Hydrostatic:

Pascal's law and Archimedes thrust.

REMINDER

1.1 HYDROSTATIC

Hydrostatic: v = 0

1-Pressure force $\vec{F}$ (normal to the surfaces), due to the impacts of the fluid particles on the surface. The molecules of the fluid have disordered movements, which causes shocks on the surfaces of objects immersed in the fluid. With each shock, there is a change in the momentum of the molecule, and therefore a force is exerted on the surface.If we change the orientation of the surface on which this force is exerted, we always obtain a force of the same standard, which leads to using another modeling, and we instead define a new scalar quantity (therefore without orientation):

the pressure: **P = F / S,**

where F is the norm of the force which would be exerted on the surface S of an object if we placed this object at this location, but it is not necessary that there be an object in the fluid to calculate a pressure.

unité :

-SI : Pascal (Pa) -l'atmosphère, 1 atm = 101 300 Pa , -1bar=10^5 Pa

Forces of cohesion and adhesion= force between the molecules of the liquid (cohesion) and between the molecules of the liquid and those of its container (adhesion).

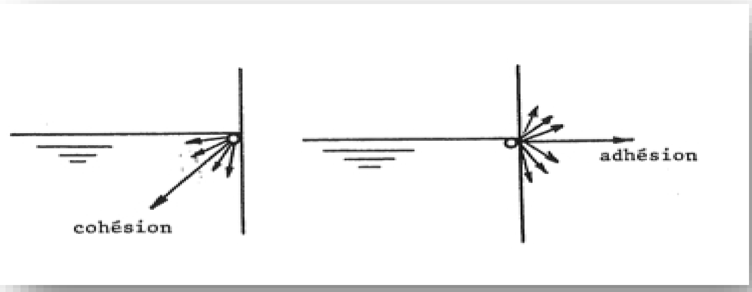
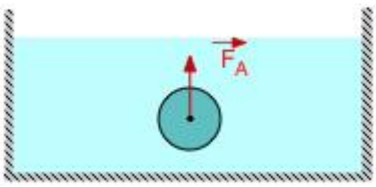


Figure 5.1 Forces of cohesion and adhesion

1.2 CONSEQUENCES OF PRESSURE FORCES -Archimedes' force-

1-Archimedes' force: resulting from pressure forces on the walls of an object. It is vertical upwards, and $\pi_A = \rho.g.V$



1.3 FUNDAMENTAL RELATIONSHIP OF HYDROSTATIC:

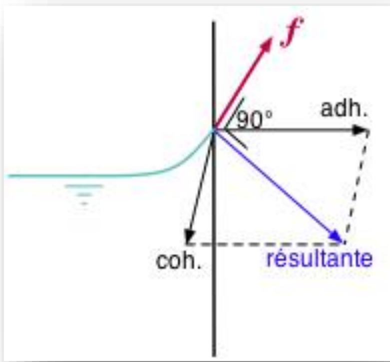
- At two points located at the same height relative to the bottom, if we have the same liquid, we will have the same hydrostatic pressure.
- If we descend from a depth x in a liquid of density ρ, we have an increase in the hydrostatic pressure: **$\Delta p = \rho. g. x$**

1.4 CONSEQUANCES OF COHESION AND ADHESION FORCES- SURFACE TENSION:-

1-SURFACE TENSION:

The liquid rises (or falls) along the wall of the container AS IF there was a force γ pulling on the surface. In this model, this force is the capillary force acting on the surface of the liquid, tangentially to it.
This force tends to minimize the surface area of the liquid.
In practice, we use surface tension instead: $f = \gamma / l$

Another definition is often preferred: $\gamma = dW / dS$ work per unit of surface that must be provided to enlarge the surface. SI unit: **N/m** or **J/m^2**



Jurin's law : Capillaries: difference in liquid level with the outside of the capillary of $h = 2\gamma\cos\theta/r\rho g$ (Jurin's law)

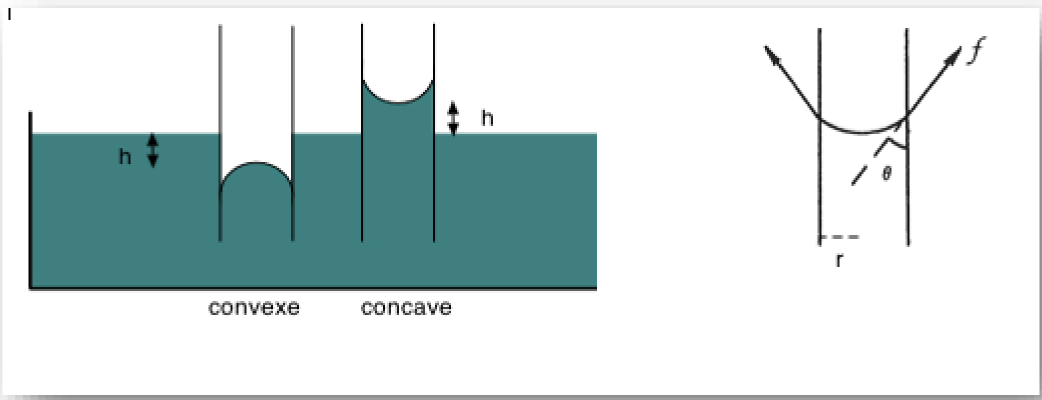
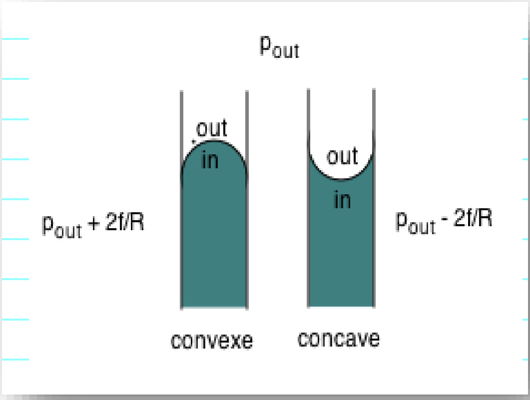


Figure 5.2 : Difference in liquid level with the outside and inside of the capillary.

Pressure variation at the surface of a liquid:



• if flat surface, $P(in) = P(out)$
• if convex surface,

$$\boldsymbol{P(in) = P(out) + 2\,\gamma/R}$$

• if concave surface,

$$\boldsymbol{P(in) = P(out) - 2\,\gamma/R}$$

In the last 2 cases, we have:

$$\boldsymbol{P(concave\ side) - P(convex\ side) = 2\,\gamma/R}$$

$$\boldsymbol{(Laplace's\ law).}$$

where R is the radius of curvature of the surface.

Bernoulli's law (hydrodynamics)

REMINDER

1.5 DYNAMICS OF INCOMPREHENSIBLE FLUIDS

Definitions:

The principle of continuity expresses the conservation of mass, which means that no fluid cannot be created nor disappear in a given volume.

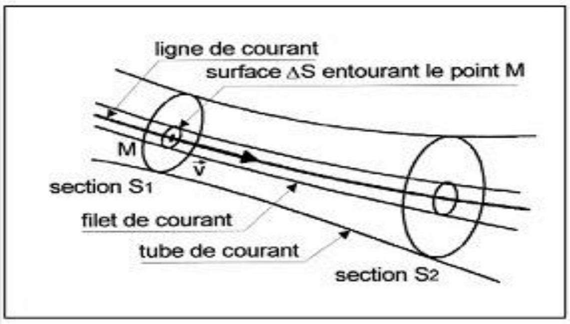


Figure 6.1: The principle of continuity expresses the conservation of mass.

- Flow: is the quantity of material that passes through a straight section of pipe during the unit of time.
- Mass flow: If dm is the elementary mass of fluid having traveled a straight section of the carried out during the time interval dt, the mass flow is written:

$$q_m = \frac{dm}{dt} [Kg.s^{-1}]$$

- Volume flow: If dV is the elementary volume of fluid having traveled a straight section of rolling during the time interval dt, the volume flow is written:

$$q_V = \frac{dV}{dt} [m^3.s^{-1}]$$

- The relation between $q_m and\ q_V$: The density ρ is given by the relation:$\rho = \frac{dm}{dV}$.

from where : $q_m = q_V$

Since the flow rate always remains constant in a steady state), **the continuity equation** is written as: $\boldsymbol{Q = S_1 V_1 = S_2 V_2}$

1.6 GENERAL Flow EQUATION OR Bernoulli EQUATION

A flow regime is said to be permanent or stationary if the parameters, which characterize (pressure, temperature, speed, density, etc.), have a constant value over time:

a- Case of Perfect Fluids (non-viscous)

Bernoulli's equation expresses that, all along a fluid stream

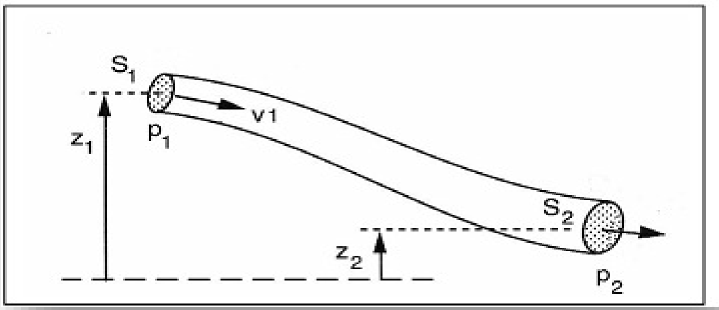


Figure 6.2: A flow regime permanent or stationary

Bernoulli's equation expresses that, throughout a fluid stream in permanent (stationary) motion, the total energy per unit weight of the fluid remains constant and the equation is written:

$$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} = H = constant$$

b- Case of real fluids (viscous)

In the case of real fluids, the energy decreases in the direction of flow. this is due to the viscous nature of the fluid which dissipates part of the energy: this loss of energy is called pressure loss and the equation is written:

$$z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h$$

h: which is the consequence of the viscosity of the fluid and the roughness of the walls of the section flow.

Tutorial N°6: Exercises on Bernoulli's law (hydrodynamics)

Exercise 6.1:

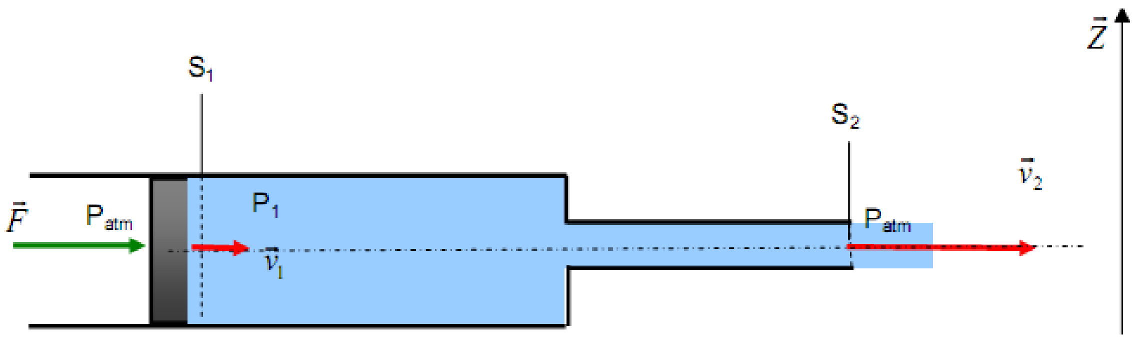
The figure below shows a piston that moves without friction in a cylinder of section S_1 and diameter $d_1 = 4\ cm$ filled with a perfect fluid of density$\rho = 1000\ kg/m^3$. A force F with an intensity of 62.84 N acts on the piston, at a constant speed V_1 .The fluid can escape to the outside through a cylinder of section S_2 and diameter $d_2 = 1\ cm$ at a speed V_2 and a pressure $P_2 = P_{atm} = 1\ bar$.

1- By applying the Fundamental Principle of Dynamics to the piston, determine the pressure P_1 of the fluid at section S_1 as a function of F, P_{atm} and d?

2- Write the continuity equation and determine the expression of the speed V_1 as a function of V_2?

3- By applying the Bernoulli equation, determine the flow speed V_2 as a function of P_1, P_{atm} and ρ?

We assume that the cylinders are in a horizontal position ($Z_1 = Z_2$)



Solution

1- Applying the Fundamental Principle of Dynamics, we obtain:

$$P_1 = \frac{4F}{\pi d_1^2} + P_{atm}\text{=1.5bar}$$

2- The continuity equation:

$$\pi {d_1}^2 V_1 = \pi {d_2}^2 V_2 \text{ and } d_1 = 4d_2$$

$$\pi 16 {d_2}^2 V_1 = \pi {d_2}^2 V_2 \Rightarrow V_1 = \frac{1}{16} V_2$$

3- By applying the Bernoulli equation

$$\frac{P_1}{\rho}+\frac{v_1^2}{2}=\frac{P_{atm}}{\rho}+\frac{v_2^2}{2}\Longrightarrow\frac{P_1}{\rho}+\frac{v_2^2}{2\times 256}=\frac{P_{atm}}{\rho}+\frac{v_2^2}{2}$$

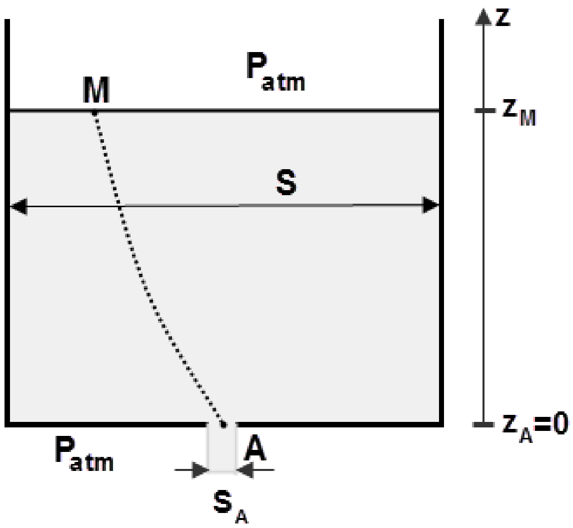
$$\frac{P_1}{\rho}-\frac{P_{atm}}{\rho}=\frac{v_2^2}{2}-\frac{v_2^2}{2\times 256}\Longrightarrow\frac{(P_1-P_{atm})}{\rho}=\frac{(256-1)\times v_2^2}{512}$$

$$\frac{(P_1-P_{atm})}{\rho}=\frac{(255)\times v_2^2}{512}$$

$$v_2^2=\frac{512\times(P_1-P_{atm})}{255\times\rho}\Rightarrow v_2=\sqrt{\frac{512\times(P_1-P_{atm})}{255\times\rho}}=10m/s.$$

Exercise 6.2:

A reservoir, cubic in shape and section $S=4m^2$ and $a=2m$ .The reservoir is filled with liquid that can be emptied through an opening A pierced at its horizontal bottom and opening into the open air. A is section $S_A=8\ cm^2$. We will assume that when it is drained, the liquid is perfect, incompressible and its flow speed is constant.



1- When emptying this reservoir, consider the streamline joining points M and A. By applying the Bernoulli relation between these two points, give the expression for the flow speed v_A liquid to the point A depending on the acceleration of gravity g and the altitude Z_M of the point M.
2- Give the relation of the volume flowQ_v at the orifice A as a function of g, Z_M and M.
3- Establish the relationship of the flow speed v_M at point M according to g, Z_M, S_A and S
4- Calculate the time necessary for the total emptying of this reservoir.

Solution

1- The Bernoulli relation between the two points $M\ and\ A$ is written:

$$z_A + \frac{P_A}{\rho g} + \frac{v_A^2}{2g} = z_M + \frac{P_M}{\rho g} + \frac{v_M^2}{2g}$$

$$z_A = 0$$

The points $M\ and\ A$ and being in direct contact with the air, their pressures are equal to atmospheric pressure: $P_A = P_M = P_{atm}$

$$\frac{P_{atm}}{\rho g} + \frac{v_A^2}{2g} = z_M + \frac{P_{atm}}{\rho g} + \frac{v_M^2}{2g} \Rightarrow\ v_A^2 = v_M^2 + 2.g.Z_M$$

For a perfect liquid the volume flow Q_v is constant:$S_A.v_A = S.v_M$

$$v_M = \frac{S_A.v_A}{S}$$

$S_A = 8 \times 10^{-4}\ m^2$
$S = 4\ m^2$

$$v_M = \frac{8 \times 10^{-4}\ .v_A}{4} \Rightarrow v_M = 2 \times 10^{-4}\ .v_A$$

$v_M^2 = 4 \times 10^{-8}\ .v_A^2 \ll v_A^2, we\ then\ neglect\ v_M^2\ compared\ to\ v_A^2$

$v_A^2 = 2.g.Z_M \Rightarrow\ v_A = \sqrt{2.g.Z_M}$

2- $Q_v = S_A.v_A = S_A.\sqrt{2.g.Z_M}$

3- $v_M = \frac{S_A.\sqrt{2.g.Z_M}}{S}$

4- Speed v_M is expressed by:

$$v_M = -\frac{dZ_M}{dt}$$

$$-\frac{dZ_M}{dt} = \frac{S_A.\sqrt{2.g.Z_M}}{S}$$

$$\frac{dZ_M}{\sqrt{Z_M}} = -\frac{S_A.\sqrt{2.g}}{S}.dt$$

We integrate from $t = 0$ until the moment T when the reservoir has been completely emptied: $Z_M = Z_A = 0\ m$

$$\int_{Z_M}^{0} \frac{dZ_M}{\sqrt{Z_M}} = -\frac{S_A.\sqrt{2.g}}{S}.\int_0^{\tau} dt$$

$$-2\sqrt{Z_M} = \frac{S_A.\sqrt{2.g}}{S}.T$$

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$$-2\sqrt{Z_M} = \frac{S_A.\sqrt{2.g}}{S}.T$$
$$T = \frac{2\sqrt{Z_M}.S}{S_A.\sqrt{2.g}} = \frac{2\sqrt{2}\times 4}{8\times 10^{-4}\times\sqrt{2\times 9.81}} \approx 3193 \text{ s}$$

Exercise 6.3:

The aorta is the largest artery in the body. It receives the blood that leaves the heart and distributes it to the arteries throughout the body. The heart rate for an adult is 80 beats per minute. With each beat, the heart injects a volume $v_b = 0.075\ L$ into the aorta.

1- Calculate total volume V_t of blood flowing through the aorta in one minute. Deduce the volume flow Q_v.

2- Calculate average speed v_{moy} of blood flow knowing that the diameter of the aorta is $d = 2cm$.

3- Calculate the Reynolds number R_e for flow in the aorta knowing that the dynamic viscosity of the blood is$\eta = 5 \times 10^{-3} Pa.s$ and its density is $\rho = 1060\ Kg.m^{-3}$. Deduce the flow regime.

4- Determine the critical speed v_{critical} at which the regime becomes turbulent.

5- The blood distributed by the aorta ultimately reaches the capillaries. A blood capillary is an extremely thin blood vessel of medium radius $r_c = 5\mu m$. The blood circulates there at an average speed $v_{cap} = 0.06\ cm\ s^{-1}$ Calculate the volume flow rate Q_{cap} of blood in this capillary.

6- Determine the average number N_{cap} of capillaries present in the body in a human being.

Solution

1- The total volume of blood flowing through the aorta in one minute is:

$$V_t = 80 \times v_b = 80 \times 0.075 = 6\ L$$
$$Q_v = 6L.minute^{-1} = \frac{6 \times\ 10^{-3} m^3}{60\ s} = 10^{-4} m^3.s^{-1}$$

2- $Q_v = v_{moy}.S$

S is the section of the aorta:

$$S = \pi.\left(\frac{d}{2}\right)^2 = \pi.\left(\frac{2\times\ 10^{-2}}{2}\right)^2 = 3.14\times\ 10^{-2}m^2$$

$$v_{moy} = \frac{Q_v}{S} = \frac{10^{-4}}{3.14\times\ 10^{-2}} = 0.32\ m.s^{-1}$$

3- The Reynolds number is defined by:

$$R_e = \frac{\rho.v_m.d}{\eta} = \frac{1060\times 0.32\times 2\times\ 10^{-2}}{5\times\ 10^{-3}} = 1356.8$$

$$R_e < 2000\ The\ flow\ regime\ is\ laminar$$

4- The regime becomes turbulent for $R_e > 3000$. The critical speed from which the flow becomes turbulent is for $R_e = 3000$.

$$v_{\text{critical}} = \frac{\eta.R_e}{\rho.d} = \frac{5\times\ 10^{-3}\times 3000}{1060\times 2\times\ 10^{-2}} = 0.71m/s$$

5- $Q_{cap} = v_{cap}.S_{cap} = v_{cap}.\pi.r^2 = 6\times\ 10^{-4}\times\pi\times(5\times\ 10^{-6})^2 = 4.7\times\ 10^{-14}m^3/s.$

6- $Q_v = N_{cap}.Q_{cap}$

$$N_{cap} = \frac{Q_v}{Q_{cap}} = \frac{10^{-4}}{4.7\times\ 10^{-14}} = 2.13\times\ 10^{10}$$