University of Biskra
Mathematics Department
Module: Analysis 2

First year license
2023/2024

Worksheet N° 1 & 2

Exercise 1

1. Calculate the following:

   1) $\int 2dx$ 2); $\int x^2+2x-1dx;$ 3) $\int \frac{\theta^2+3}{\sqrt{\theta}}d\theta;$

   4) $\int xy^2+y^2\sqrt[3]{y}dy;$ 5) $\int x^2+\sqrt{x}+\frac{1}{x^4\sqrt{x}}dx;$ 6) $\int \sin(\theta)-\cos(\theta)d\theta.$

2. Find the real function f that satisfying the following conditions

   1) $f'(x)=3x^2+e^{-x}$ and $f(0)=1;$ 3) $f''(x)=sin(x)-e^{-2x},$ $f'(0)=\frac{5}{2}$ and $f(0)=0;$

   2) $f'(x)=\sqrt[3]{x^2}-\frac{1}{x^2}$ and $f(1)=3;$ 4) $f''(x)=sin(x)-cos(x),$ $f'(0)=0$ and $f(\frac{\pi}{2})=0.$

Exercise 2 *(by substitution)* Find the antiderivatives of the following functions:

$f(x)=\tan(x),$ $f(x)=\sin^3(x)\cos^2(x),$ $f(x)=\frac{\sin(x)cos(x)}{(1+\cos(2x))^2},$ $f(x)=\frac{\arcsin(x)}{\sqrt{1-x^2}},$

$f(x)=\frac{x}{x^4+a^2},$ $f(x)=\frac{1}{x\ln(x)},$ $f(x)=xa^{x^2},$ $f(x)=\frac{(a^x-b^x)^2}{a^xb^x}$ with $a,\ b>0.$

$\int \sin^{2p+1}(x)\cos^q(x)dx$ and $\int \sin^p(x)\cos^{2q+1}(x)dx,$ with $p,q\in\mathbb{N}$

$\int \frac{1}{x\ln^n(x)}$ with $n\in\mathbb{Z}^*$

Exercise 3 *(by parts)* Let $n\in\mathbb{N}^*$. Calculate the following using integration by parts.

1. • $\int \arctan(x)dx,$ • $\int \arcsin(x)dx,$ • $\int \frac{\arcsin(\sqrt{x})}{\sqrt{x}}dx,$ • $\int \frac{x\arctan(x)}{(1+x^2)^2}dx$
2. • $\int \ln(x)dx,$ • $\int x\ln(x)dx,$ • $\int (\ln(x))^2dx,$ • $\int x^n\ln(x)dx$
3. • $\int x^2e^{-x}dx,$ • $\int x^ne^{-x}dx,$
4. • $\int \sin(x)e^xdx,$ • $\int \cos(\beta x)e^{\alpha x}dx,$ • $\int \sin(\beta x)e^{\alpha x}dx,$
5. • $\int \sqrt{a^2-x^2}dx,$ • $\int \cos^2(x)dx,$ • $\int \sin^4(x)dx$

Exercise 4 *(Rational and irrational functions)*Calculate the following integrals

- $\int \frac{x+1}{x^2+x+2}dx,$ • $\int \frac{x+1}{x^2+x-2}dx,$ • $\int \frac{x+1}{(x-1)(x-2)}dx,$ • $\int \frac{x+1}{(x-1)^2(x-2)}dx,$
- $\int \frac{x3+2x^2+3x-1}{(x^2+1)(x^2-4)}dx,$ • $\int \frac{x^5+x^4-8}{x^3-4x}dx,$ • $\int \frac{x+1}{\sqrt{(x-1)(x-2)}}dx,$ • $\int \frac{x+1}{\sqrt{(1-x)(x-2)}}dx,$
- $\int \frac{x3+2x^2+3x-1}{x^2+2x+1}dx,$ • $\int \frac{1}{2\cos^2(x)+\cos(x)\sin(x)+\sin^2(x)}dx$ • $\int \ln(x^2+2x-3)dx,$
- $\int \frac{1}{(x^2-4)^2}dx,$ • $\int \frac{1}{(x^2+4)^2}dx,$ • $\int \frac{1}{(x^2-4)^4}dx,$ • $\int \frac{1}{(x^2+4)^4}dx,$

$\int \frac{2x^{1/2}+3x^{1/4}}{1+x^{1/4}}dx$, • $\int \frac{2x^{1/2}+3x^{1/3}}{1+x^{1/3}}dx$, • $\int \sqrt[3]{5x-1}dx$, • $\int \frac{dx}{\sqrt{\sqrt{x}-2}}$, • $\int \sqrt[3]{\frac{x+1}{x-1}}dx$.

Exercise 5 Calculate the following integrals:

1. $\int_0^{+\infty} e^{-x}dx$ and $\int_0^{+\infty} x^2e^{-x}dx$.
2. $\int_1^{e} \ln(x)dx$ and $\int_e^{2e} x^2\ln(x)dx$.
3. $\int_0^{2\pi} \sin(x)cos^2(x)dx$ and $\int_0^{\frac{\pi}{2}} sin(x)e^x dx$.

Exercise 6 Let's consider the function f defined by:

$$f(x) = \begin{cases} \alpha\left(\frac{1}{x}\right)^{\alpha+1} & \text{if } x > 1 \\ 0 & \text{else.} \end{cases}$$

With α is a positive real number.

Question: Discuss the existence of the integral $\int_a^{+\infty} x^n f(x)dx$ according to n with $n \in N^*$.

Exercise 7 *(Leave the exercise to the students)*

Using the definite integration, show that the surface delimited by a circle of radius R and center (x_0, y_0) is πR^2.