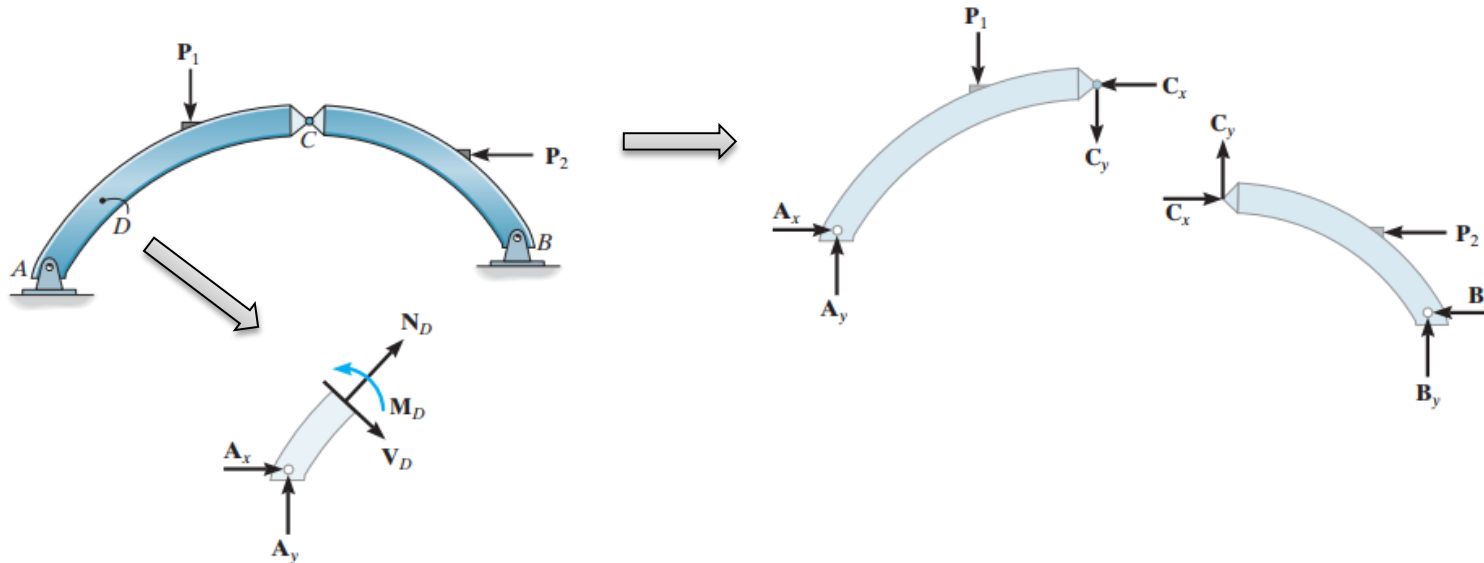


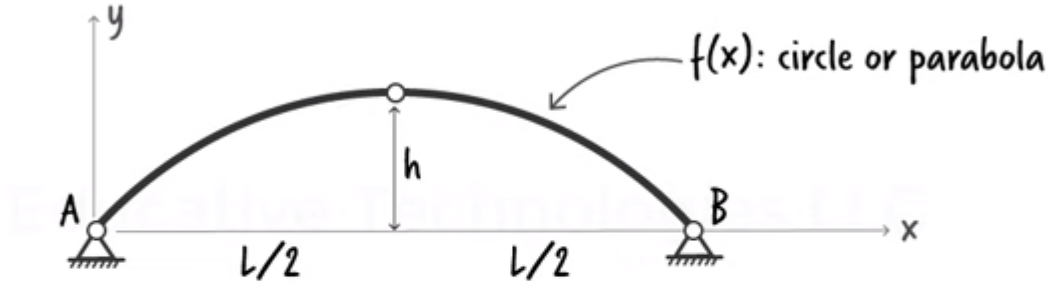
TD N°03:

Arch Analysis “Three-Hinged Arch”

Three-hinged arches are statically determinate and can be analyzed by separating the two members and applying the equations of equilibrium to each member.



Equation of the arch:



$$f(x) = ax^2 + bx + c$$

$$f(x) = \frac{4hx}{L^2}(L-x)$$

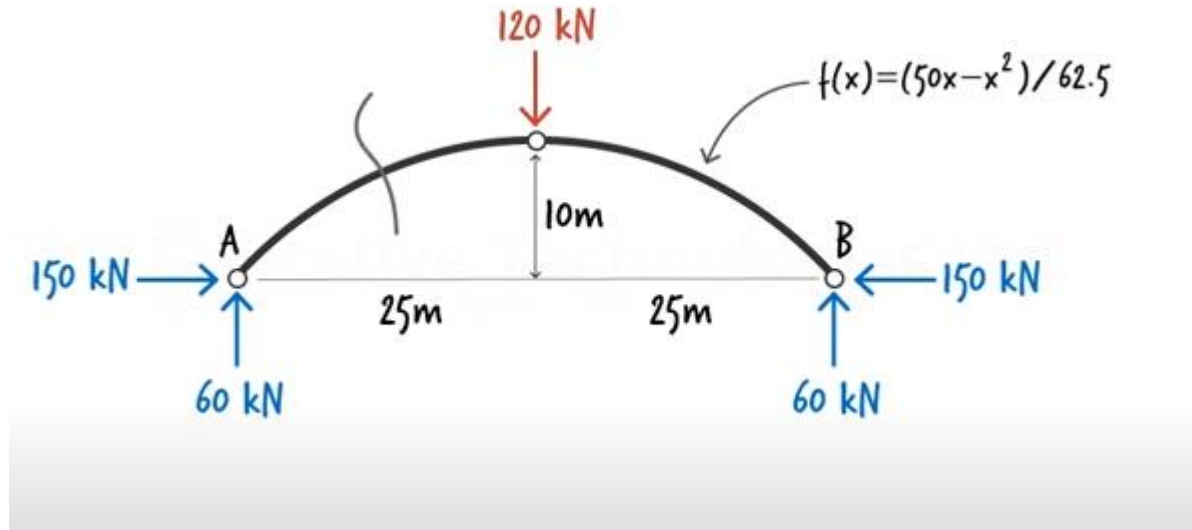
$$f(0) = 0 + 0 + c = 0 \Rightarrow c = 0$$

$$f(L/2) = a(L/2)^2 + b(L/2) = h$$

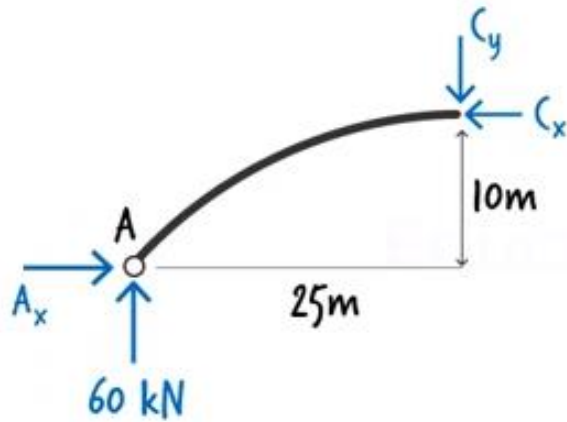
$$f(L) = aL^2 + bL = 0 \Rightarrow aL + b = 0$$

$$\begin{cases} a(L/2)^2 + b(L/2) = h \\ aL + b = 0 \end{cases} \Rightarrow a = \frac{-4h}{L^2}, b = \frac{4h}{L}$$

Example N°01:



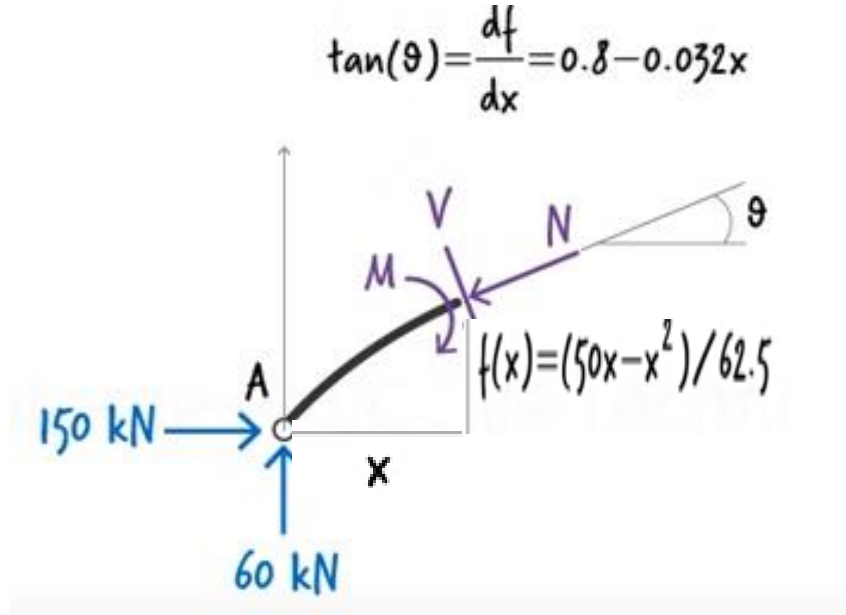
Support reactions:



$$\sum_{\partial C} M = 0 \Rightarrow 25(60) - 10A_x = 0$$

$$A_x = 150\text{ kN}$$

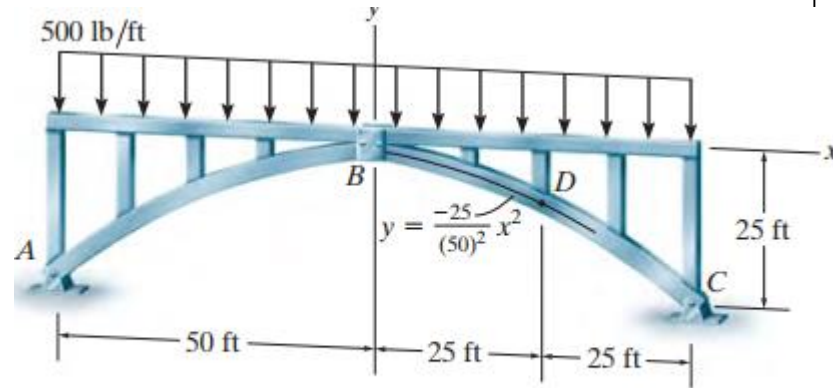
Internal forces at distance x:



Applying the equilibrium equations to determine N, V & M.

Example N°02 :

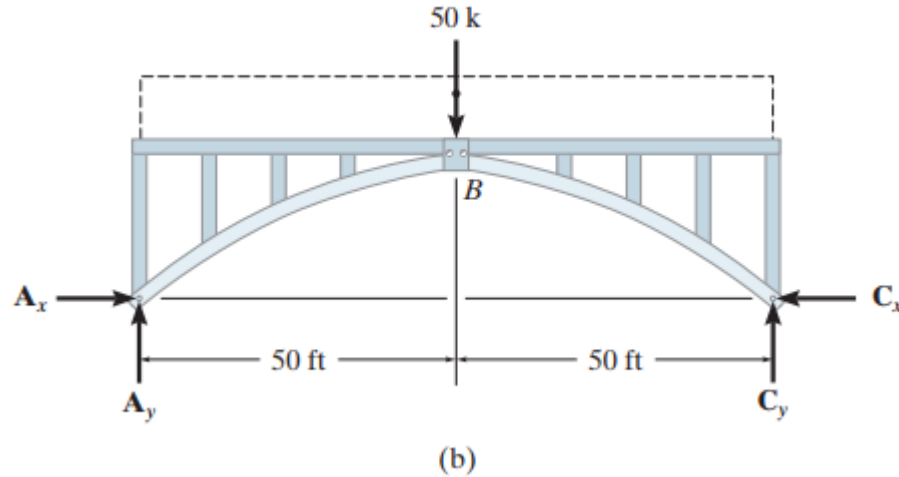
The three-hinged open-spandrel arch bridge like the one shown in the photo has a parabolic shape. If this arch were to support a uniform load and have the dimensions shown in Figure a, show that the arch is subjected only to axial compression at any intermediate point such as point D. Assume the load is uniformly transmitted to the arch ribs.



(a)

Solution:

Here the supports are at the same elevation. The free-body diagrams of the entire arch and part BC are shown in Figure b and Figure c. Applying the equations of equilibrium, we have:



Entire arch:

$$\downarrow + \sum M_A = 0; \quad C_y(100 \text{ ft}) - 50 \text{ k}(50 \text{ ft}) = 0$$

$$C_y = 25 \text{ k}$$

Arch segment BC :

$$\downarrow + \sum M_B = 0; \quad -25 \text{ k}(25 \text{ ft}) + 25 \text{ k}(50 \text{ ft}) - C_x(25 \text{ ft}) = 0$$

$$C_x = 25 \text{ k}$$

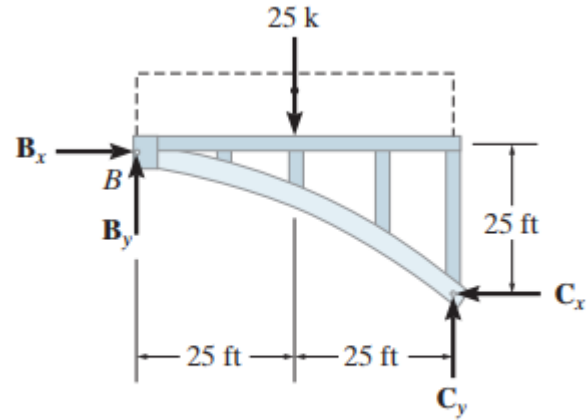
$$\rightarrow \sum F_x = 0;$$

$$B_x = 25 \text{ k}$$

$$+\uparrow \sum F_y = 0;$$

$$B_y - 25 \text{ k} + 25 \text{ k} = 0$$

$$B_y = 0$$



(c)

A section of the arch taken through point D, $x = 25 \text{ ft}$, $y = -25 (25)^2 / (50)^2 = -6.25 \text{ ft}$, is shown in Figure d. The slope of the segment at D is:

$$\tan \theta = \frac{dy}{dx} = \frac{-50}{(50)^2} x \Big|_{x=25 \text{ ft}} = -0.5$$

$$\theta = -26.6^\circ$$

Applying the equations of equilibrium, Fig. 5–10d we have:

$$\rightarrow \Sigma F_x = 0; \quad 25 \text{ k} - N_D \cos 26.6^\circ - V_D \sin 26.6^\circ = 0$$

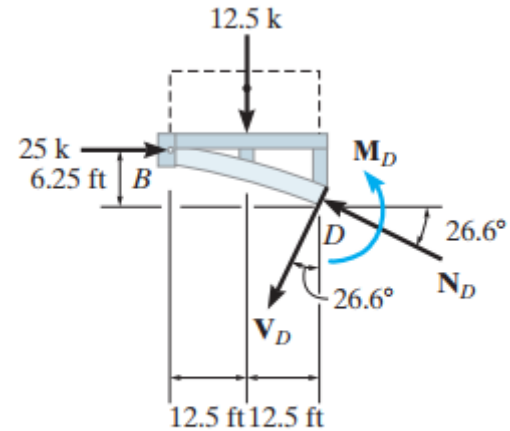
$$+\uparrow \Sigma F_y = 0; \quad -12.5 \text{ k} + N_D \sin 26.6^\circ - V_D \cos 26.6^\circ = 0$$

$$\curvearrowright \Sigma M_D = 0; \quad M_D + 12.5 \text{ k}(12.5 \text{ ft}) - 25 \text{ k}(6.25 \text{ ft}) = 0$$

$$N_D = 28.0 \text{ k}$$

$$V_D = 0$$

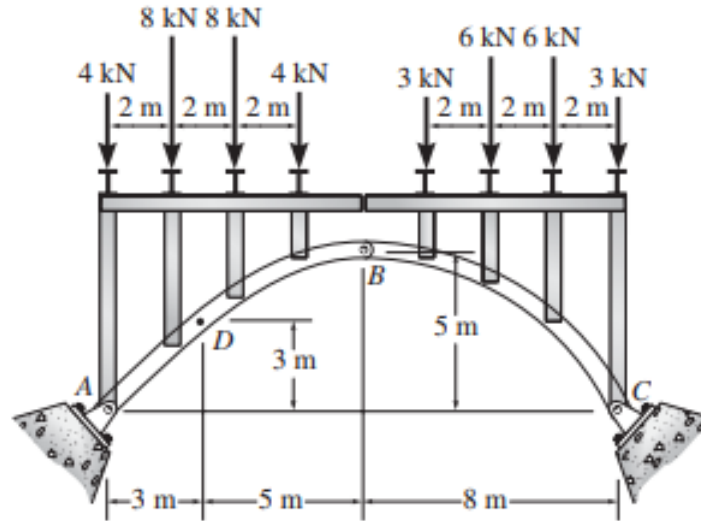
$$M_D = 0$$



(d)

Example N°02 :

The three-hinged spandrel arch is subjected to the loading shown. Determine the internal moment in the arch at point D.



Solution:

$$M_D = 10.8 \text{ kN} \cdot \text{m}$$

References

1. Hibbeler, R C. 2012. Structural Analysis. Eighth Edition, Pearson Prentice Hall, New Jersey.
2. Dr.Structure (2018). Analysis of Three-Hinged Arches (video). Youtube.
<https://youtu.be/pJKfOvN36J0?si=rglivOUVRFXOJWiz>
<https://youtu.be/pJKfOvN36J0?si=rglivOUVRFXOJWiz>