Modeling and Simulation

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Introduction to Probability

- Probability is a fundamental concept in mathematics and statistics that quantifies the likelihood of an event occurring. It provides a framework for reasoning about uncertainty and randomness in various scenarios.
- At its core, probability seeks to answer questions such as: What is the chance of a coin landing heads up? Will it rain tomorrow? How likely is it for a stock to increase in value? By assigning probabilities to outcomes, we gain insight into the inherent unpredictability of the world around us.

Applications in Various Fields

- Probability finds widespread applications across diverse fields, from finance to engineering to healthcare. In finance, probabilistic models are used to forecast asset prices, assess investment risks, and design trading strategies.
- Engineers employ probability theory to analyze the reliability of systems, optimize processes, and predict failures.
- In healthcare, probabilities inform medical diagnoses, treatment decisions, and epidemiological predictions.

What is the probability?

- Definition: Probability is a fundamental concept used to quantify uncertainty or likelihood in various situations. It is a measure of the likelihood that an event will occur, ranging from 0 (impossible) to 1 (certain). In between, probabilities are expressed as decimals or percentages.
  Example: Probability of rolling a six-sided die and getting a 3.
- Illustration: Explain that if the die is fair, each outcome (1 through 6) is equally likely, so the probability of rolling a 3 is 1/6.

Sample Space

- Definition: This refers to the set of all possible outcomes of an experiment. For example, when rolling a fair six-sided die, the sample space is 1, 2, 3, 4, 5, 6.

Figure 1: Die

Events

- Definition: An event is a subset of the sample space, i.e., a collection of outcomes. Events can range from single outcomes to combinations of outcomes.
- Example: Getting an even number when rolling a six-sided die (Event: 2, 4, 6).
- The probability of an event A, denoted as P(A), is the likelihood of that event occurring. It is calculated by dividing the number of outcomes favorable to event A by the total number of outcomes in the sample space.
- $$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes in the sample space}}$$

Probability Axioms

- Axiom 1: Probability of any event is a number between 0 and 1 ($0 \leq P(E) \leq 1$).
- Axiom 2: Probability of the sample space is 1 (P(S) = 1).
- Axiom 3: If A and B are disjoint events, then $P(A \cup B) = P(A) + P(B)$.

Basic Probability Rules

- Complement Rule: The probability of the complement of an event (not A) happening is 1 minus the probability of the event A. P(A') = 1 - P(A).
- Union Rule: The probability of either event A or event B occurring is the sum of their individual probabilities, minus the probability of their intersection if they are not mutually exclusive. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- Intersection Rule: The probability of both event A and event B occurring is the product of their individual probabilities, if they are independent events. $P(A \cap B) = P(A)xP(B)$.

Conditional Probability:

- This is the probability of an event occurring given that another event has already occurred. It's denoted as P(A|B) and is calculated as the probability of both events A and B occurring, divided by the probability of the conditioning event B.
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$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Bayes' Theorem

- P(A | B) = [P(B | A) * P(A)] / P(B).
  Example: A medical test for a disease and the probability of having the disease given a positive test result.
- Let $X = A_1, A_2, ..., A_n$ be a complete system of states (also called events).
  For any event B such that P(B) $\neq 0$, *then* :
- $P(A_i/B) = [P(B/A_i) * P(A_i)]/[(P(B/A_1) * P(A_1)) + ... + (P(B/A_n) * P(A_n))]$

Examples

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Markov Chains

- Markov Chains are powerful mathematical models used to represent systems that transition from one state to another in a probabilistic manner.
- These chains are named after the Russian mathematician Andrey Markov and find applications in various fields including finance, biology, engineering, and computer science.
- In this presentation, we will delve into the fundamentals of Markov Chains and explore their practical applications in modeling and simulation.

Basics of Markov Chains

- Markov Chains are characterized by their memoryless property, where the future state of the system depends solely on its current state and not on the sequence of events leading up to it.
- They consist of a finite or countably infinite set of states and transition probabilities between these states.
- The probability of transitioning from one state to another is encapsulated in a transition matrix, which fully describes the dynamics of the system.

Components of a Markov Chain

- State Space: Represents the set of all possible states that the system can occupy.
- Transition Matrix: A square matrix where each element P_{ij} represents the probability of transitioning from state i to state j.
- Initial State Distribution: Specifies the probabilities of the system starting in each state, typically represented as a probability vector.

Markov Chain properties

- Markov Chains are suggested to model phenomena:
- that can be described by a set of states (situations):
- that change over time
- according to a probabilistic process
- where we can know the probabilities of transition from one state at time t to another state at time t+1
- and this through a study of statistics

Markov Chains - Formalism: (1)

- A Markov Chain is described by:
- $T = 0, 1, 2, 3, \&$
- A set of states $X = x1, x2, \&, xi, \&, xm$
- All transition probabilities $P_i j$ from state xi at time t to state xj at time t+1,
- These probabilities $Pij(t)$ are represented in a Matrix P called the Markov matrix (or transition matrix), of dimension m*m

Example of a Markov Chain (1)

- A study of the weather in a city located in a country north of the planet, at the end of summer, showed the following observations:
- If it snows today, it is possible to snow tomorrow with a probability of 60%, however in the remaining cases, there is an equal probability of having either a sunny day or rain the next day,
- If we have a sunny day or rain, then we have a one in four chance (1/4) of having the same thing the next day,
- If there is a change from a sunny day or rain, only half of the remaining cases involve this change to a snowy day.

State transitions through Markov chains

- In order to predict future situations for a probabilistic dynamic system, a MC can be used,
- A forecast by a MC is based on processing (calculation) probabilities on the transition matrix;
- Explanation through example:

Example: Taxi delivery in a city (1)

- Let's assume a taxi agency with three (3) branches in a city. The branches are: 1. Downtown (A) 2. East (B) 3. West (C)
- Statistics on deliveries gave the following values:
- Phone calls to the downtown branch: 30% are delivered downtown, 30% are delivered in the East, and 40% are delivered in the West
- Calls to the East branch: 40% are delivered downtown, 40% are delivered in the East, and 20% are delivered in the West.
- Calls to the West branch: 50% are delivered downtown, 30% are delivered in the East, and 20% are delivered in the West.

Example: Taxi delivery in a city (2)

- After making a delivery, the taxi goes to the nearest location to make the next delivery,
- The delivery time is estimated to be 15 minutes,
- $T = 0, 1 * U, 2 * U, 3 * U, \& / U = 15min$
- Question:
- If we assume that a taxi starts at branch C, what is the probability that it will be in branch B after two (2) deliveries?

Markov Chain Properties

- Ergodicity: A property where the system tends to visit all states with positive probability over time and the transition probabilities stabilize.
- Stationary Distribution: Represents a distribution over the states that remains unchanged by the transition process.
- Absorbing States: States from which there is no way to leave once entered, often used in models with terminal conditions.

Applications of Markov Chains

- Finance: Modeling stock prices, portfolio optimization, risk assessment.
- Biology: Modeling genetic mutations, population dynamics, epidemiology.
- Engineering: Reliability analysis, fault diagnosis, stochastic processes in manufacturing.
- Computer Science: PageRank algorithm, network protocols, machine learning (e.g., Hidden Markov Models).

Conclusion

- Markov Chains provide a versatile framework for modeling and simulating systems with stochastic behavior.
- Understanding their properties and applications enables the analysis and prediction of complex phenomena in various domains.
- Continued research and development in Markov Chain theory contribute to advancements in modeling, simulation, and decision-making processes.

1. Set Theory
2. Linear Algebra
3. Calculus
4. Probability
5. Modeling and Simulation of Dynamic Systems