# 2014/2 ENGINEERING DEPARTMENTS PHYSICS 2

## RECITATION 1

## (Electrical Fields)

1.  A conductive sphere is charged with **-0.4 nC**. The conductive sphere is touched to another identical uncharged sphere. The spheres are then separated from each other. Second sphere is allowed to touch third uncharged sphere and then they are separated from each other. As a result, calculate the number of electrons and charges of the spheres.

The spheres are identical. Therefore, the charges of 1st and 2nd spheres are (after 1st touching);

$$q_1 = \frac{1}{2}(-0.4 \times 10^{-9}) = -2 \times 10^{-10} \, C$$

The number of electrons;

$$N_1 = \frac{2 \times 10^{-10}}{1.602 \times 10^{-19} \, C/electron} = 1.25 \times 10^{9} \, elektron$$

Charges of 2nd and 3rd spheres are (after 2nd touching);

$$q_2 = q_3 = \frac{1}{2}(-2 \times 10^{-10}) = -1 \times 10^{-10} \, C$$

The number of electrons of 2nd and 3rd spheres ;

$$N_2 = N_3 = \frac{1 \times 10^{-10}}{1.602 \times 10^{-19}} = 6.2 \times 10^{8} \, elektron$$

**2.** Three point charges are located at the corners of an equilateral triangle as shown in **Figure 1.** Calculate the resultant electric force on the **7μC** charge.
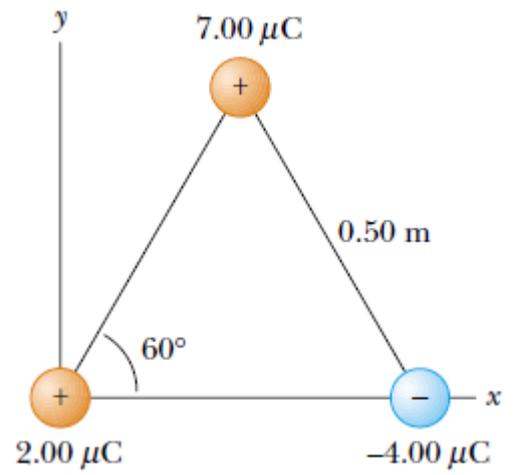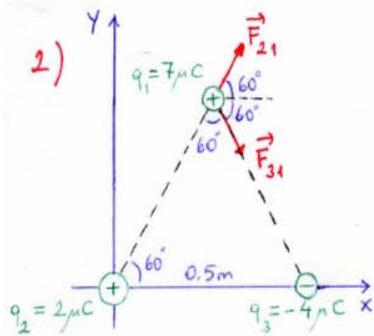


**Figure 1**

**2)**



The resultant electric force on the 7μC charge;

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31}$$

Find the $F_{21}$ and $F_{31}$ forces

$$|\vec{F}_{21}| = F_{21} = k\frac{q_2 q_1}{r_{21}^2} \qquad k = 9 \cdot 10^9 \, N \, m^2/C^2$$

$$F_{21} = 9 \cdot 10^9 \frac{2 \cdot 10^{-6} \cdot 7 \cdot 10^{-6}}{0,5^2} = 0,504 \, N$$

$$\vec{F}_{21} = F_{21} \cos 60 \, \hat{\imath} + F_{21} \sin 60 \, \hat{\jmath} = 0,252 \, \hat{\imath} + 0,436 \, \hat{\jmath} \quad (N)$$

$$F_{31} = k\frac{q_3 q_1}{r_{31}^2} = 9 \cdot 10^9 \frac{4 \cdot 10^{-6} \cdot 7 \cdot 10^{-6}}{0,5^2} = 1,008 \, N$$

$$\vec{F}_{31} = F_{31} \cos 300 \, \hat{\imath} + F_{31} \sin 300 \, \hat{\jmath} = F_1 \cos 60 \, \hat{\imath} + F_{31} \sin (-60) \, \hat{\jmath}$$

$$\vec{F}_{31} = 0,504 \, \hat{\imath} - 0,873 \, \hat{\jmath} \quad (N)$$

The resultant electric force;

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31}$$

$$\vec{F}_1 = (0,252 \, \hat{\imath} + 0,436 \, \hat{\jmath}) + (0,504 \, \hat{\imath} - 0,873 \, \hat{\jmath})$$

$$\vec{F}_1 = 0,756 \, \hat{\imath} - 0,437 \, \hat{\jmath} \quad (N)$$

$$|\vec{F}_1| = F_1 = \sqrt{(0,756)^2 + (-0,437)^2} = 0,873 \, N$$

3. Four identical particles, each having charge **+q**, are fixed at the corners of a square of side **L**. A fifth point charge **-Q** (at **P** point) lies a distance **z** along the line perpendicular to the plane of the square and passing through the center of the square (**Figure 2**). Determine the force exerted by the other four charges on **-Q**?
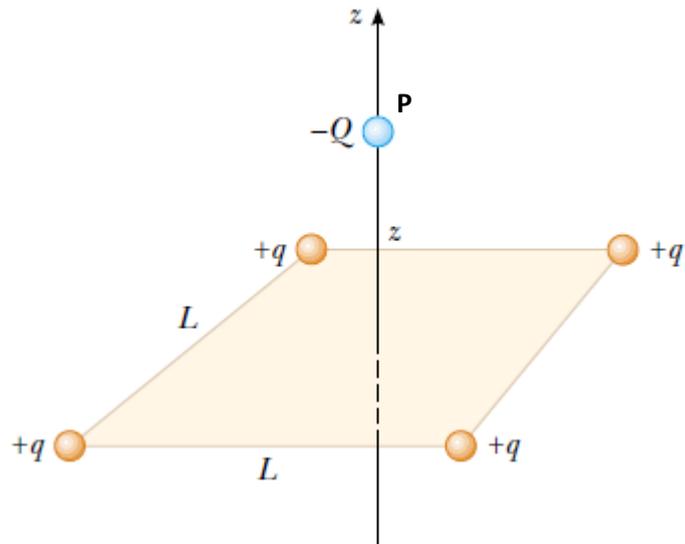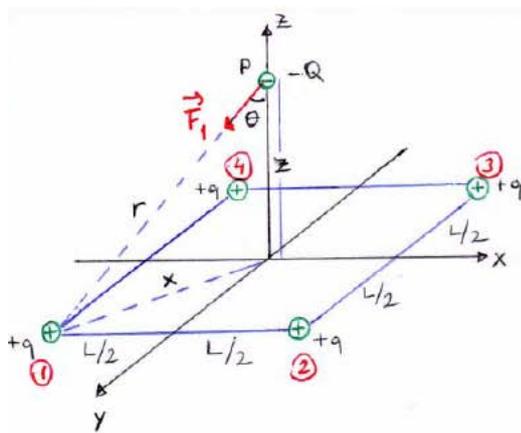


**Figure 2**



$$F_1 = F_2 = F_3 = F_4 = k\frac{qQ}{r^2}$$

Due to the system symmetrically, total $F_x=0$

$$\vec{F}_{12} = -k\frac{qQ}{r^2}\cos\theta\,\hat{k}$$

$$\cos\theta = \frac{z}{r} \qquad \vec{F}_{12} = -k\frac{qQ}{r^3}z\,\hat{k}$$

$$\sum\vec{F}_z = 4\vec{F}_{12} \qquad \left\{x = \frac{\sqrt{2}}{2}L \quad ve \quad r = \sqrt{z^2 + \left(\frac{L\sqrt{2}}{2}\right)^2}\right\}$$

$$\sum\vec{F}_z = -4k\frac{qQ}{\left(z^2 + \frac{L^2}{2}\right)^{3/2}}z\,\hat{k}$$

**4.** A small, **2 g** plastic ball is suspended by a **20 cm** long string in a uniform electric field as shown in **Figure 3**. If the ball is in equilibrium when the string makes a **15.0°** angle with the vertical, what is the net charge on the ball?
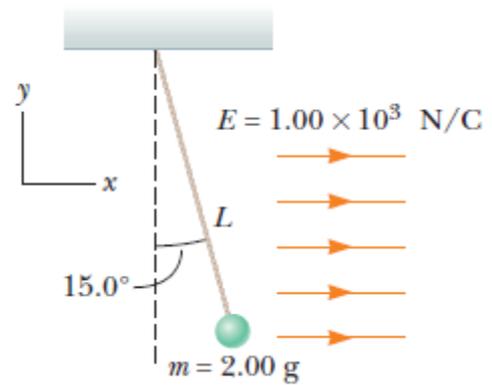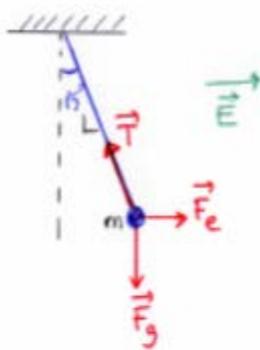
$E = 1.00 \times 10^3$ N/C

$L$

$15.0°$

$m = 2.00$ g

**Figure 3**

From free-body diagram,

$$\sum \vec{F_y} = 0$$

$$T\cos 15° - F_g = 0$$

$$T = \frac{mg}{\cos 15°} = \frac{2 \cdot 10^3 \cdot 9,8}{\cos 15} = 2,03 \cdot 10^{-2} N$$

Same way,

$$\sum \vec{F_x} = 0$$

$$T\sin 15° - qE = 0$$

$$q = \frac{T\sin 15}{E} = \frac{2,03 \cdot 10^{-3} \sin 15}{1 \cdot 10^3} = 5,25 \cdot 10^{-6} C$$

$$q = 5,25 \mu C$$

**5.** Three charged particles are aligned along the **y** axis as shown in **Figure 4**. Find the electric field at **P** point. Determine the electric force on a **Q** charge placed on the **P** point.
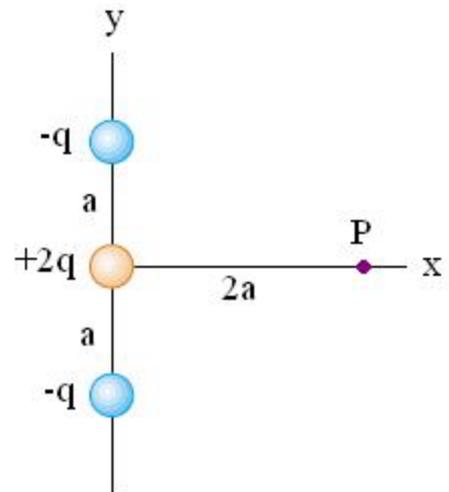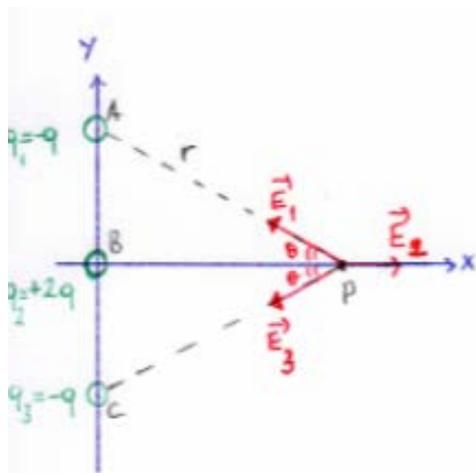


**Figure 4**



## The electric field at P point

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$E_1 = E_3 = k\frac{q}{r^2} = k\frac{q}{(a\sqrt{5})^2} = k\frac{q}{5a^2}$$

$$\vec{E}_1 = E_1 \cos\theta\,(-\hat{\imath}) + E_1 \sin\theta\,(\hat{\jmath})$$

$$\vec{E}_3 = E_3 \cos\theta\,(-\hat{\imath}) + E_3 \sin\theta\,(-\hat{\jmath})$$

$$E_2 = k\frac{2q}{(2a)^2} = k\frac{q}{2a^2}$$

$$\vec{E}_2 = k\frac{q}{2a^2}\,\hat{\imath}$$

$$r = |AP| = |CP| = \sqrt{a^2 + (2a)^2} = a\sqrt{5}$$

$$|AB| = |BC| = a$$

$$|BP| = 2a$$

$$\cos\theta = \frac{2a}{a\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$\sin\theta = \frac{a}{a\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\vec{E}_p = \frac{kq}{2a^2}\,\hat{\imath} - \frac{4kq}{5a^2\sqrt{5}}\,\hat{\imath} = \left(\frac{1}{2a^2} - \frac{4}{5a^2\sqrt{5}}\right)kq\,\hat{\imath} \quad \left(\tfrac{N}{C}\right)$$

$$\vec{F}_p = Q\cdot\vec{E}_p = \left(\frac{1}{2a^2} - \frac{4}{5a^2\sqrt{5}}\right)kqQ\,\hat{\imath} \quad (N)$$

6. **a)** Two point charges are located on **x-axis** (**Figure 5.a**). Determine the electric field exerted by **+q** at a point on a distance of **d.** Determine the electric force on **Q**.

**b)** Thin rod of length **2L** carry equal charges **q** uniformly distributed along their length. Determine the electric field exerted by the rod at a point on a distance of **d.** Determine the electric force on a **Q** charge placed on that point (**Figure 5.b**).

**c)** Identical thin rods of length **2L** carry equal charges **q** uniformly distributed along their lengths. The rods lie along the **x** axis with their centers separated by a distance **b – 2L** (**Figure 5.c**). Determine the magnitude of the force exerted by the right rod on the left one.
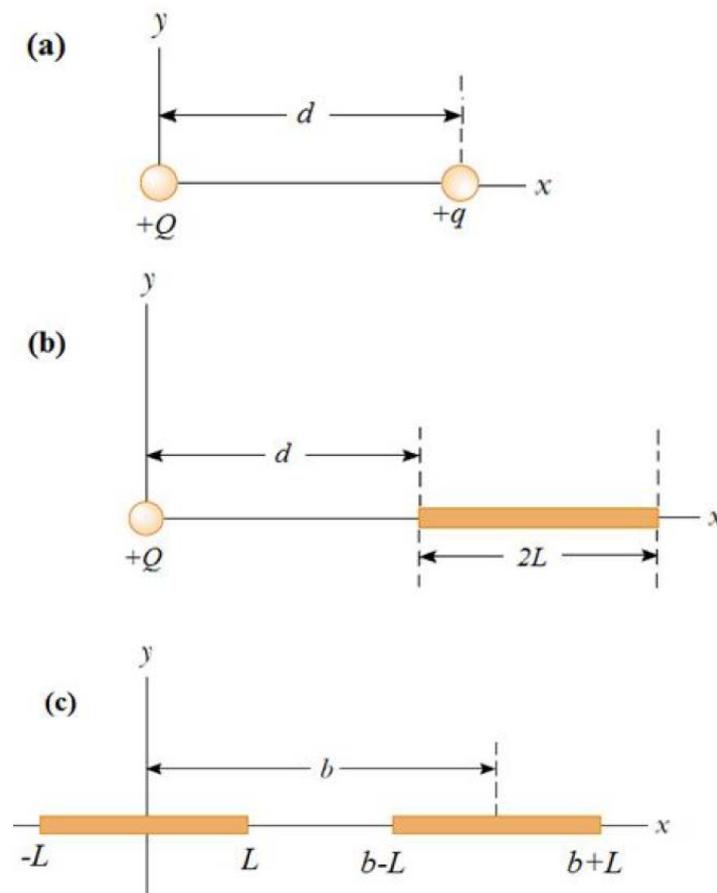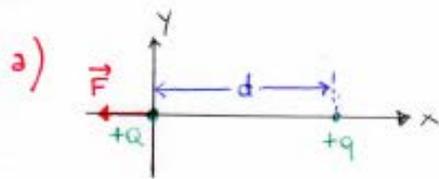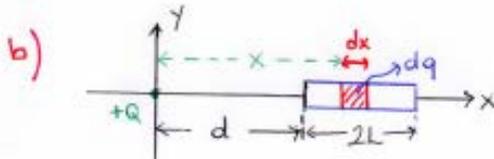


**Figure 5**

**a)**



$$E = k\frac{q}{d^2}$$

$$F = QE = k\frac{qQ}{d^2}$$

**b)**



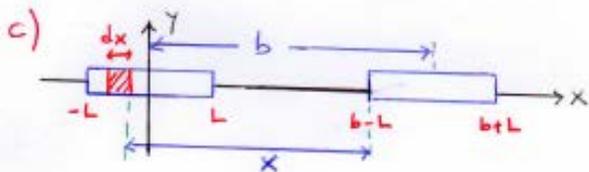$$E = \int dE = \int_d^{d+2L} k\frac{dq}{x^2}$$

$$dq = \lambda\, dx$$

$$\lambda = \frac{q}{2L}$$

$$E = k\lambda \int_d^{d+2L} \frac{dx}{x^2} = k\lambda \left| -\frac{1}{x} \right|_d^{d+2L}$$

$$E = k\frac{q}{2L}\left(-\frac{1}{d+2L} + \frac{1}{d}\right) = \frac{kq}{d(d+2L)}$$

$$F = QE = \frac{kqQ}{d(d+2L)}$$

**c)**



the left-hand rod creates this field at a distance $d$ from its right-hand end:

$$F = k\frac{qQ}{d(d+2L)}$$

$$dF = k\frac{q\,dq}{x(x+2L)} \qquad dq = \lambda\,dx = \frac{q}{2L}dx$$

$$F = \int_{b-2L}^{b} dF = \frac{kq^2}{2L}\int_{b-2L}^{b} \frac{dx}{x(x+2L)} \qquad \text{BİLGİ: } \int \frac{dx}{x(x+a)} = \frac{1}{a}\ln\left(\frac{x}{x+a}\right) + c$$

$$F = \frac{kq^2}{2L}\left[\frac{1}{2L}\ln\left(\frac{x}{x+2L}\right)\right]_{b-2L}^{b}$$

$$F = \frac{kq^2}{4L^2}\left[\ln\left(\frac{b}{b+2L}\right) - \ln\left(\frac{b-2L}{b-2L+2L}\right)\right] = \frac{kq^2}{4L^2}\ln\left(\frac{b}{b+2L}\cdot\frac{b}{b-2L}\right)$$

$$F = \frac{kq^2}{4L^2}\ln\left(\frac{b^2}{b^2-4L^2}\right)$$

**7.** A uniformly charged insulating rod of length **14 cm** is bent into the shape of a semicircle as shown in **Figure 6**. The rod has a total charge of **-7.5 μC**. Find the magnitude and the direction of the electric field at **O**, the center of the semicircle. Calculate the electric force on a **3 μC** charge placed on the **O** point.
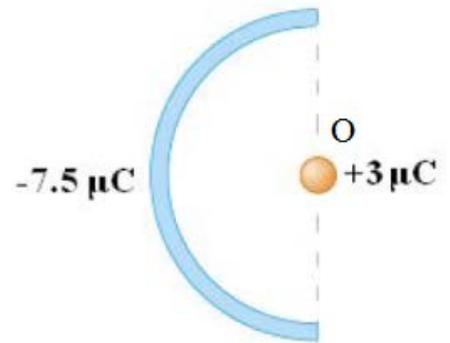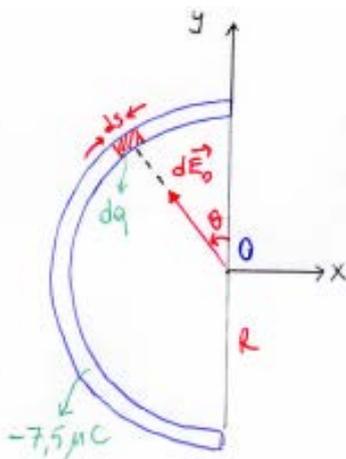
-7.5 μC   O   +3 μC

**Figure 6**

$$\int d\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\int d\vec{E}_o = \int d\vec{E}_{ox}(\hat{x}) + \int d\vec{E}_{oy}(\hat{y})$$

From symmetry, "$\int d\vec{E}_{oy}(\hat{y}) = 0$"

$$E_o = E_{ox} = \int dE_{ox} = \int dE_o \sin\theta$$

$$E_o = \int \frac{kdq}{R^2}\sin\theta = \int \frac{k \lambda R d\theta}{R^2}\sin\theta$$

$$E_o = \frac{k\lambda}{R}\int_0^\pi \sin\theta \, d\theta = \frac{k\lambda}{R}\left(-\cos\theta \Big|_0^\pi\right)$$

$$E_o = \frac{2k\lambda}{R}$$

$dq = \lambda ds = \lambda R d\theta$

$L = \frac{2\pi R}{2} = \pi R$

$\lambda = \frac{q}{L}$

$$E_o = \frac{2kq\pi}{L^2} = \frac{2 \cdot 9 \cdot 10^9 \cdot 7.5 \cdot 10^{-6} \cdot \pi}{0.14^2}$$
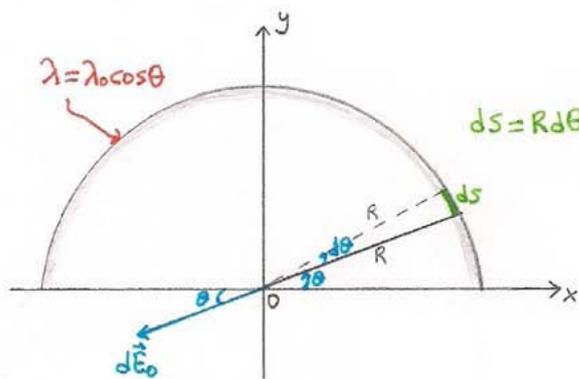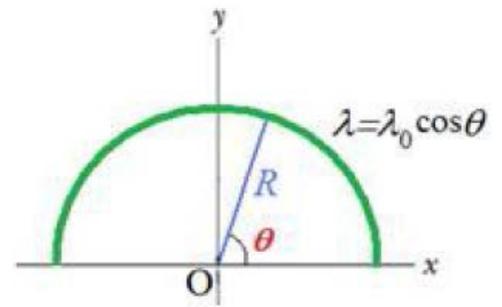
$$E_o = 2.16 \cdot 10^7 \,(N/C) = 21.6 \quad (MN/C)$$

Since the rod has a negative charge $\Rightarrow$ $\vec{E}_o = -21.6 \, \hat{x} \,(MN/C)$

$$\vec{F}_{+3\mu C} = q \cdot \vec{E}_o = 3 \cdot 10^{-6} \cdot 21.6 \cdot 10^6 \,(-\hat{x}) = -64.8 \, \hat{x} \,(N)$$

**8.** A line of positive charge is formed into semicircle of radius R as shown in Figure. The charge per unit length along the semicircle is described by the expression $\lambda = \lambda_0 \cos\Theta$.
Final;

  **a.** The total charge on the semicircle

  **b.** The electric field vector on point 'O'

a) $Q = \int \lambda \, ds$

$Q = \int_0^{\pi} \lambda_0 \cos\Theta \, R d\Theta$

$Q = \lambda_0 R \int_0^{\pi} \cos\Theta d\Theta$

$Q = \lambda_0 R \left[ \sin\Theta \right]_0^{\pi}$

$\boxed{Q = 0}$

$ds = R d\Theta$

b) $\vec{E_o} = \int_0^{\pi} d\vec{E_o} = \int_0^{\pi} (-dE_o \cos\Theta \, \hat{i} - dE_o \sin\Theta \, \hat{j})$

$dE_o = k \frac{dQ}{R^2} = k \frac{\lambda ds}{R^2} = k \frac{\lambda_0 \cos\Theta \, R d\Theta}{R^2} = k \frac{\lambda_0}{R} \cos\Theta d\Theta$

$\vec{E_o} = -k \frac{\lambda_0}{R} \int_0^{\pi} (\cos^2\Theta \, \hat{i} + \cos\Theta \sin\Theta \, \hat{j}) d\Theta$

$\cos 2\Theta = 2\cos^2\Theta - 1 = 1 - 2\sin^2\Theta$

$\sin 2\Theta = 2\sin\Theta \cos\Theta$

$\vec{E_o} = -k \frac{\lambda_0}{R} \left( \int_0^{\pi} \frac{1 + \cos 2\Theta}{2} d\Theta \, \hat{i} + \int_0^{\pi} \frac{\sin 2\Theta}{2} d\Theta \, \hat{j} \right)$

$\vec{E_o} = -k \frac{\lambda_0}{R} \left\{ \left[ \frac{\Theta}{2} + \frac{\sin 2\Theta}{4} \right]_0^{\pi} \hat{i} + \left[ -\frac{1}{2} \frac{\cos 2\Theta}{2} \right]_0^{\pi} \hat{j} \right\}$

$\boxed{\vec{E_o} = -k \frac{\pi \lambda_0}{2R} \hat{i}}$

**9.** A proton in a uniform electric field **640 N/C** accelerates from the quiescent. After a while, its speed reaches to **1.2 x 10⁶ m/s.**

(a) Calculate the acceleration of proton.

(b) How much time passed to reach this speed?

(c) What is the distance at the end of this period?

(d) Find the kinetic energy at the end of this period?

a) $F_e = m \cdot a \implies a = \dfrac{qE}{m} = \dfrac{1,602 \cdot 10^{-19} \cdot 640}{1,67 \cdot 10^{-27}} = 6,14 \cdot 10^{10} \, m/s^2$

b) $v_s = \cancel{v_i}^{0} + a t \implies t = \dfrac{v_s}{a} = \dfrac{1,2 \cdot 10^{6}}{6,14 \cdot 10^{10}} = 1,95 \cdot 10^{-5} s$

c) $x_s - \cancel{x_i}^{0} = \dfrac{1}{2} (\cancel{v_i}^{0} + v_s) t$

$\quad x_s = \dfrac{1}{2} (1,2 \cdot 10^{6})(1,95 \cdot 10^{-5})$

$\quad x_s = 11,7 \, m$

d) $K = \dfrac{1}{2} m v^2 = \dfrac{1}{2} \, 1,67 \cdot 10^{-27} (1,2 \cdot 10^{6})^2$

$\quad K = 1,2 \cdot 10^{-15} \, J$