Math reminders

I- Cartesian coordinates:

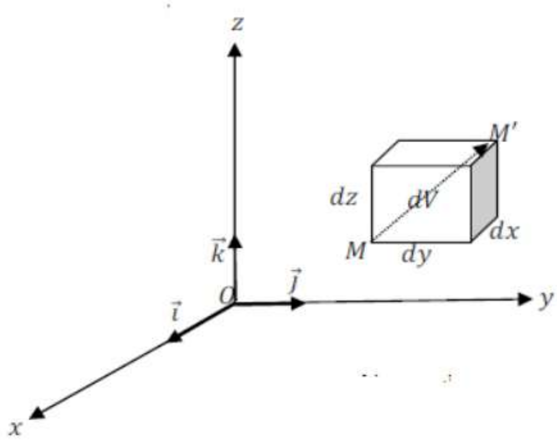
I-1-Vector position and vector displacement

In a Cartesian landmark (Oxyz) with a fixed base$(\vec{\imath}, \vec{\jmath}, \vec{k})$, the position vector $\overrightarrow{OM}$ done by :

$$\overrightarrow{OM} = \vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

$$d\vec{r} = \frac{\partial \vec{r}}{\partial x}.dx + \frac{\partial \vec{r}}{\partial y}dy + \frac{\partial \vec{r}}{\partial z}dz$$

$$= dx\vec{\imath} + dy\vec{\jmath} + dz\vec{k}$$



I-2-Surface element and volume element:

In Cartesian coordinates, the elementary surfaces are (in index the coordinates vary on the surface):

X=Cste dS=dy.dz

Y=Cste dS=dx.dz

Z=Cste dS=dx.dy

I-3-The volume element is

dV=dx.dy.dz

II-The Cylindrical coordinates

II-1-Vector position and vector displacement

In the cylindrical base$(\overrightarrow{U_\rho}; \overrightarrow{U_\varphi}; \vec{k})$ the position vector is written by :

$$\overrightarrow{OM} = \vec{r} = \rho\overrightarrow{U_\rho} + z\vec{k}$$

$$d\overrightarrow{OM} = d\vec{r} = d\rho\overrightarrow{U_\rho} + \rho d\varphi\overrightarrow{U_\varphi} + dz\vec{k}$$

II-2-Surface element and volume element:

The surface element takes the following forms as appropriate:

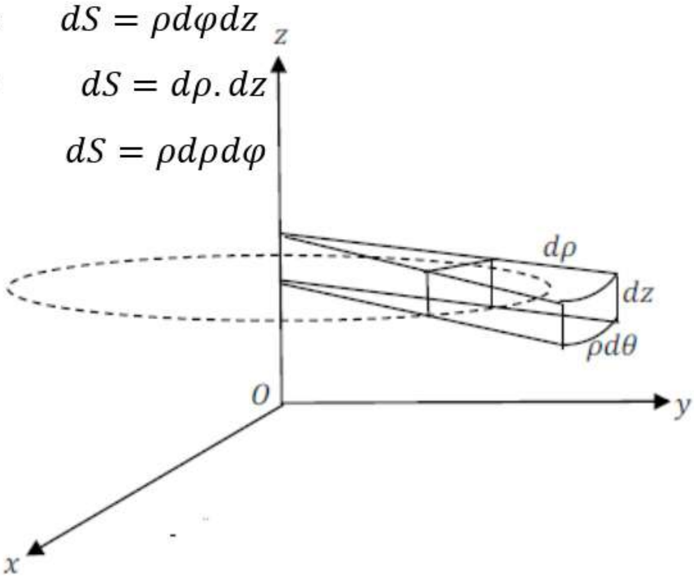
$$\rho = Cste \qquad dS = \rho d\varphi dz$$

$$\varphi = Cste \qquad dS = d\rho . dz$$

$$z = Cste \qquad dS = \rho d\rho d\varphi$$

II-3-The volume element is :

$dV = \rho d\rho d\varphi dz$



III-The spherical coordinates

III-1-Vector position and vector displacement

In the spherical base$(\overrightarrow{U_r};\ \overrightarrow{U_\theta};\ \overrightarrow{U_\varphi})$ the position vector is :

$$\overrightarrow{OM} = \vec{r} = r\overrightarrow{U_r}$$

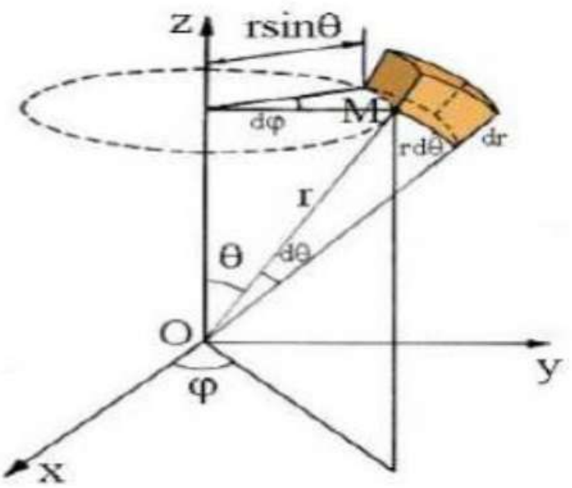
$$d\vec{r} = dr\overrightarrow{U_r} + rd\theta\overrightarrow{U_\theta} + r\sin\theta\, d\varphi\overrightarrow{U_\varphi}$$

III-2-Surface element and volume element:

The surface element dS in spherical coordinates is:

$$dS = R^2 \sin\theta\, d\theta . d\varphi$$

III.The volume element is: $\mathbf{dV} = \mathbf{dS}.\mathbf{dr} = r^2 dr \sin\theta\, d\theta . d\varphi$



IV. Operators

IV.1.Gradient

IV.1.1.Cartesian coordinate :

The gradient of a scalar field f(x,y,z) is the vector:

$$\overrightarrow{\mathrm{grad}}.\mathrm{f} = \vec{\nabla}.\mathrm{f} = \frac{\partial \mathrm{f}}{\partial \mathrm{x}}\vec{\mathrm{i}} + \frac{\partial \mathrm{f}}{\partial \mathrm{y}}\vec{\mathrm{j}} + \frac{\partial \mathrm{f}}{\partial \mathrm{z}}\vec{\mathrm{k}}$$

The vector quantity $\vec{\nabla}$ is the nabla operator defined by :

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

IV.1.2.In cylindrical coordinates:

The gradient of a scalar field ($f(\rho, \varphi, z)$) is the vector:

$$\overrightarrow{\mathrm{grad}}.\mathrm{f} = \frac{\partial \mathrm{f}}{\partial \rho}\overrightarrow{\mathrm{U}_\rho} + \frac{1}{\rho}.\frac{\partial \mathrm{f}}{\partial \varphi}\overrightarrow{\mathrm{U}_\varphi} + \frac{\partial \mathrm{f}}{\partial \mathrm{z}}\vec{\mathrm{k}}$$

IV.1.3.In spherical coordinates :

The gradient of a scalar field ($f(r, \theta, \varphi)$) is the vector:

$$\overrightarrow{grad}f = \frac{\partial f}{\partial r}\overrightarrow{U_r} + \frac{1}{r}\frac{\partial f}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\overrightarrow{U_\varphi}$$

V.The divergence of vector $\vec{A}$, the scalar:

$$\vec{A} = X(x,y,z)\vec{i} + Y(x,y,z)\vec{j} + Z(x,y,z)\vec{k}$$

V.1.Cartesian coordinate :

$$\mathrm{div}\vec{\mathrm{A}} = \vec{\nabla}.\vec{\mathrm{A}} = \frac{\partial \mathrm{X}}{\partial \mathrm{x}} + \frac{\partial \mathrm{Y}}{\partial \mathrm{y}} + \frac{\partial \mathrm{Z}}{\partial \mathrm{z}}$$

The result of the divergence of a vector is a **SCALAR.**

V.2.In cylindrical coordinates:

$$\vec{\mathrm{A}} = \mathrm{A}_\rho\overrightarrow{\mathrm{U}_\rho} + \mathrm{A}_\varphi\overrightarrow{\mathrm{U}_\varphi} + \mathrm{z}\vec{\mathrm{k}}$$

$$\vec{\nabla}.\vec{A} = \frac{1}{\rho}\frac{\partial A_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial z}{\partial z}$$

V.3.In spherical coordinates : Be a vector

$$\vec{A} = A_r\vec{u}_r + A_\theta\vec{u}_\theta + A_\varphi\vec{u}_\varphi$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r sin\theta}\frac{\partial(A_\theta sin\theta)}{\partial \theta} + \frac{1}{r sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

VI. The rotational of a vector

VI.1. Cartesian coordinate :

$$\overrightarrow{rot}\vec{A} = \vec{\nabla} \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{\imath} - \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)\vec{\jmath} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{k}$$

The result of the rotational of a vector is a **VECTOR.**

VI.2. In cylindrical coordinates:

$$\vec{\nabla} \times \vec{A} = \left(\frac{1}{\rho}\frac{\partial z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\overrightarrow{U_\rho} - \left(\frac{\partial z}{\partial \rho} - \frac{\partial A_\rho}{\partial z}\right)\overrightarrow{U_\varphi} + \frac{1}{\rho}(\frac{\partial (\rho A_\varphi}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi})\vec{k}$$

VI.3. In spherical coordinates :

$$\vec{\nabla} \times \vec{A} = \frac{1}{rsin\theta}\left(\frac{\partial\left(A_\varphi sin\theta\right)}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi}\right)\vec{u}_r - \left(\frac{1}{r}\frac{\partial\left(rA_\varphi\right)}{\partial r} - \frac{1}{rsin\theta}\frac{\partial A_r}{\partial \varphi}\right)\vec{u}_\theta +$$
$$\frac{1}{r}\left(\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\vec{u}_\varphi$$