = F_{cf} \times z_1$$

or

$$\begin{aligned} M_f &= 0.567 f_{ck} b_f h_f (d - h_f/2) \\ &= 0.567 \times 25 \times 400 \times 100 (350 - 100/2) \times 10^{-6} \\ &= 170 \text{ kN m} < 180 \text{ kN m, the design moment} \end{aligned}$$

Therefore, the stress block must extend below the flange.

It is now necessary to determine the depth, s_w , of the web in compression, where

$$s_w = s - h_f.$$

For equilibrium:

Applied moment

$$\begin{aligned}180 &= F_{cf} \times z_1 + F_{cw} \times z_2 \\ &= 170 + 0.567 f_{ck} b_w s_w \times z_2 \\ &= 170 + 0.567 \times 25 \times 200 s_w (250 - s_w/2) \times 10^{-6} \\ &= 170 + 2835 s_w (250 - s_w/2) \times 10^{-6}\end{aligned}$$

This equation can be rearranged into

$$s_w^2 - 500 s_w + 7.05 \times 10^3 = 0$$

Solving this quadratic equation

$$s_w = 15 \text{ mm}$$

so that the depth of neutral axis

$$\begin{aligned}x &= (h_f + s_w)/0.8 = (100 + 15)/0.8 \\ &= 144 \text{ mm} = 0.41d\end{aligned}$$

As $x < 0.45d$ compression reinforcement is not required.

For the equilibrium of the section

$$F_{st} = F_{cf} + F_{cw}$$

or

$$0.87 f_{yk} A_s = 0.567 f_{ck} b_f h_f + 0.567 f_{ck} b_w s_w$$

$$0.87 \times 500 \times A_s = 0.567 \times 25(400 \times 100 + 200 \times 15) = 610 \times 10^3$$

Therefore

$$A_s = \frac{610 \times 10^3}{0.87 \times 500}$$
$$= 1402 \text{ mm}^2$$

Then chose the number and diameter of bar from table A.1

Exercise N°08:

Determine the ultimate moment of resistance of the T-section shown in the bellow figure. The characteristic material strengths are $f_{yk} = 500 \text{ N/mm}^2$ and $f_{ck} = 25 \text{ N/mm}^2$. Assume the reinforcement is strained to the yield, so that $f_{st} = 0.87 f_{yk}$.

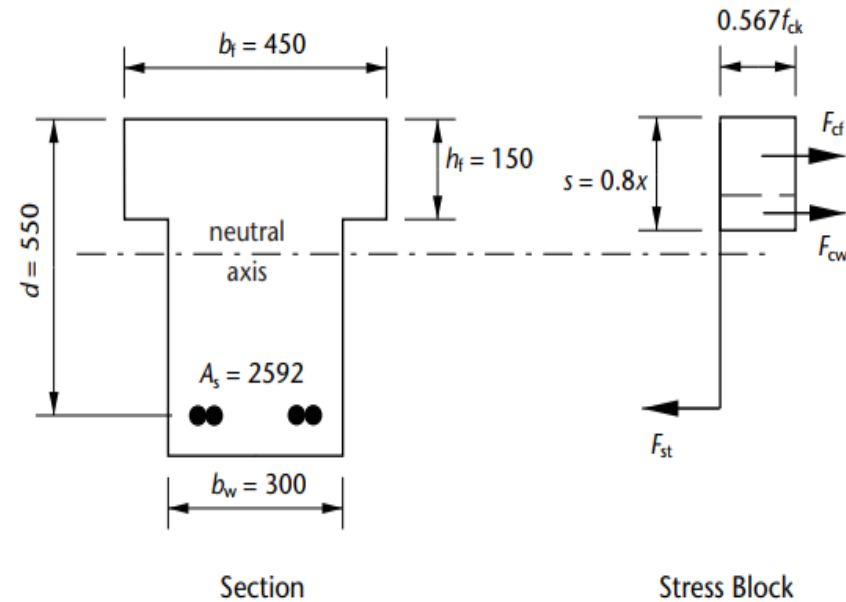


Fig.2

Solution:

Analysis:

The compressive force in the flange is

$$\begin{aligned} F_{cf} &= 0.567 f_{ck} b_f h_f \\ &= 0.567 \times 25 \times 450 \times 150 \times 10^{-3} = 957 \text{ kN} \end{aligned}$$

Then tensile force in the reinforcing steel, assuming it has yielded, is

$$\begin{aligned} F_{st} &= 0.87 f_{yk} A_s \\ &= 0.87 \times 500 \times 2592 \times 10^{-3} = 1128 \text{ kN} \end{aligned}$$

Therefore $F_{st} > F_{cf}$ so that $s > h_f$ and the force in the web is

$$\begin{aligned}F_{cw} &= 0.567 f_{ck} b_w (s - h_f) \\ &= 0.567 \times 25 \times 300 (s - 150) \times 10^{-3} \\ &= 4.25 (s - 150)\end{aligned}$$

For equilibrium

$$F_{cw} = F_{st} - F_{cf}$$

or

$$4.25 (s - 150) = 1128 - 957$$

Hence

$$s = 190 \text{ mm}$$

$$x = s/0.8 = 238 \text{ mm} = 0.43d$$

With this depth of neutral axis the reinforcement has yielded, as assumed, and

$$F_{cw} = 4.25 (190 - 150) = 170 \text{ kN}$$

$$\begin{aligned}M &= F_{cf} (d - h_f/2) + F_{cw} (d - s/2 - h_f/2) \\ &= [957 (550 - 150/2) + 170 (550 - 190/2 - 150/2)] \times 10^{-3} \\ &= 519 \text{ kN m}\end{aligned}$$

Table A.1 Sectional areas of groups of bars (mm²)

<i>Bar size (mm)</i>	<i>Number of bars</i>									
	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>10</i>
6	28.3	56.6	84.9	113	142	170	198	226	255	283
8	50.3	101	151	201	252	302	352	402	453	503
10	78.5	157	236	314	393	471	550	628	707	785
12	113	226	339	452	566	679	792	905	1020	1130
16	201	402	603	804	1010	1210	1410	1610	1810	2010
20	314	628	943	1260	1570	1890	2200	2510	2830	3140
25	491	982	1470	1960	2450	2950	3440	3930	4420	4910
32	804	1610	2410	3220	4020	4830	5630	6430	7240	8040
40	1260	2510	3770	5030	6280	7540	8800	10100	11300	12600

Reference:

Mosley,B. Bunjey, J. Hulse, R. 2012. Reinforced concrete design to Eurocode 2. SEVENTH EDITION. PALGRAVE MACMILLAN