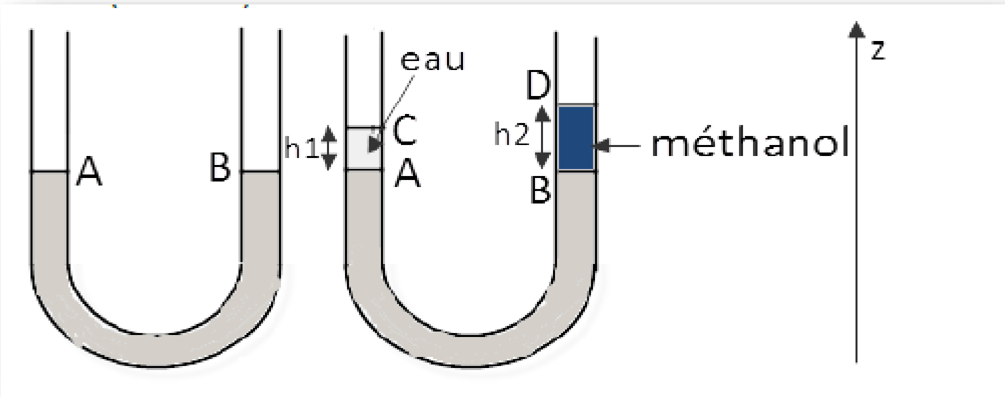
Tutorial N°5: Exercises on Pascal's law and Archimedes thrust. (Hydrostatic)

Exercise 5.1:

To determine the volume mass of ethanol ρ_{ethanol}, glycerin is introduced into a U tube. In the left branch, water of density $\rho_{\text{water}} = 1000\ kg.m^{-3}$ is poured over a height $h_1 = 10cm$, which causes a difference in level between the points A and B . To bring the points A and B back to the same height, methanol is poured over a height $h_1 = 12.5\ cm$ (diagram).



1. Write the fundamental hydrostatic relationship for the three fluids.

2. Deduce the volumic mass (density) of ethanol ρ_{ethanol}

Solution

1- The fundamental hydrostatic relationship for the three fluids:

Glycerin: $P_A - P_B = 0$ (1)

Water: $P_A - P_C = \rho_{water}.h_1 . g$ (2)

Methanol $P_B - P_D = \rho_{\text{Methanol}}.h_2 . g$ (3)

2- So we have:

from (1) $P_A = P_B$

from (2) $P_A = P_C + \rho_{water}.h_1 . g$

from (3) $P_B = P_D + \rho_{\text{Methanol}}.h_2 . g$

We also have: $P_C = P_D = P_{atm}$

from where: $\rho_{\text{Methanol}}.h_2 .g = \rho_{water}.h_1 .g$

$$\rho_{\text{Methanol}} = \frac{\rho_{water}.h_1}{h_2} = \frac{1000 \times 10}{12.5} = 800Kg.m^{-3}$$

Exercise 5.2:

A hollow steel sphere of density $\rho_{steel} = 7600\ Kg.m^{-3}$ and radius $r = 20cm$ and thickness$e = 5mm.$

1- Determine the weight of this sphere.

2- Determine the Archimedes' thrust that would be exerted on this sphere if it were totally immersed in water.

3- Determine the force that Archimedes would exert on this ball if it were completely submerged in water.

4- Could this sphere float on the surface of water? If yes, then what is the fraction of its submerged volume?

Solution

1-The volume of the hollow sphere V_{HC} is

Volume of the hollow sphere V_{HC} = Volume of the sphereV_S − vacuum volume V_V

$$V_{HC} = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi(r-e)^3 = \frac{4}{3}\pi[r^3 - (r-e)^3] = \frac{4}{3}\pi[(0.2)^3 - (0.2 - 0.008)^3]$$

$$V_{HC} = 3.86 \times 10^{-3} m^3$$

The weight P_{HC} of the hollow sphere is then:

$$P_{HC} = mg = \rho_{steel}.V_{HC}.g = 7600 \times 3.86 \times 10^{-3} \times 9.81 = 287.79N$$

2. The Archimedes thrust for the totally submerged ball is the weight of the displaced volume of the water therefore:

$$\boldsymbol{\pi = \rho_{water}.V_S.g = \rho_{water}.}\frac{4}{3}\pi r^3\boldsymbol{.g = 1000.}\frac{4}{3}\pi.(0.2)^3.9.81 = 328.74N$$

The ball will float because the Archimedes π thrust is greater than its weightP_{HC}:

$$\pi > P_{HC}$$

3- The volume of the submerged part of the ball is equal to the volume of the water V_{water} displaced. At equilibrium, the Archimedes thrust π is equal to the weight of the volume of water displaced: $\pi = P_{HC}$

$$\rho_{water} V_{water} \cdot g = P_{HC}$$

$$V_{water} = \frac{P_{HC}}{\rho_{water} \cdot g} = \frac{287.79}{1000 \times 9.81} = 2.93 \times 10^{-2} m^3$$

1- Knowing that the volume of the sphere is:

$$V_S = \frac{4}{3} \times \pi \times (0.2)^3 = 3.35 \times 10^{-2} m^3$$

The fraction of the submerged volume of the ball compared to its volume:

$$\frac{V_{water}}{V_S} = \frac{2.93 \times 10^{-2} m^3}{3.35 \times 10^{-2} m^3} = 0.87 = 87\%$$

Exercise 5.3:

A very fine capillary tube with a radius r is introduced into a tank filled with water.

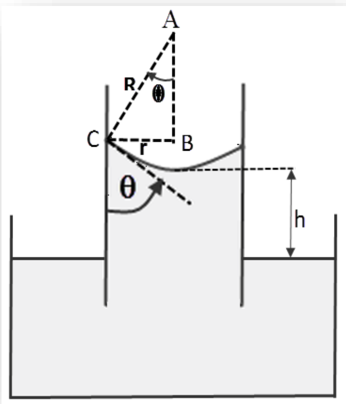
1. What phenomenon do we observe? Explain the phenomenon.

2. Demonstrate Jurin's law; the height h as a function of surface tension γ, contact angle θ, radius r of the capillary tube, density of water ρ and acceleration of gravity g.

3. Assuming that the raw sap is perfectly wetting and has the same properties as water: and, calculate the height of ascent: $\rho = 1000\ Kg.m^{-3}$ and $\gamma = 73.10^{-3} N/m$. calculate the height of sap rise in rayon xylene channels $r = 25 \mu m$

Solution

1. We observe the rise of water in the capillary tube of an height h due to the Laplace pressure difference. The reverse pressure due to the weight of the riser in the capillary will limit the rise of the water to a height h



2- The pressure difference ΔP Laplace due to surface tension γ is expressed by:

$$\Delta P = \frac{2\gamma}{R}$$

R : is the radius of curvature of the meniscus (interface between water and air).

In the triangle ABC the cosine of the contact angle θ verifies $\cos\theta = \frac{r}{R}$ hence $R = \frac{r}{\cos\theta}$

r is the radius of the capillary tube. ΔP is then written:

$$\Delta P = \frac{2\gamma\ \cos\theta}{r}$$

The water rises to a height h until the hydrostatic pressure π of the water riser in the balance tube ΔP, knowing that $\pi = \rho.g.h$, at pressure equilibrium we have: $\pi = \Delta P\ \Rightarrow \rho.g.h = \frac{2\gamma\ \cos\theta}{r}$

hence Jurin's law: $h = \frac{2\gamma\ \cos\theta}{r.\rho.g}$

3. The sap is perfectly wet: $\theta^0 = 0\ r = 25\mu m = 25.10^{-6}m$

$$h = \frac{2\times\ 73.10^{-3}\times\cos 0}{25.10^{-6}\times 1000\times 9.81} = 0.595m$$