Mohamed Khider University of Biskra

Faculty of FSESNV
Department of SM
University Year 2023/2024

Module: Series and Diff. Eq
Level: 2^{nd} Year LMD
Specialty: Physics

Dirigated Work N°4

(Numerical series, Function sequences and series, Integer series)

Exercise 1 *Study the convergence of the following numerical series:*

$$\sum_{n=1}^{+\infty} \frac{1}{n(n+1)}, \quad \sum_{n=2}^{+\infty} \ln(1+\frac{1}{n}), \quad \sum_{n=0}^{+\infty} \frac{4n+3}{n+1}, \quad \sum_{n=1}^{+\infty} \frac{2^n n!}{n^n}, \quad \sum_{n=2}^{+\infty} \frac{(-1)^n}{\ln(n)},$$

$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n-1)^2}, \quad \sum_{n=1}^{+\infty} \frac{2n}{n+2^n}, \quad \sum_{n=1}^{+\infty} \left(\frac{n}{n+1}\right)^{n^2}, \quad \sum_{n=1}^{+\infty} \frac{2^{n+1}}{3^n},$$

Exercise 2 *Consider the following sequence of functions:*

$$f_n(x) = \frac{\sqrt{n}x}{1+\sqrt{n}x^2}, x \in \left]0, +\infty\right].$$

1) - Study its simple convergence.
2) - Study its uniform convergence on $[b, +\infty[\,, b > 0$.

Exercise 3 *Let the sequence of functions be defined by:*

$$f_n(x) = \frac{e^{-x}}{1+n^2x}, \ n \geq 1.$$

1) - Study its simple convergence and its uniform convergence on $[1, +\infty[$.
2) - Study the simple convergence of the series $\sum_{n\geq 1} f_n(x)$ *on* $[1, +\infty[$.
3) - Show that $\sum_{n\geq 1} f_n(x)$ *converges normally on* $[1, +\infty[$.

Exercise 4 *1) - Determine the domain of convergence of the following integer series:*

$$\sum_{n>1} \frac{(-1)^n}{n^3} x^n.$$

2) - We consider the entiger series $f(x) = \sum_{n>0} n(-2)^n x^n$.

a) - What is the radius of convergence of $f(x)$.
b) - What is the radius of convergence of $g(x) = \sum_{n>0} (-2)^n x^n$.
c) - Deduce that $f(x) = xg'(x)$.

Charged of courses
Dr. OUAAR, F