Optical systems

An optical system is a set of surfaces which reflect (mirrors) or refract (diopter) light rays. A centered system have a symmetrical axis. Systems dotted with diopters only are called dioptrics (lenses, spectacles, microscopes). Systems composed of diopters and mirrors are said to be catadioptric (telescopes)

Stigmatic image of an luminous point in an optical system

Let us consider a point A in a first space called "object space". From A let us run a set of luminous rays going through the system. If all these rays converge to the same point A' of the image space, we can write :

- ✓ A' is the image of A through the system. It is also said that A' is the conjugate of A
- ✓ The system is said to be stigmatic for AA' conjugation

We demonstrate that stigmatism implies a constant value for the optical path (AA')

The case of a **real image** :

The image may be observed on a screen in the image space.

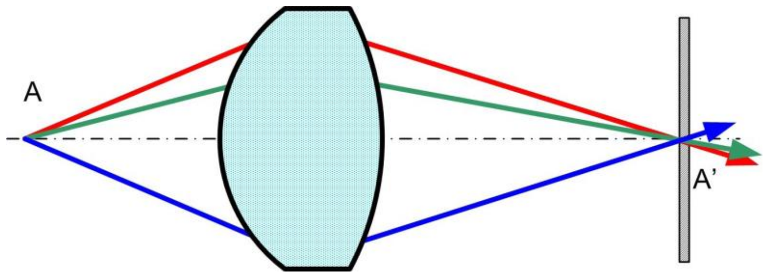

Figure1

The case of a **virtual image** :

The image cannot be observed on a screen. It is nevertheless visible by an observer situated in the image space.

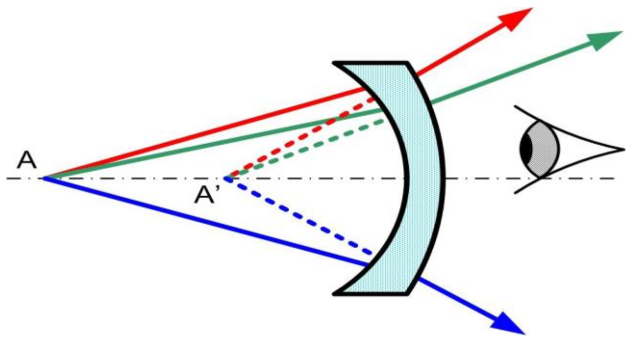

Figure2

Diopters planes, conjugate formula

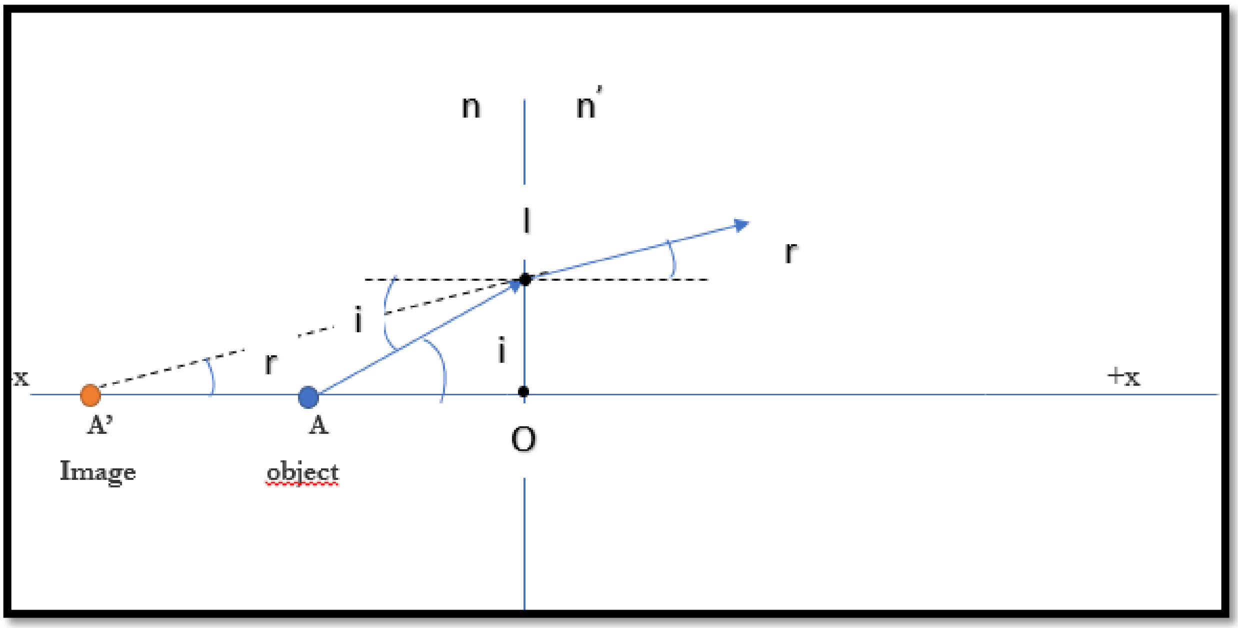

Figure3

$$\overline{OI} = \overline{OA}.\tan i = \overline{OA'}.\tan r \Longrightarrow \overline{OA'} = \overline{OA}.\frac{\tan i}{\tan r} = \overline{OA}.\frac{\sin i}{\cos i}.\frac{\cos r}{\sin r}$$

$$n\sin i = n'\sin r \Rightarrow \sin i = \frac{n'}{n}.\sin r \Rightarrow \sin r = \frac{n}{n'}\sin i$$

$$\Rightarrow \begin{cases} \sin^2 r + \cos^2 r = 1, \quad \cos r = \sqrt{1 - \sin^2 r} \\ \cos r = \sqrt{1 - \left(\frac{n}{n'}\right)^2 \sin^2 r} \\ \sin^2 i + \cos^2 i = 1, \quad \cos i = \sqrt{1 - \sin^2 i} \end{cases}$$

$$\overline{OA'} = \overline{OA}.\frac{n'}{n}.\sqrt{\frac{1-\left(\frac{n}{n'}\right)^2 \sin^2 r}{1-\sin^2 i}}$$

$$\boldsymbol{\frac{\overline{OA'}}{n'} = \frac{\overline{OA}}{n}}$$

Parallel plates

The parallel plate is composed of two distant dioptric planes of e, n is the medium index. An object point A has, as an image, a point A' situated on the perpendicular led from A to the plates sides. We show that :

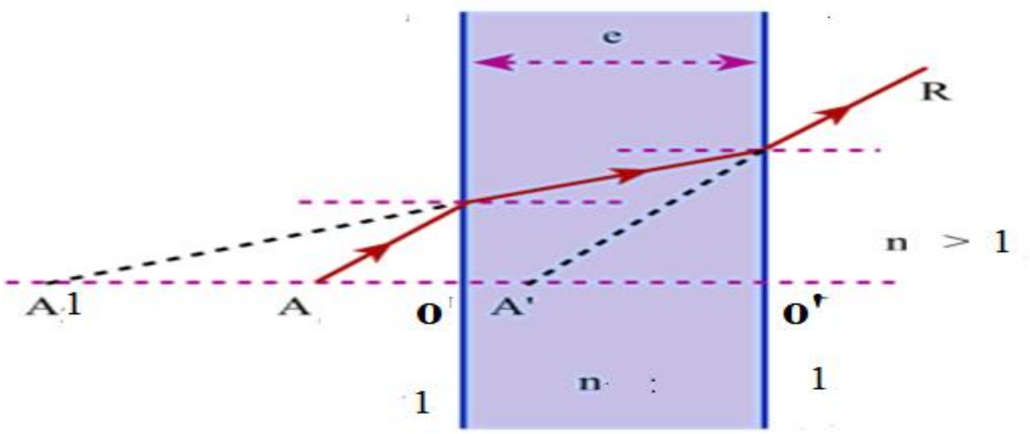

Figure4

The shift of the image relative to the object is $AA' = e\left(1 - \frac{1}{n}\right)$

Démonstration

Regarding diopter D1 :

Air	dioptre D1	Milieu 1
n=1	→	n
A objet	$\frac{\overline{OA_1}}{n} = \frac{\overline{OA}}{1}$	A1 l'image à partir d'un diopter

Regarding diopter D2 :

Milieu 1	dioptre D2	Air
n	→	1
A1 objet	$\frac{\overline{O'A'}}{1} = \frac{\overline{O'A_1}}{n}$	A' l'image à partir d'un diopter D2

$$\overline{AA'} = \overline{AO} + \overline{OO'} + \overline{O'A'}$$

$$= \frac{\overline{A_1O}}{n} + e + \frac{\overline{O'A_1}}{n} = e + \frac{\overline{O'A_1} + \overline{A_1O}}{n} = e + \frac{\overline{O'O}}{n} = e - \frac{e}{n}$$

$$AA' = e\left(1 - \frac{1}{n}\right)$$

Calculate d (shift between the first and final ray

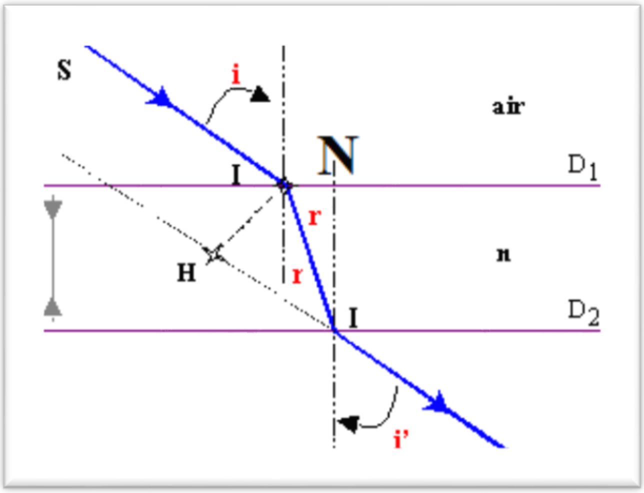

Figure5

$$\begin{cases} \sin(i-r) = \frac{\overline{IH}}{\overline{II'}} = \frac{d}{\overline{II'}} \\ \cos r = \frac{\overline{IN}}{\overline{II'}} = \frac{e}{\overline{II'}} \end{cases}$$

$$d = e.\frac{\sin(i-r)}{\cos r}$$

https://www.youtube.com/watch?v=Fu4H8SE1-y4

Prisms

A prism of index n is composed of two dioptric planes forming an angle A. Following figure 6 a luminous ray enters from side 1 under incidence i and comes out of side 2 under incidence i', the corresponding refraction angles in the prism are r and r', D is the deviation from the ray provoqued by the prism. The angular sign convention is normal for side 1 and inversed for side 2.

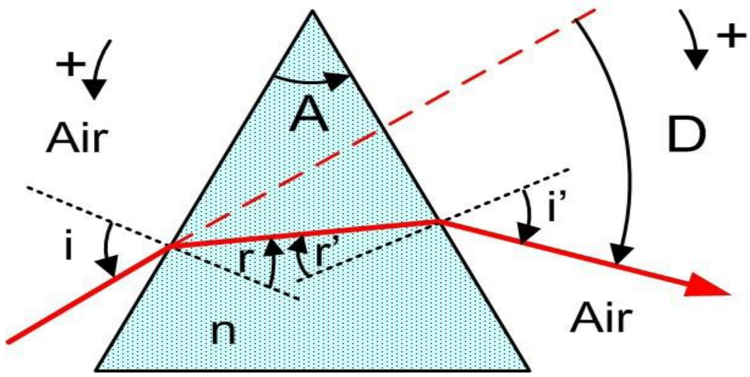

Figure6

$$\begin{cases} \sin i = n \sin r \\ n \sin r' = \sin i' \\ A = r + r' \\ D = i + i' - A \end{cases}$$

At the minimum of deviation: i = i' et r = r', we obtain a relationship between n, A and D, allowing index measures of optical material:

$$\begin{cases} \sin i = n \sin r \\ n \sin r' = \sin i' \\ A = 2r \\ D = 2i \end{cases}$$

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Spherical Diopter

A spherical diopter is a portion of refractive spherical surface separating two homogeneous and transparent media of different indices. It is characterized by:

- The center C of the sphere called the diopter center

- The point S called the top of the diopter.

- The optical axis, the axis of symmetry of revolution of the diopter, passing through points C and S.

- The radius of the sphere $R = \overline{SC}$, called the radius of curvature, an algebraic quantity which is negative for a concave spherical diopter $\overline{SC} < 0$ and positive for a convex spherical diopter $\overline{SC} > 0$.

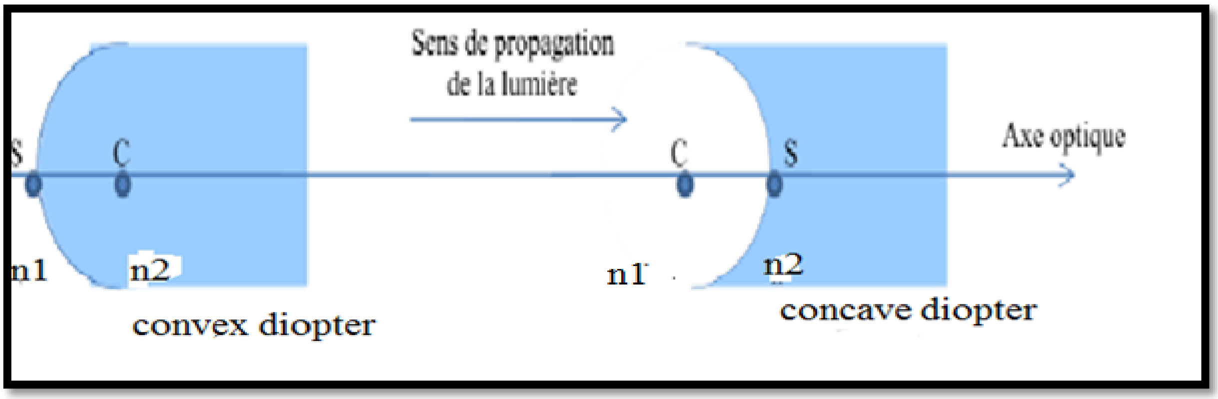

Figure7

Note: in geometric optics, the measurement of distances is algebraized. Along the optical axis, we choose as positive direction the direction of propagation of the light (generally from left to right).

Four possible cases for the spherical diopter:

- $\overline{SC} < 0$ and $n1 < n2 \rightarrow$ Diverging concave spherical diopter.

- $\overline{SC} < 0$ and $n1 > n2 \rightarrow$ Spherical concave convergent diopter.

- $\overline{SC} > 0$ and $n1 < n2 \rightarrow$ Convergent convex spherical diopter.

- $\overline{SC} > 0$ and $n1 > n2 \rightarrow$ Divergent convex spherical diopter.

Conjugation relationships

The image of a luminous point in a diopter

Let us consider a spherical diopter separating two mediums of indexes n and n', defined by its curvature center as C, its vertex as S, its curvature ray $R = \overline{SC}$. All lengths and angles are orientated in accordance with the trigonometry convention A point A is situated on the object space on line SC. The ray arising from A through S is perpendicular to the diopter, it is not deviated. Another ray arising from A going through any point I from the diopter is subject to refraction, the ray arising cuts SC at a point A'.. According to figure ….:

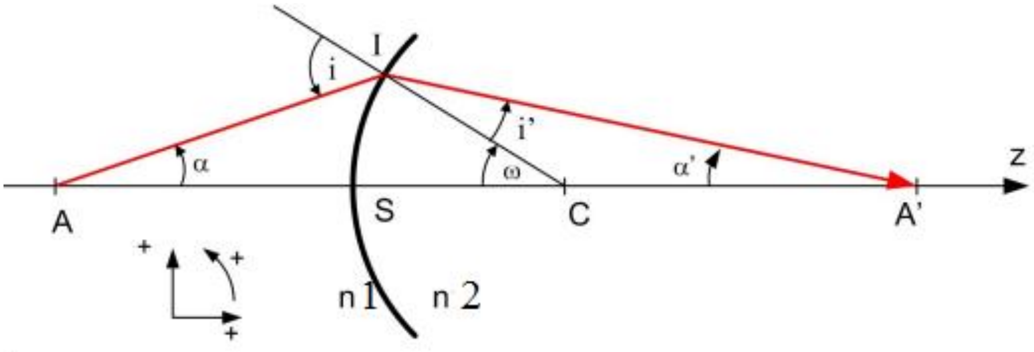

Figure8

i is the incidence angle of the ray on the dioptic.

i' is the refraction angle and, after (5), $n.sin(i) \;=\; n'.sin(i')$.

An usual formula in the triangle CAI gives:

$$\frac{\overline{CA}}{\sin(\pi - i)} = \frac{\overline{CA}}{\sin(i)} = -\frac{\overline{IA}}{\sin(\omega)}$$

$$\overline{CA} < 0, \overline{IA} < 0, \omega < 0, i < 0$$

One can also write :

$$\frac{\overline{CA'}}{\sin(i')} = -\frac{\overline{IA'}}{\sin(\omega)}$$

Therefore :

$$\frac{\overline{CA}}{\overline{CA'}} = \frac{\overline{IA}.\sin(i)}{\overline{IA'}.\sin(i')} = \frac{n}{n'}.\frac{\overline{IA}}{\overline{IA'}}$$

Conjugate stigmatism would mean that A' does not depend on the position of I. It is thus necessary that CA' remains fixed, similarly for the IA/IA' ratio. This is obtained only in a particular position of A and is not achieved in general cases.

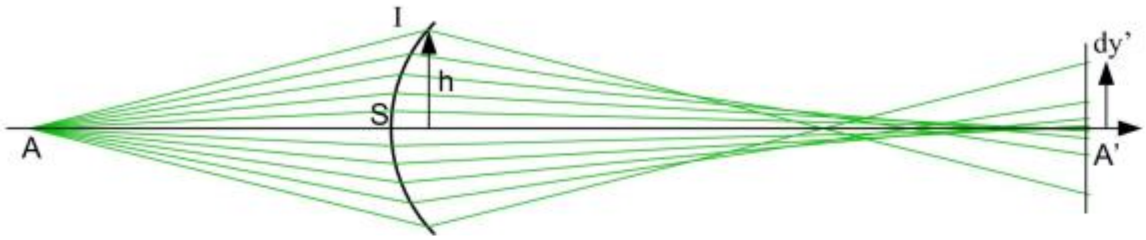

Figure9

Paraxial approximation

Figure 9 shows that the ray tranversal aberration increases with the height of incidence h of I on the diopter. Let us seek the limit A' of the intersection of the refracted ray when h leads to 0. When I leads towards S, the relationship becomes :

$$\frac{\overline{CA}}{\overline{CA'}} = \frac{n_1}{n_2}.\frac{\overline{SA}}{\overline{SA'}}$$

The study of aberrations shows that the distance gap dy' between the refracted ray and A' on a plane going through A' and perpendicular to the axis is approximately proportional to h 3 . For small values of h, dy' is very short, there is an approximate stigmatism. In this case, rays incidences i and i' on the surface of the diopter are low, sinus and radian angles are very close, the relationship becoming:

$$n_1.i \;=\; n_2.i'$$

$$\overline{SC} = R$$

$$\overline{CA} = \overline{SA} - \overline{SC}$$

$$\overline{CA'} = \overline{SA'} - \overline{SC}$$

$$\frac{\overline{SA} - \overline{SC}}{\overline{SA'} - \overline{SC}} = \frac{n_1}{n_2}.\frac{\overline{SA}}{\overline{SA'}}$$

$$\boldsymbol{\frac{n_2}{\overline{SA'}} - \frac{n_1}{\overline{SA}} = \frac{n_2 - n_1}{\overline{SC}} = V}$$

V: vergence or power of the diopter (unit: Diopter = m^{-1}).

- If V > 0: Converging diopter

- If V < 0: Divergent diopter

Focus, focal distance, refracting power

The image focus is the image of the point towards infinity on the axis: $\frac{1}{\overline{SA}} = 0$

$$\boldsymbol{F' = \overline{SF'} = \frac{n_2}{n_2 - n_1}.\overline{SC} = \frac{n_2}{V}}$$

F' is the image focus, SF' = f ' is the image focal distance of the diopter.

<u>The object focus</u> is such that its image is to infinity on the axis:

$$\boldsymbol{F = \overline{SF} = \frac{-n_1}{n_2 - n_1}.\overline{SC} = -\frac{n_1}{V}}$$

F is the object focus SF= f is the object focal distance of the diopter.

We notice that:

$$\frac{\overline{SF'}}{\overline{SF}} = -\frac{n_2}{n_1} < 0$$

$\overline{SF}$ and $\overline{SF'}$ have opposite signs, F and F' belong to two different medium.

And so:

$$\overline{SF} + \overline{SF'} = \overline{SC}$$

<u>Axial magnification</u>

$$\boldsymbol{\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{n_1}{n_2}.\frac{\overline{SA}}{\overline{SA'}}}$$

- If $\gamma > 0$ (+) the image is straight (it has the same direction as the object).
- If $\gamma < 0$ (-) the image is reversed (reverse direction).
- If $|\gamma| > 1$ the image is larger than the object.
- If $|\gamma| < 1$ the image is smaller than the object.
- If $\gamma = 1$ the image and the object have the same size

The characteristics of the image:

- The position of the image: $\overline{SA'}$

- If $\overline{SA'} > 0$ the image is real

- If $\overline{SA'} < 0$ the image is virtual

The nature of the image:

→ Say if it is real or virtual.

→ Say if it is straight or upside down:

→ Say if it is enlarged, reduced or the same size as the object

Geometric construction of the image

- You must place the object AB: real or virtual

- Construct the image B' of point B: simply consider two rays coming from this point:

the incident ray parallel to the optical axis passes through F’

the incident ray which passes through F leaves the diopter parallel to the optical axis

The ray which passes through the center C of the diopter is not deviated

- A’ is the orthogonal projection of B’ on the optical axis.

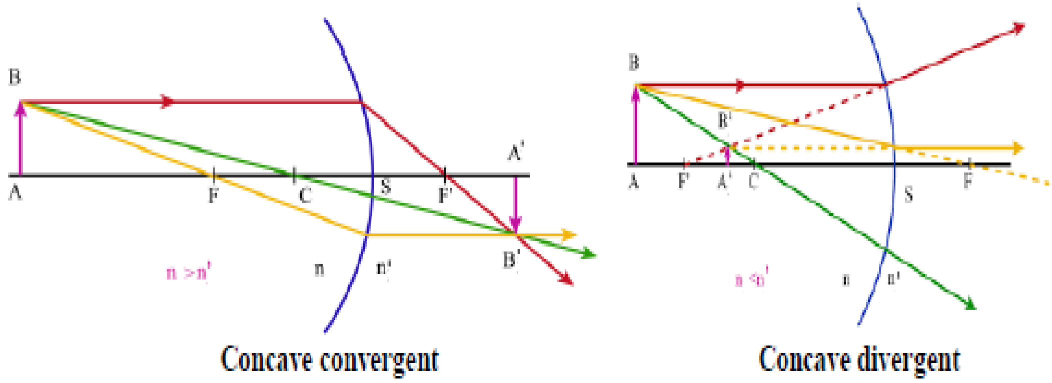

Figure10