University of Biskra
Mathematics Department
Module: Analysis 2

First year license
2023/2024

Continuous assessment
Test N° 1

Exercise: (./8pts) Calculate the following integrals:

1. $$\int \cos(x)e^x dx$$

   Indication: use the integration by parts.

2. $$\int \frac{x^4+2x^2-1}{(x-1)(x^2+1)}dx$$

   Indication: use the decomposition of the fraction.

3. $$\int \cos(x)\sin(2x)dx$$

   Indication: use the trigonometric functions proprieties.

Good luck

• $I_1 = \int \cos(x)\, e^x\, dx = ?$

$\begin{cases} u = \cos(x) \\ v' = e^x \end{cases} \Rightarrow \begin{cases} u' = -\sin(x) \\ v = e^x \end{cases}$ 0,1

$\Rightarrow I_1 = \cos(x)\, e^x + \int \sin(x)\, e^x\, dx$ 0,1

$\begin{cases} u = \sin(x) \\ v' = e^x \end{cases} \Rightarrow \begin{cases} u' = \cos(x) \\ v = e^x \end{cases}$ 0,1

$\Rightarrow I_1 = \cos(x)\, e^x + e^x \sin(x) - \int \cos(x)\, e^x\, dx$

$\Rightarrow I_1 = (\cos(x) + \sin(x))\, e^x - I_1$ 0,1

$\Rightarrow \boxed{I_1 = \frac{1}{2}(\cos(x) + \sin(x))\, e^x + C}$ 0,1

• $I_2 = \int \frac{P_4(x)}{P_3(x)}\, dx$, such as $P_4(x) = x^4 + 2x^2 - 1$

$P_3(x) = (x-1)(x^2+1) = x^3 - x^2 + x - 1$

So $\frac{P_4(x)}{P_3(x)} = ax + b + \frac{c}{x-1} + \frac{dx+e}{x^2+1}$. 0,1

* we have

$$\begin{array}{l|l} x^4 + 2x^2 - 1 & x^3 - x^2 + x - 1 \\ \hline \quad\vdots & x+1 \\ 2x^2 & \end{array}$$

0,1

So $\boxed{a = b = 1}$

• $\frac{2x^2}{(x-1)(x^2+1)} = \frac{c}{x-1} + \frac{dx+e}{x^2+1}$ — $(*)$

$\begin{cases} (*) * (x-1) \Rightarrow \frac{2x^2}{x^2+1} = c + \frac{(dx+e)(x-1)}{x^2+1} \\ x = 1 \Rightarrow \boxed{1 = C} \end{cases}$ 0,1 from $(*)$ if we put $x=0$ we have:

$$0 = -c + e \Rightarrow e = c \Rightarrow \boxed{e = 1}$$ 0.5

from $(*)$ if we put $x=-1$, we have:

$$-\frac{1}{2} = \frac{-c}{2} + \frac{-d+e}{} \Rightarrow \boxed{d = 1}$$ 0.5

So $I_2 = \int \frac{x^4+2x^2-1}{(x-1)(x^2+1)} dx$

$$= \int x+1+\frac{1}{x-1}+\frac{x+1}{x^2+1} dx$$ 0.5

$$= \int x+1\, dx + \int \frac{1}{x-1} dx + \frac{1}{2}\int \frac{2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$= \frac{1}{2}x^2 + x + \ln(|x-1|) + \frac{1}{2}\ln(x^2+1) + \arctan(x) + C.$$ 1

$I_3 = \int \cos(x)\sin(2x)\, dx$ (the first way).

we have $\sin(2x) = \sin(x+x) = 2\sin(x)\cos(x)$ 0.5

So $I_3 = \int 2\sin(x)\cos^2(x)\, dx$; we put

$$t = \cos(x) \Rightarrow dt = -\sin(x)\, dx.$$ 0.5

$$\Rightarrow I_3 = -2\int t^2 dt = -\frac{2}{3}t^3 + C.$$

$$= -\frac{2}{3}\cos^3(x) + C.$$ 0.5

the second way (see the work home).

we have
$$\begin{cases} \sin(2x+x) = \sin(2x)\cos(x) + \sin(x)\cos(2x) \\ \sin(2x-x) = \sin(2x)\cos(x) - \sin(x)\cos(2x) \end{cases} +$$

1

$$\sin(3x) + \sin(x) = 2\sin(2x)\cos(x)$$

$$\Rightarrow \sin(2x)\cos(x) = \frac{1}{2}\left(\sin(3x) + \sin(x)\right)$$

So $I_3 = \int \sin(2x)\cos(x)\,dx = \frac{1}{2}\int \sin(3x) + \sin(x)\,dx$

$$\boxed{I_3 = \frac{-1}{6}\cos(3x) - \frac{1}{2}\cos(x) + C}$$

0,5