4. THE SPHERICAL MIRROR AND MIRROR PLAN

4.1THE SPHERICAL MIRROR

❖ DESCRIPTION

A spherical mirror is a spherical cap with center C and vertex S made reflective. The axis of symmetry is the optical axis of the mirror. This axis is usually oriented from left to right because light arrives from the left (by convention). There are two types of spherical mirrors:

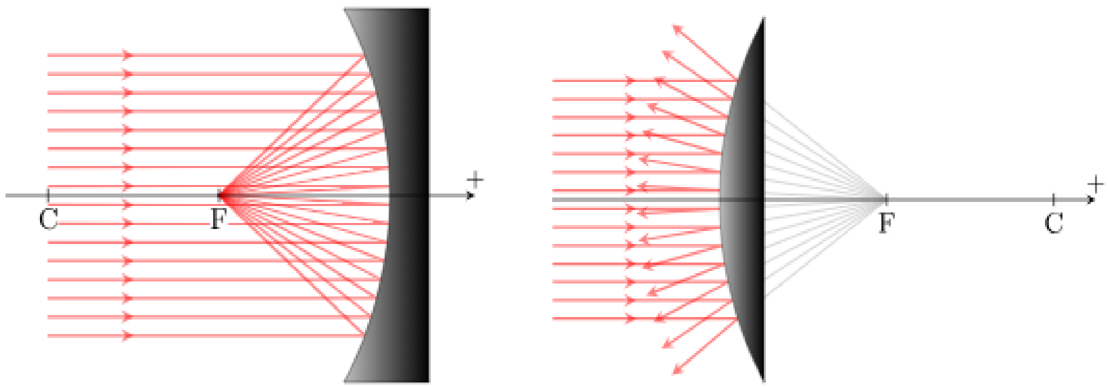

Figure1: *Concave* and convex spherical mirrors.

- the concave mirror is a spherical mirror such that $\overline{SC} < 0$
- the convex mirror is a spherical mirror such that $\overline{SC} > 0$

A spherical mirror is a concave or convex reflecting surface defined by the center of curvature C and a vertex S located on the surface. The curvature ray is $R = \overline{SC}$.

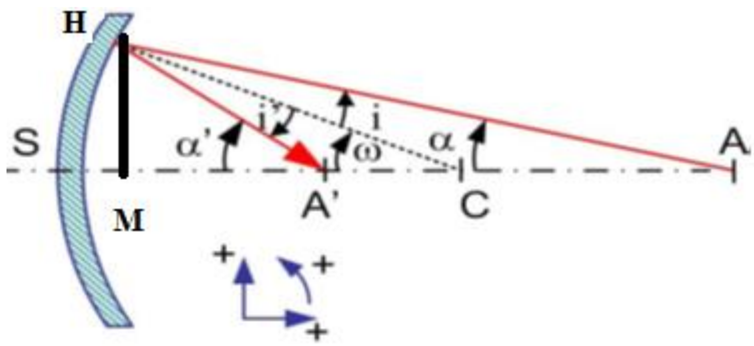

Figure2

Let us consider a point A of the line SC. A luminous ray arising from A reflects itself on a point I of the mirror and cuts its line SC in A'. If we carry out the same operation as for the diopters, we have:

$$i = i' \ \ \textcolor{red}{\boldsymbol{refraction\ law}}$$

$$\alpha + i = \omega \quad and \quad \alpha' = i' + \omega$$

Therefore:$\omega - \alpha = \alpha' - \omega$

And

$$\begin{cases} \tan\omega \approx \omega = \dfrac{\overline{HM}}{\overline{CH}} = \dfrac{\overline{HM}}{\overline{CS}} \\ \tan\alpha \approx \alpha = \dfrac{\overline{HM}}{\overline{AH}} = \dfrac{\overline{HM}}{\overline{AS}} \\ \tan\alpha' \approx \alpha' = \dfrac{\overline{HM}}{\overline{A'H}} = \dfrac{\overline{HM}}{\overline{A'S}} \end{cases}$$

$$\frac{\overline{HM}}{\overline{CS}} - \frac{\overline{HM}}{\overline{AS}} = \frac{\overline{HM}}{\overline{A'S}} - \frac{\overline{HM}}{\overline{CS}}$$

$$\frac{1}{\overline{CS}} - \frac{1}{\overline{AS}} = \frac{1}{\overline{A'S}} - \frac{1}{\overline{CS}}$$

$$\frac{2}{\overline{CS}} = \frac{1}{\overline{A'S}} + \frac{1}{\overline{AS}}$$

In the case of spherical mirrors, the principle of inverse return of light implies: $f = f'$

- The formula conjugate relation for a spherical mirror:

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{1}{\overline{SF}} = \frac{1}{f'} \qquad \overline{SF} = \overline{SF'} = f'$$

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{2}{\overline{SC}}$$

The expression below is analogous to the diopters by replacing $\overline{SA} = P$,$\overline{SA'} = P'$,$\overline{SC} = R$

$$\frac{1}{P'} + \frac{1}{P} = \frac{2}{R}$$

- The magnification

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = -\frac{\overline{SA'}}{\overline{SA}} = -\frac{P'}{P}$$

- The focal lengths $\overline{SF}$ and $\overline{SF'}$ have the expressions:

$$\overline{SF} = \overline{SF'} = \frac{\overline{SC}}{2}$$

Case of the mirror plan:

For a plane mirror $\overline{SC} = \infty$

The conjugation relation is then written: $\overline{SA'} = -\overline{SA}$

The object and the image are equidistant from the mirror.

The magnification γ in this case is: $\gamma = 1$

The image $A'B'$is the same size as the object AB

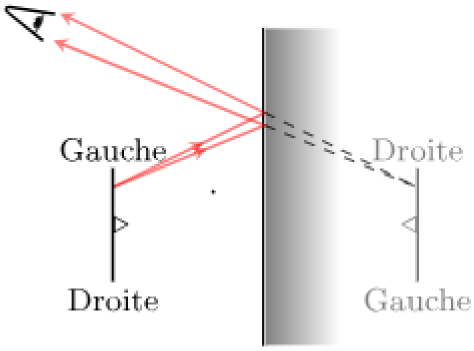

Figure3

Formation of an image with a plane mirror.
The image is reversed (left/right)

Exemple : Plane Mirror

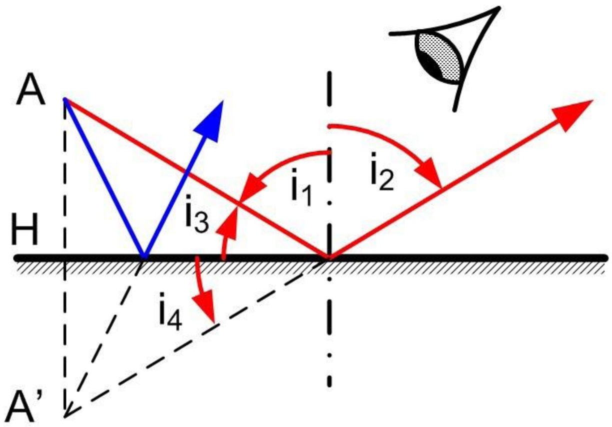

Figure 4

Let an object point be A and its projection on the mirror H. Any ray out of A follows the law ($i_2 = -i_1$). The prolongation of the reflected ray cuts the line AH at a point AH'. On figure 4 we easily demonstrate that if the absolute values of i1 and i2 are equal, those of i3 and i4 also are and HA=HA'. Point A' is the symmetry of A in relation to the mirror. This is true for all luminous rays issued of A, the image is stigmatic, figure 1 show a real object and a virtual image. If we inverse the direction of the luminous rays, the object becomes A', virtual, and the image becomes A, real.

Image construction in a mirror

From point B of object AB, the ray going through C (Red) reflects to itself, the ray going through F (Green) reflects itself in parallel to the axis, the ray parallel to the axis (Blue) reflects itself by going through F and the ray going through S (Orange) reflects itself symmetrically in relation to the axis.

B' is at the intersection of emerging rays, A' is on the **projection** of B' on the axis.

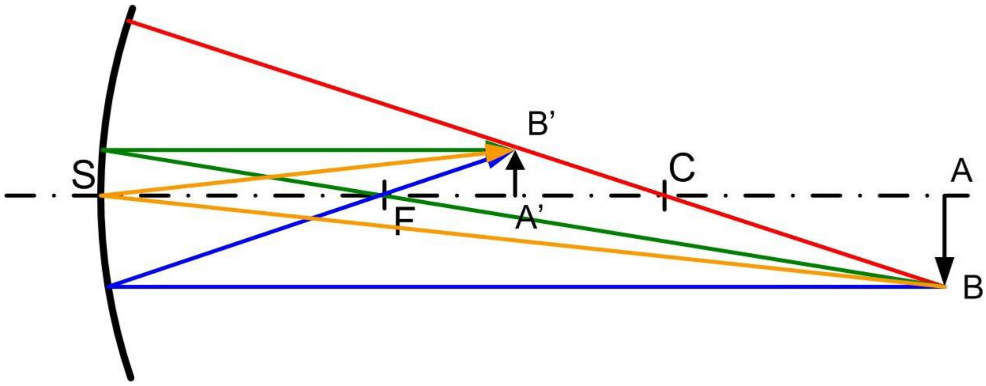

Figure 5 : *Concave Mirror*

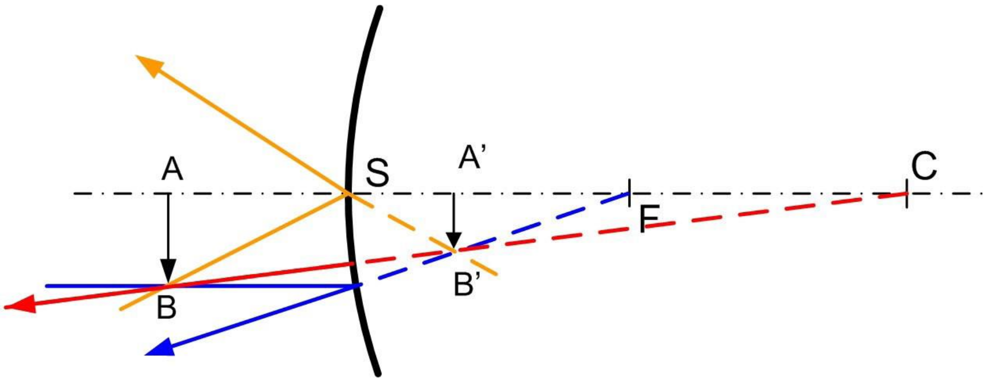

Figure 6 : *Convex Mirror*