Mohamed Khider University of Biskra
Faculty of Exact Sciences and Natural and Life Sciences
Department of Mathematics / Computer science
Course : Machine Structure 2
Level / Year: L1 (2023/2024)

Assignment N° 1

Combinational Circuits

Exercise 01: Three switches S1, S2, and S3 control the starting of two motors M1 and M2 according to the following conditions:

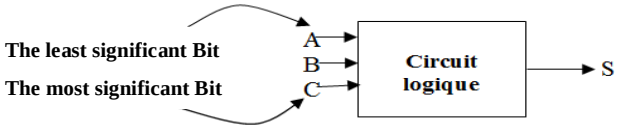
- motor M1 should only start if at least two switches are closed ($S_i = 1$),
- as soon as one or more switches are activated ($S_i = 1$), motor M2 must start.

Design the logic circuit that implements M1 and M2 functions with NAND operators (logic gates).

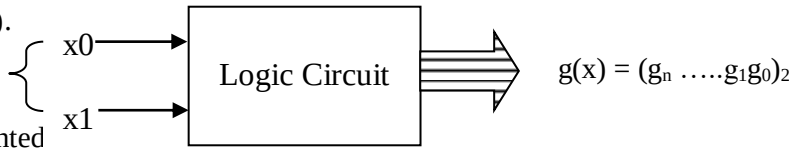
Exercise 02: Consider the following figure: Output S is set to 1 if:

- The number $(CBA)_2$ is odd (عدد فردي) ("0" is considered an odd number), or
- The number $(CBA)_2 = 4$

1. Draw the corresponding truth table?
2. Simplify this expression using the Karnaugh map?



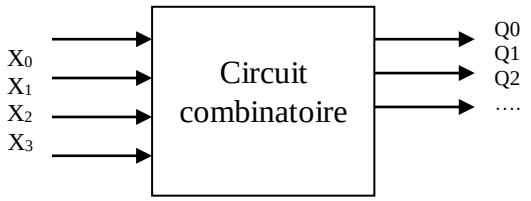
Exercise 03: Consider the function $g(x) = 2x+1$, where x is a natural decimal number of two bits: $x=(x_1x_0)_2$. We want to realize the circuit which determines the function g(x).



1) How many bits, the output function g is represented
2) Express by a truth table each of the outputs of the function g
3) Perform the simplification of each of the outputs using the Karnaugh map
4) Represent the logic circuit using the NOT, AND, and OR logic gates.

Exercise 04: We want to design a combinational circuit which calculates the result (Quotient) of division by 3 of a pure binary number represented on 4 bits ($X_3X_2X_1X_0$). Such as the Quotient: (…..$Q_0Q_1Q_2$).

1) On how many bits is represented the quotient
2) Give the truth table of the output functions of this circuit.
3) Simplify output functions using Karnaugh maps
4) Draw the simplified logic diagram of this circuit using the logic gates (and, or, not).



Exercise 05: Four Managers of a company (A, B, C, D) can have access to a digital safe (strongbox) which is equipped with an electric lock. Each of the managers has a different key (a, b, c, d) and it has been agreed that:

- "A" can only open the safe box if at least "B" or "C" is present.
- "B", "C", "D" can only open it if at least two of the other managers are present.

1. Draw the truth table corresponding to the opening of the lock (output S) according to the keys (inputs: a, b, c and d).
2. Using the Karnaugh map, simplify the output S according to the inputs: a, b, c and d.

Exercise 06: Each of the three alarms a, b, c is activated when it becomes 1.

- Lamp L turns on when only one alarm is activated (minor alarm!).
- Bell B sounds when at least two alarms are activated (major alarm!)

For this, we will consider two cases:

1) L and B must not be activated at the same time.
2) L is "on" (and stays on) when the bell is ringing.

According to the previous cases "1" and "2" :

a) Give the truth table.
b) Simplify the output expressions, i.e., L and B.

Exercise 07: If we add two signed numbers using our n-bit adder circuit, then the result might give an error in the following sense: it could happen that we add two positive numbers and the result is a negative number, or we add two negative numbers and the result is a positive number, or we subtract a positive number from a negative number and the result is positive, or we subtract a negative number from a positive number and the result is negative. In all these cases, the error occurs because the adder defines addition on a circle, not on the (infinite) set of integers.

For example: suppose n = 8 and consider adding 64 + 64, i.e. $64 = 01000000_2$ in binary. The result we wish is 128; however, if the result (10000000_2) is treated as signed then it is interpreted as -128 (not 128). Such errors are called overflow. We would like to detect such errors automatically. Define a binary variable overflow which takes the value 1 when an overflow (error) occurs and 0 when no overflow occurs (no error). This overflow variable depends on the four variables Binvert, A_{n-1}, B_{n-1} and S_{n-1} where B_{invert} says whether we are doing addition or subtraction: 0 or 1 respectively.

1. Construct the truth table, showing how overflow depends on the four variables B_{invert}, A_{n-1}, B_{n-1} and S_{n-1} ?
2. Using Karnaugh map, find the expression of the overflow?

Exercise 08:

- Design a circuit that adds two 2-bits natural numbers, A(a_2a_1) and B(b_2b_1), providing a 3-bits output ($c_3c_2c_1$).
- Design a circuit that compares two 2-bits natural numbers A(a_2a_1) and B(b_2b_1) providing a 2-bits output (S_2S_1) such that:
  - $S_2S_1 = 01$ if A < B
  - $S_2S_1 = 10$ if A > B
  - $S_2S_1 = 00$ if A = B

Exercise 09: A bridge can support as a maximum 7 tons, so we have to monitor the weight of vehicles on both extremities A and B (edges of the bridge). To measure the respective weight of these vehicles, we have two scales “a” and “b”. It is assumed that each vehicle weighs less than 7 tons.

The operation is then as follows:

- If only one vehicle arrives, the barrier (A or B) opens;
- If a+b <= 7 tons, barriers A and B open;
- If a+b > 7 tons, the barrier corresponding to the lightest vehicle will open,
- If a=b barrier A opens first,

a and b are not binary variables, it is necessary to create 2 binary variables x and y, and to reformulate the statement of the problem.

- Find the equations of A and B, as a function of x and y, and give the diagram using AND, OR, NOT gates.

Exercise 10:

A hot drink dispenser (distributor) can dispense (distribute) coffee or tea, with or without milk, or milk alone. Three buttons are used to control the dispenser: "coffee", "tea", "milk". To get one of these drinks alone, just press the corresponding button. To obtain a drink with milk, press the button corresponding to the chosen drink and the "milk" button at the same time.

In addition, the distributor only works if a token has previously been inserted into the slot of the device. A wrong move after inserting the token (for example, pressing simultaneously on “coffee” and “tea”) causes the token to be returned. Since milk is free, the token is also returned if milk alone is chosen.

- Calculate and simplify the functions of returning the token (K), for distributing coffee (C), tea (T), and milk (M). It should be noted that the function of returning the token can be either active or not when no token is introduced into the device.