Tutorial N°1: Exercises on dimensional analysis and error calculation

Exercise 1.1:

Establish from simple formulas, the dimensions and fundamental units of quantities following speed v, acceleration a, force F, surface S, volume V, density p, energy E, pressure P.

Solution

The dimension of the physical quantity <u>speed</u> v is

$$\dim v = \frac{length}{time} = \frac{L}{T} = LT^{-1} \text{ unit } m.s^{-1}$$

The dimension of the physical quantity acceleration a is

$$\dim a = \frac{length}{T^2} = \frac{L}{T^2} = T^{-2}L \text{ unit } m.s^{-2}$$

The dimension of the physical quantity <u>force</u> F is

$$\dim F = mass\ x\ acceleration = mass\ x\ \frac{length}{T^2} = \frac{LM}{T^2} = T^{-2}LM \text{ unit } \mathrm{kg}.m.s^{-2}$$

The dimension of the physical quantity surface S is

$$[S] = [a]^2 = L^2 \text{ unit } \mathrm{m}^2$$

The dimension of the physical quantity energy E is

$$E = \frac{1}{2}mv^2 \quad \dim E = [E] = [1/2]\,[m][v]^2 = ML^2T^{-2}\ unit\ kg.m^2.s^{-2}\ or\ joule$$

The dimension of the physical quantity <u>pressure</u> P is

$$\dim P = \frac{force}{area} = \frac{LMT^{-2}}{L^2} = T^{-2}L^{-1}M \text{ unit } \mathrm{kg}.m^{-1}.s^{-2}$$

Exercise 1.2:

The average value $< \boldsymbol{E} >$ of the total kinetic energy of translation of the molecules of a gas is given by:

$$< \boldsymbol{E} >= \frac{\mathbf{3}}{\mathbf{2}} \boldsymbol{k_B \theta}$$

θ represents the absolute temperature.

What are the dimensions of Boltzmann's constant $\boldsymbol{k_B}$?

Solution

$\boldsymbol{k_B} = \frac{2<E>}{3\theta}$ $[\boldsymbol{k_B}] = \left[\frac{2}{3}\right]\frac{[<E>]}{[\theta]}$, $[\boldsymbol{k_B}] = \boldsymbol{ML^2T^{-2}\theta^{-1}}$ unit $\boldsymbol{kg.m^2.s^{-2}k^{-1}}$ (J/K)

Exercise 1.3:

Experience shows that the force with which a liquid act on a ball immersed in it is proportional to the radius of the ball r as well as its speed v. We write its expression:

$F = 6\pi \mu^x r^y v^z$

where μ is a dimension coefficient : $\mu = ML^{-1}T^{-1}$

1- Find x, y and z

When the speed is a little high, the expression for the force becomes $F = kSv^2$,where k is a constant and S is the area of the great circle.

2- Find the dimension k.

3- Demonstrate that the kinetic energy ($Ec = \frac{1}{2}mv^2$) has the same dimension as a work $\omega = FL$.

Solution

1- $F = 6\pi \mu^x r^y v^z$, $\mu = ML^{-1}T^{-1}$

$$[F] = [\mu]^x[r]^y[v]^z = M^xL^{-x}T^{-x}L^yL^zT^{-z} = M^xL^{-x+y+z}T^{-(x+z)} \qquad (1)$$

On the other hand, we have

$$[F] = [ma] = MLT^{-2} \qquad (2)$$

$$(1) = (2) \begin{cases} x = 1 \\ -x + y + z = 1 \\ -(x+z) = -2 \end{cases}$$

$$x = 1, y = 1, z = 1$$

So $F = 6\pi \mu r v$

2- Dimension of $k = \frac{F}{Sv^2}$ $[k] = \frac{[F]}{[S][v]^2} = \frac{MLT^{-2}}{L^2L^2T^{-2}} = ML^{-3}\ unit\ kgm^{-3}$

3- $E_c = \frac{1}{2}mv^2 \rightarrow [E_c] = \left[\frac{1}{2}\right][m][v]^2 = ML^2T^{-2}$ (1)

The work $\boldsymbol{\omega = FL}$ $\rightarrow$ $[\omega] = [F][L] = MLT^{-2}L = ML^2T^{-2}$ (2)

$(1) = (2) \Rightarrow [E_c] = [\omega]$

$(Kg.m^2.s^{-2})$

Exercise 1.4:

The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. L is about 10 cm and is known to 1 mm accuracy. The period of oscillations is about 0.634 second. The time of 100 oscillations is measured with a wristwatch of 1s resolution. What is the accuracy in the determination of g?

Solution

The accuracy in determination of g is found in terms of minimum percentage error in calculation. The percentage error in g = $\frac{\Delta g}{g} x\ 100\%$, where $\frac{\Delta g}{g}$ the relative error in determination of g.

$T = 2\pi\sqrt{\frac{L}{g}}\ or\ T^2 = 4\pi^2\frac{L}{g}\ or\ g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2\ 0.1}{(0.634)^2} = 9.81 m/s^2;$

Now, $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2 \times \frac{\Delta T}{T}$

In terms of percentage, $100 \times \frac{\Delta L}{L} = 100 \times \frac{0.1}{10} = 1\%$

Percentage error in $T\ is\ 100 \times \frac{\Delta T}{T} = 100 \times \frac{1}{100 \times 0.634} = 1.57\%$

Thus, percentage error in g = $\frac{\Delta g}{g} \times 100\% = 1\% + 2 \times 1.57\% = 4.14\%$

$g = (g \mp \Delta g)\ m/s^2$

$g = (9.81 \mp 0.0414)\ m/s^2$

Exercise 1.5:

The error in measuring the radius of the sphere is 0.5%. What is the permissible percentage error in the measurement of its (a) surface area and (b) volume?

Solution

Percentage error in determination of any quantity = Relative error in determination of quantity ×100%. The relative error in area and volume of sphere are:
$\frac{\Delta A}{A} = \frac{2\Delta r}{r}$ *and* $\frac{\Delta V}{V} = \frac{3\Delta r}{r}$ respectively.

Given $\frac{\Delta r}{r} = 0.5\%$

(a) The surface area of a sphere of radius r is $A = 4\pi r^2$

Percentage error in $A = \frac{\Delta A}{A} \times 100 = \frac{2\Delta r}{r} \times 100 = 2 \times 0.5\% = 1\%$

(b) The volume of a sphere with radius r is $V = \frac{4\pi}{3} r^3$

Percentage error in $V = \frac{\Delta V}{V} \times 100 = \frac{3\Delta r}{r} \times 100 = 3 \times 0.5\% = 1.5\%$