Introduction aux probabilités et statistique descriptiv - les paramètres de dispersion et les paramètres de forme

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5.Dispersion parameter (spread)

These parameters aim, in the case of a quantitative nature, to characterize the variability of the data in the sample..

The fundamental dispersion indicators are

1. The variance
2. The standard deviation.
3. The .mean deviation.
4. The Coefficient of variation.
5. The interquartile, interdecile and interpercentile.

- Soit un échantillon de n valeurs observées $x_1, x_2, \ldots., x_i, \ldots., x_n$ d'un caractère quantitatif X et soit $\bar{x}$ sa moyenne observée. On définit la variance observée notée V ou S^2 comme la moyenne arithmétique des carrés des écarts à la moyenne.

$$V = S^2 = \frac{1}{n}\sum_{i=1}^{k} n_i \left(x_i - \bar{X}\right)^2$$

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- La formule de la variance qui résulte du théorème de Koenig est donc:

$$V = S^2 = \frac{1}{n} \sum_{i=1}^{k} n_i x_i^2 - \bar{X}^2$$

- Dans le cas de données regroupées en k classes d'éffectif n_i (variable continue regroupée en classes), la formule de la variance est la suivante:

$$V = S^2 = \frac{1}{n}\sum_{i=1}^{k} n_i \left(c_i - \bar{X}\right)^2 ,$$

ou c_i est le centre de classe

- The observed standard deviation corresponds to the square root of the observed variance:

$$\sigma = \sqrt{V} = S = \sqrt{S^2}$$

- Let us be a sample of n observed values $x_1, x_2, \ldots, x_n$ of a quantitative character X and let $\bar{x}$ be its observed mean. We define the mean deviation denoted $E.M$ as the arithmetic mean of the deviations from the mean

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- In the case of data grouped into k classes of frequency n_i (continuous variable grouped into classes), the formula is as follows:

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- Observed variance and standard deviation are absolute dispersion parameters that measure the absolute variation of the data independent of the order of magnitude of the data.

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- The coefficient of variation denoted $C.V.$ is a relative dispersion index taking into account this bias and is equal to:

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- The interquartile range is a measure of variation that is not influenced by extreme values. Its definition is simple: the interquartile range measures the extent of the 50% of values located in the middle of a series of classified data.

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- The interdecile interval measures the extent of the 80% of values located in the middle of a series of classified data.

$$ID = D_9 - D_1$$

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6.Shape parameters

We define the shape parameters for a quantitative statistical variable, discrete or continuous, with real values.

1. **Skewness coefficient.(Asymmetry)**
2. **Kurtusis coefficient (heaviness of tail)**

1. The Pearson Skewness coefficient involves the *Mo* mode: when it exists, it is defined by:

$$S_k = \frac{\bar{X} - Mo}{\sigma}$$

- Si S_k est égal à 0, the reduced variable has the same flattening as a bell curve, we say that the variable is normal.

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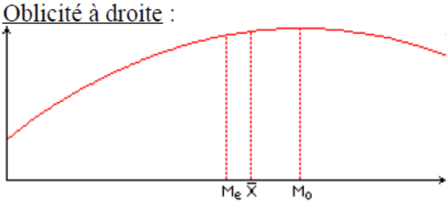

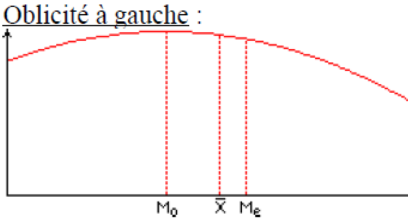

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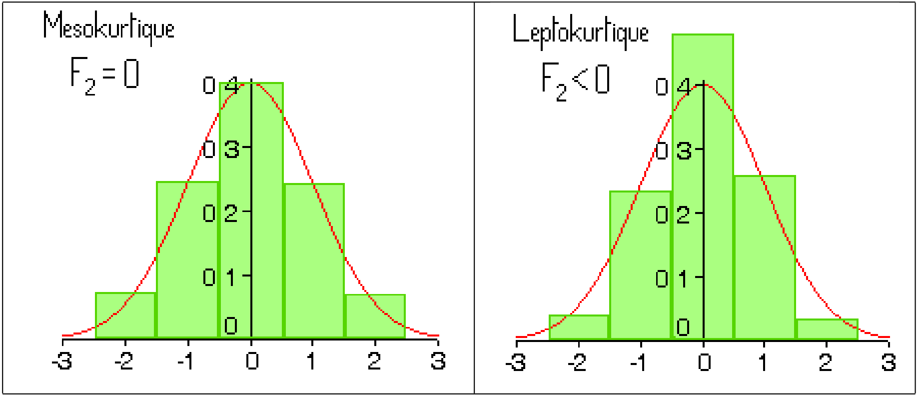
Shape parameters
6.2 Kurtusis coefficient:



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