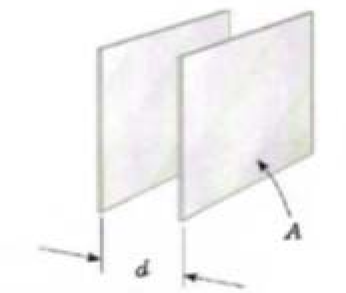
## 2014/2 ENGINEERING DEPARTMENTS PHYSICS 2
## RECITATION 4
## (CAPACITANCE AND DIELECTRICS/ CURRENT&RESISTANCE and DIRECT CURRENT CIRCUITS)

**1.** An air-filled capacitor consists of two parallel plates, each with an area of **200 cm²**, separated by a distance of **0.4 cm**.

**a)** Calculate the capacitance.

**b)** If the capacitor had been connected to a 500-V battery, calculate the charge on each plate, the stored energy, the electric field between the plates and the energy density of the capacitor.

**c)** If air had been replaced with a liquid of dielectric constant $\kappa$ =2.6, how much charge would have been flowed to the capacitor from the 500-V battery?




$A = 200\ cm^2 = 2.10^{-2}\ m^2$

$d = 0{,}4\ cm = 4.10^{-3}\ m$

a) $C_0 = \epsilon_0 \dfrac{A}{d}$

$C_0 = 8{,}85.10^{-12} . \dfrac{2.10^{-2}}{4.10^{-3}}$

$\boxed{C_0 = 4{,}4.10^{-11}\ (F)}$

$C_0 = 44\ (pF)$

b) $C_0 = \dfrac{Q_0}{\Delta V}$

$Q_0 = C_0\ \Delta V$

$Q_0 = 4{,}4.10^{-11} . 500$

$\boxed{Q_0 = 2{,}2.10^{-8}\ (C)}$

$Q_0 = 22\ (nC)$

$U_0 = \dfrac{Q_0^2}{2C_0}$

$U_0 = \dfrac{(2{,}2.10^{-8})^2}{2.\ 4{,}4.\ 10^{-11}}$

$\boxed{U_0 = 5{,}5.10^{-6}\ (J)}$

*or*

$U_0 = \frac{1}{2} C_0 (\Delta V)^2$

$U_0 = \frac{1}{2} . 4{,}4.10^{-11} . 500^2$

$U_0 = 5{,}5.10^{-6}\ (J)$

$E = \dfrac{\Delta V}{d}$

$E = \dfrac{500}{4.10^{-3}}$

$\boxed{E = 1{,}25.10^{5}\ (V/m)}$

$E = 125\ (kV/m)$

$u_E = \frac{1}{2} \epsilon_0 E^2$

$u_E = \frac{1}{2}\ 8{,}85.10^{-12} . (1{,}25.10^{5})^2$

$\boxed{u_E = 6{,}9.10^{-2}\ (J/m^3)}$

c) $C = \kappa C_0$

$C = 2{,}6 .\ 4{,}4.10^{-11}$

$C = 11{,}4.10^{-11}\ (F)$

$C = \dfrac{Q}{\Delta V}$

$Q = C\ \Delta V$

$Q = 11{,}4.10^{-11} . 500$

$Q = 5{,}7.10^{-8}\ (C)$

$Q = 57\ (nC)$

$\Delta Q = Q - Q_0$

$\Delta Q = 57 - 22$

$\Delta Q = 35\ (nC)$

$\boxed{\Delta Q = 3{,}5.10^{-8}\ (C)}$

**2.** For the system of capacitors shown in **Figure 1**,

**a)** Find the total energy stored by the group.

**b)** When the discharge takes place on $C_3$ capacitor to convert to a conductor, how much charge and potential on $C_1$ would have been changed?

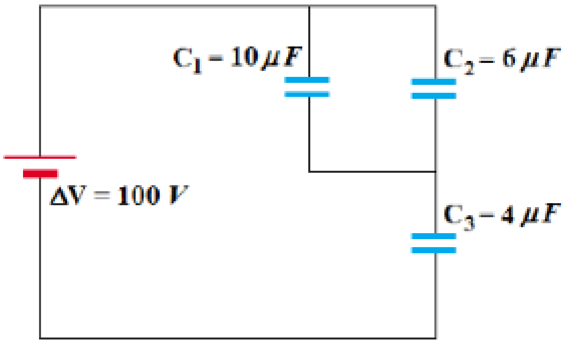
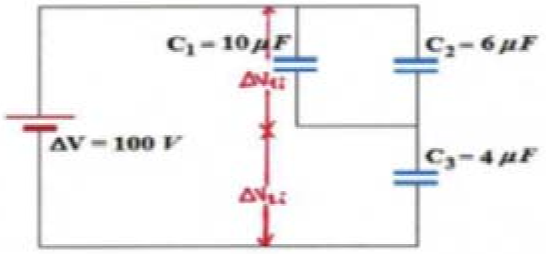



**Figure 1**




a) $U = \frac{1}{2} C_e (\Delta V)^2$

$$\frac{1}{C_e} = \frac{1}{C_3} + \frac{1}{C_1 + C_2}$$

$$\frac{1}{C_e} = \frac{1}{4} + \frac{1}{10+6}$$

$C_e = 3{,}2\ (\mu F)$

$U = \frac{1}{2}\, 3{,}2 \cdot 10^{-6} \cdot 100^2$

$\boxed{U = 1{,}6 \cdot 10^{-2}\ (J)}$

b) $Q = C_e \cdot \Delta V$

$Q = 3{,}2 \cdot 10^{-6} \cdot 100$

$Q = 3{,}2 \cdot 10^{-4}\ (c)$

Due to the charges on capacitors connected in series are the same;

$q_{1i} + q_{2i} = q_{3i} = Q$

$q_{1i} + q_{2i} = Q$

$C_1 \Delta V_{1i} + C_2 \cdot \Delta V_{1i} = Q$

$$\Delta V_{1i} = \frac{Q}{C_1 + C_2}$$

$\Delta V_{1i} = \frac{3{,}2 \cdot 10^{-4}}{10+6}$ ; $\Delta V_{1i} = 20\ (v)$

$q_{1i} = C_1 \Delta V_{1i}$

$q_{1i} = 10 \cdot 10^{-6} \cdot 20$ ; $q_{1i} = 2 \cdot 10^{-4}\ (c)$

After turning to the conductor,

Potential difference on $C_3$ is equal to zero

*Initial potential difference on $C_1$ is equal to* $\boxed{\Delta V_{1i} = 20\ (V)}$

*After $C_3$ capacitor turns to the conductor, final potential difference on $C_1$ is equal to* $\boxed{\Delta V_f = 100\ (V)}$

$\Delta V_f - \Delta V_{1i} = 100 - 20 = 80\ (v)$

$q_{1s} = C_1 \Delta V_f$

$q_{1s} = 10 \cdot 10^{-6} \cdot 100 = 10 \cdot 10^{-4}\ (c)$

$\Delta q = q_{1s} - q_{1i}$

$\Delta q = (10 - 2) \cdot 10^{-4}$ ; $\boxed{\Delta q = 8 \cdot 10^{-4}\ (c)}$

**3.** A parallel-plate capacitor has a plate separation of **1.2 cm** and a plate area of **0.12 m²**. The plates are charged to a potential difference of **120 V** and disconnected from the source. A dielectric slab having thickness **0.4 cm** and a dielectric constant of $\kappa = 2$ is inserted exactly halfway between the plates as shown in **Figure 2**.

**a)** What is the capacitance before the dielectric being placed?

**b)** Calculate the capacitance after the slab is introduced using these equations:

$$C = \frac{Q}{\Delta V} \text{ ve } \Delta V = V_b - V_a = -\int_a^b \vec{E}.d\vec{s}$$

**c)** Find the charge on the plates. Determine the electric fields in the region of the dielectric and absence of the dielectric?

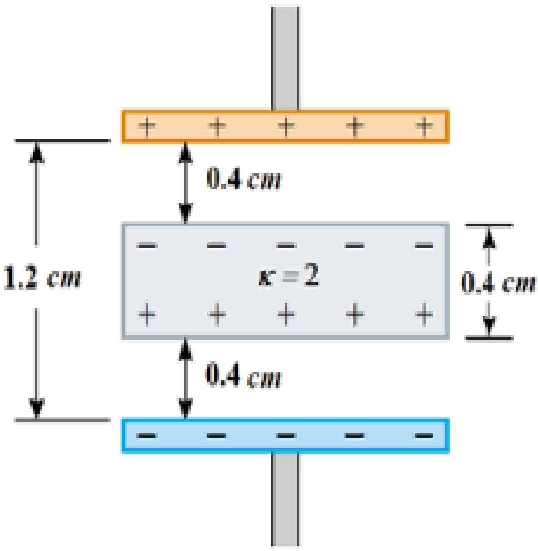
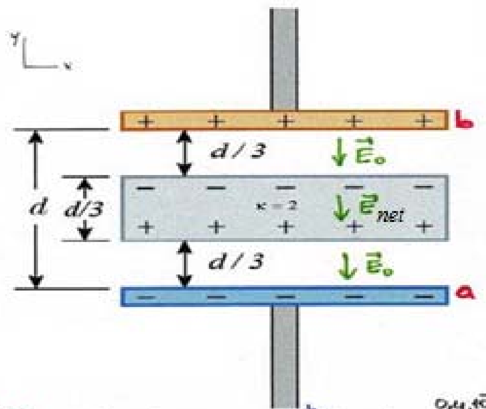



**Figure 2**




$d = 1,2 \text{ cm} = 1,2.10^{-2} \text{ m}$ $\kappa = 2$

$A = 0,12 \text{ m}^2$

$\Delta V = 120 \text{ V}$

a) $C_o = \epsilon_o \dfrac{A}{d}$

$C_o = 8,85.10^{-12} . \dfrac{0,12}{1,2.10^{-2}}$

$C_o \cong 9.10^{-11} \text{ (F)}$

$\boxed{C_o = 90 \text{ (pF)}}$

b) $\Delta V = V_b - V_a = -\int_a^b \vec{E}.d\vec{s} = -\int_0^{0,4.10^{-2}} E_o . dy . \cos 180° - \int_0^{0,4.10^{-2}} E_{net} dy . \cos 180° - \int_0^{0,4.10^{-2}} E_o . dy . \cos 180°$

$E_o = \dfrac{\sigma}{\epsilon_o} = \dfrac{Q}{A \epsilon_o}$ , $E_{net} = E_o - E_{ind} = \dfrac{E_o}{\kappa} = \dfrac{Q}{2 A \epsilon_o}$

$\Delta V = \dfrac{Q}{\epsilon_o A} . \dfrac{5}{2} \int_0^{0,4.10^{-2}} dy$

$\Delta V = \dfrac{5Q}{2 \epsilon_o A} . 0,4.10^{-2} = \dfrac{5 . Q . 0,4.10^{-2}}{2 . 8,85.10^{-12} . 0,12}$

$\Delta V = 0,94.10^{10} Q$

$C = \dfrac{Q}{\Delta V} = \dfrac{Q}{0,94.10^{10} Q}$ ; $C = 1,06.10^{-10} \text{ (F)}$

$\boxed{C = 106 \text{ (pF)}}$

c) $Q = C_o . \Delta V$

$Q = 90.10^{-12} . 120$

$\boxed{Q = 1,08.10^{-8} \text{ (C)}}$

$E_o = \dfrac{Q}{A \epsilon_o}$

$E_o = \dfrac{1,08.10^{-8}}{0,12 . 8,85.10^{-12}}$

$\boxed{E_o \cong 1.10^4 \text{ (V/m)}}$

$E_{net} = \dfrac{Q}{2 A \epsilon_o} = \dfrac{E_o}{\kappa}$

$E_{net} = \dfrac{1.10^4}{2}$

$\boxed{E_{net} = 5.10^3 \text{ (V/m)}}$

**4.** A conducting spherical shell has inner radius **a** and outer radius **c**. The space between these two surfaces is filled with a dielectric for which the dielectric constant is $\kappa_1$ between **a** and **b**, and $\kappa_2$ between **b** and **c** (**Figure 3**). Determine the capacitance of this system.

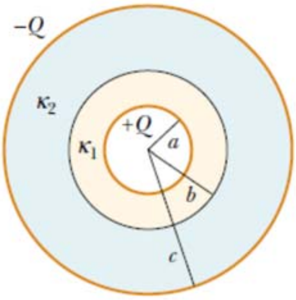



**Figure 3**

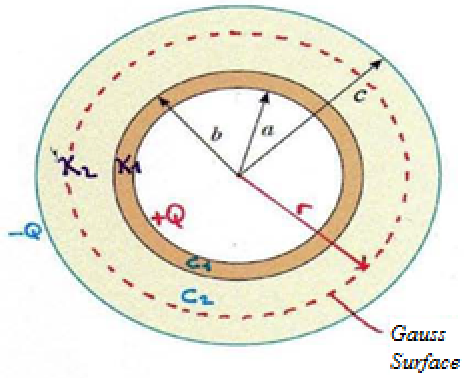
Electric field in the region between the conductors;

$(c<r<a)$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E\cdot(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = k\frac{Q}{r^2}$$




For the region with dielectric $K_1$

$$V_b - V_a = \Delta V_{ab} = -\int_a^b \vec{E}\cdot d\vec{s}$$

$$\Delta V_{ab} = -\int_a^b k\frac{Q}{r^2}dr = -kQ\left[-\frac{1}{r}\right]_a^b = kQ\left(\frac{1}{b}-\frac{1}{a}\right)$$

$$\Delta V_{ab} = kQ\frac{(a-b)}{ab} \qquad a-b<0 \qquad \Delta V_{ab}<0$$

For the region with dielectric $K_2$

$$V_c - V_b = \Delta V_{bc} = -\int_b^c \vec{E}\cdot d\vec{s}$$

$$\Delta V_{bc} = -\int_b^c k\frac{Q}{r^2}dr = -kQ\left[-\frac{1}{r}\right]_b^c = kQ\left(\frac{1}{c}-\frac{1}{b}\right)$$

$$\Delta V_{bc} = kQ\frac{(b-c)}{bc} \qquad b-c<0 \qquad \Delta V_{bc}<0$$

$$C = \frac{Q}{|\Delta V|} \quad ; \quad C_1 = K_1\frac{Q}{|\Delta V_{ab}|} = K_1\frac{ab}{k(b-a)}$$

$$C_2 = K_2\frac{Q}{|\Delta V_{bc}|} = K_2\frac{bc}{k(c-b)}$$

$$\frac{1}{C_e} = \frac{1}{C_1}+\frac{1}{C_2} \quad ; \qquad \boxed{C_e = \frac{K_1K_2\,abc\,(4\pi\epsilon_0)}{K_2bc - K_1ab + ac(K_1-K_2)}}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

5. A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in **Figure 4.** You may assume that $\ell >> d$

**a)** Find an expression for the capacitance of the device in terms of the plate area $A$ and $d, \kappa_1, \kappa_2$ and $\kappa_3$.

**b)** Calculate the capacitance using the values $A = 3cm^2, d = 1.5mm, \kappa_1 = 6, \kappa_2 = 3, \kappa_3 = 5$ and $\Delta V = 16V$.

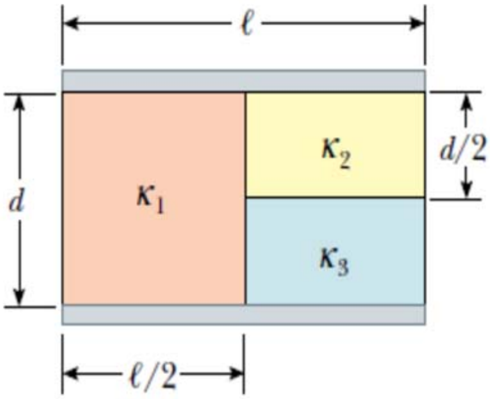



**Figure 4**

a) $C_2$ and $C_3$ capacitors are connected in series each other and parallel to $C_1$ capacitor

$$C_1 = \kappa_1 \epsilon_0 \frac{A/2}{d} \quad ; \quad C_2 = \kappa_2 \epsilon_0 \frac{A/2}{d/2} \quad ; \quad C_3 = \kappa_3 \epsilon_0 \frac{A/2}{d/2}$$

$$\left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \frac{C_2 C_3}{C_2 + C_3} = \frac{\epsilon_0 A}{d}\left(\frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3}\right)$$

$$C_e = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\epsilon_0 A}{d}\left(\frac{\kappa_1}{2} + \frac{\kappa_2 \kappa_3}{\kappa_2 + \kappa_3}\right)$$

b) Using the given values;

$$C_e = \frac{8{,}85 \cdot 10^{-12} \cdot 3 \cdot 10^{-4}}{1{,}5 \cdot 10^{-3}}\left(\frac{6}{2} + \frac{3 \cdot 5}{3+5}\right) = 8{,}63 \cdot 10^{-12} F$$

$$C_e = 8{,}63 \, pF$$

$$U = \frac{1}{2} C_e (\Delta V)^2 = \frac{1}{2} 8{,}63 \cdot 10^{-12} \cdot (16)^2 = 1{,}10 \cdot 10^{-9} J$$

$$U = 1{,}10 \, nJ$$

**6.** A copper wire ***2m*** long and ***4mm*** in diameter carries a current of ***6A***. If the conductor is copper with a free charge density of ***8.5x10²⁸(1/m³)*** and a resistivity of ***ρ=1.6x10⁻⁶ Ωcm,*** calculate,

**a)** the current density,
**b)** the electric field,
**c)** the resistance,
**d)** the average drift velocity of free electrons,
**e)** the power dissipated as heat in this wire. (***e=1.6x10⁻¹⁹ C, π=3***)

$2r = 4\ mm = 4.10^{-3}\ m$

$\ell = 2m$

$I = 6A$

$n = 8,5.10^{28}\ (1/m^3)$

$\rho = 1,6.10^{-6}\ \Omega.cm = 1,6.10^{-8}\ \Omega.m$

$A = \pi r^2 \qquad (\pi = 3)$

$A = \pi (2.10^{-3})^2$

$A \cong 1,26.10^{-5}\ (m^2)$

a) $J = \frac{I}{A}$

$J = \frac{6}{1,26.10^{-5}}$

$\boxed{J = 4,77.10^5\ (A/m^2)}$

b) $J = \sigma E$

$\rho = \frac{1}{\sigma}$

$J = \frac{E}{\rho} \qquad E = \rho J$

$E = 1,6.10^{-8} . 4,77.10^5$

$\boxed{E = 7,6.10^{-3}\ (V/m)}$

c) $R = \rho \frac{\ell}{A}$

$R = 1,6.10^{-8}\ \frac{2}{1,26.10^{-5}}$

$\boxed{R \cong 2,54.10^{-3}\ (\Omega)}$

d) $v_s$ : the average drift speed of free electrons in an electric field

$J = n e v_s$

$v_s = \frac{J}{ne}$

$v_s = \frac{4,77.10^5}{8,5.10^{28}.1,6.10^{-19}}$

$\boxed{v_s \cong 3,51.10^{-5}\ (m/s)}$

e) $P = I^2 R$

$P = 6^2 . 2,54.10^{-3}$

$\boxed{P \cong 9,1.10^{-2}\ (W)}$

1. Material with uniform resistivity $\boldsymbol{\rho}$ is formed into a wedge as shown in **Figure 5**. Find the resistance between face **A** and face **B** of this wedge.

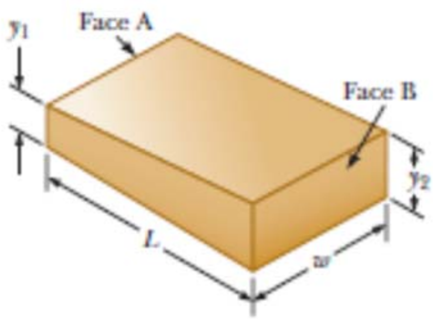
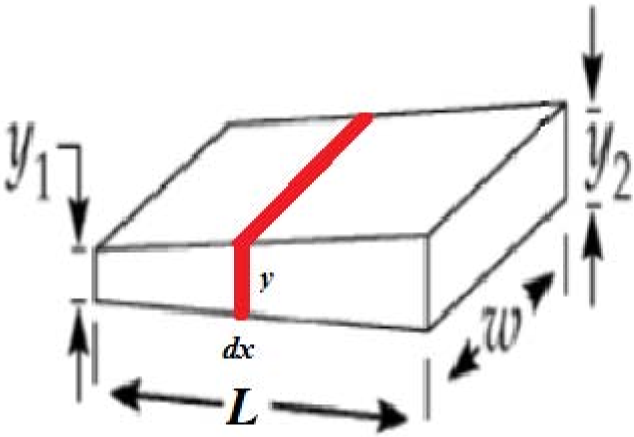



**Figure 5**




$$R = \int \frac{\rho\, dx}{A} = \int \frac{\rho\, dx}{w y}$$

where,

$$y = y_1 + \frac{y_2 - y_1}{L} x$$

$$R = \frac{\rho}{w} \int_0^L \frac{dx}{y_1 + \frac{y_2 - y_1}{L} x} = \frac{\rho L}{w(y_2 - y_1)} \ln \left[ y_1 + \frac{y_2 - y_1}{L} x \right] \Bigg|_0^L$$

$$R = \frac{\rho L}{w(y_2 - y_1)} \ln \left( \frac{y_1 + y_2 - y_1}{y_1} \right)$$

$$R = \frac{\rho L}{w(y_2 - y_1)} \ln \left( \frac{y_2}{y_1} \right)$$

**2.** For the circuit in **Figure 6**, find

**a)** the dissipated power for each resistance ($R_1$, $R_2$ and $R_3$).

**b)** the power supplied by $\varepsilon_1$ and $\varepsilon_2$ generators.

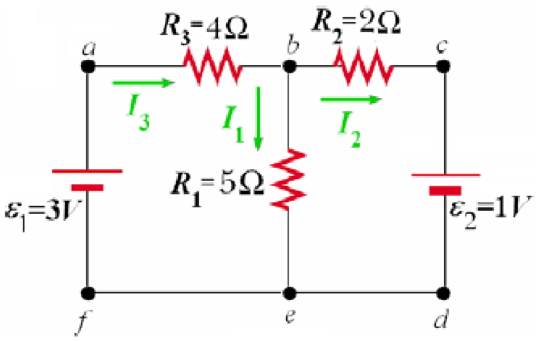



**Figure 6**

**1. Junction rule** $\sum_{\text{junction}} I = 0$

**2. Loop rule** $\sum_{\text{closed loop}} \Delta V = 0$

Kirchhoff's Rules

for abefa loop: $-I_3R_3 - I_1R_1 + \mathcal{E}_1 = 0$

$-4I_3 - 5I_1 + 3 = 0 \quad (2)$

for bcdeb loop: $-I_2R_2 - \mathcal{E}_2 + I_1R_1 = 0$

$-2I_2 - 1 + 5I_1 = 0 \quad (3)$

for b junction: $I_3 = I_1 + I_2 \quad (1)$

from (1), (2) and (3) equations; $I_1 = \frac{5}{19}(A)$, $I_2 = \frac{3}{19}(A)$, $I_3 = \frac{8}{19}(A)$

$P_{R_1} = I_1^2 R_1 = \boxed{\frac{125}{361}(W)}$ $\quad P_{R_2} = I_2^2 R_2 = \boxed{\frac{18}{361}(W)}$ $\quad P_{R_3} = I_3^2 R_3 = \boxed{\frac{256}{361}(W)}$

b) $P_{\mathcal{E}_1} = \mathcal{E}_1 I_3 = \boxed{\frac{24}{19}(W)}$ $\quad P_{\mathcal{E}_2} = \mathcal{E}_2 I_2 = \boxed{\frac{3}{19}(W)}$

**3.** In the circuit the capacitor is uncharged, the switch ***S*** closes at ***t = 0***, as in **Figure 7**.

**a)** Express the current $I$ in the circuit as functions of time and sketch $I = f(t)$ graph.

**b)** After the circuit becomes the steady-state, the switch ***S*** is opened. Find the time interval required for the charge on the capacitor to fall to one-second its initial value.

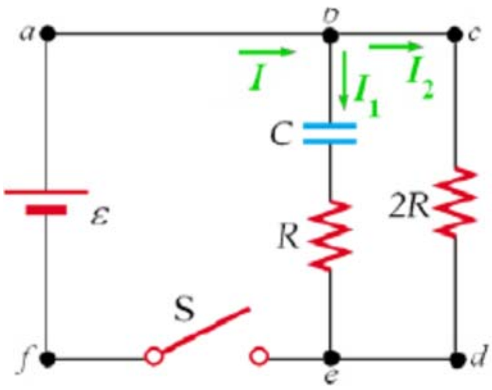



**Figure 7**

a) for b junction : $I = I_1 + I_2$

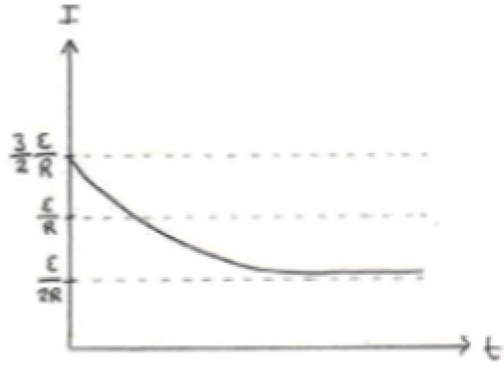
for acdfa loop : $\varepsilon - I_2 \cdot 2R = 0$ ; $I_2 = \frac{\varepsilon}{2R}$

$I(t) = I_1(t) + I_2$

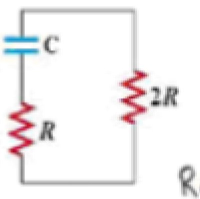
$$\boxed{I(t) = \frac{\varepsilon}{R} e^{-t/RC} + \frac{\varepsilon}{2R}}$$

$t = 0 \Rightarrow I(0) = \frac{\varepsilon}{R} + \frac{\varepsilon}{2R} = \frac{3}{2}\frac{\varepsilon}{R}$

$t \to \infty \Rightarrow I(\infty) = \frac{\varepsilon}{2R} \quad (e^{-\infty} = 0)$




b) $q(t) = Q e^{-t/RC}$




$R_e = R + 2R = 3R$

$q(t) = \frac{Q}{2}$ ; $\frac{Q}{2} = Q e^{-t/R_{e}C}$

$\frac{1}{2} = e^{-t/3RC}$

$\ln\left(\frac{1}{2}\right) = -\frac{t}{3RC}$

$t = -3RC \ln\left(\frac{1}{2}\right)$

$$\boxed{t = 3RC \ln 2}$$

4. If no charges exist on the capacitor before switch ***S*** is closed ***t = 0*** as in **Figure 8**.
**a)** Shortly after the switch ***S*** is closed, find the currents $I_1$, $I_2$ and $I_3$.
**b)** After the switch ***S*** has been closed for a length of time sufficiently long, find the currents $I_1$, $I_2$ and $I_3$.
**c)** After the switch ***S*** has been closed for long time, find the potential difference between ***a*** and ***b*** points.
**d)** Find the charge on the capacitor after the switch ***S*** has been closed for long time.

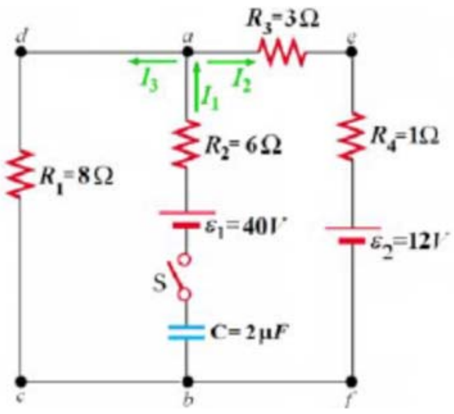



**Figure 8**

a) for a junction: $I_1 = I_2 + I_3$ (1)

for adcba loop: $-I_3R_1 + \mathcal{E}_1 - I_1R_2 = 0$

$-8I_3 + 40 - 6I_1 = 0$

$6I_1 + 8I_3 = 40$ (2)

for abfea loop: $I_1R_2 - \mathcal{E}_1 + \mathcal{E}_2 + I_2R_4 + I_2R_3 = 0$

$6I_1 - 40 + 12 + I_2 + 3I_2 = 0$

$6I_1 + 4I_2 = 28$ (3)

from (1), (2) and (3) equations; $\boxed{I_1 \cong 3{,}7(A)}$ $\boxed{I_2 \cong 1{,}5(A)}$ $\boxed{I_3 \cong 2{,}2(A)}$

b) In the steady-state, there is no current through the capacitor (ab). $\boxed{I_1 = 0}$

for defcd loop: $-I_2R_3 - I_2R_4 - 12 - I_2R_1 = 0$

$-3I_2 - I_2 - 12 - 8I_2 = 0$

$\boxed{I_2 = -1\,(A)}$ $\boxed{I_3 = 1\,(A)}$

c) $V_a - I_3R_1 = V_b$

$\boxed{V_a - V_b = 8\,(V)}$

d) $V_a - \mathcal{E}_1 + V_c = V_b$

$V_a - V_b = \mathcal{E}_1 - V_c$

$8 = 40 - V_c$

$\boxed{V_c = 32\,(V)}$

$Q = CV_c$

$Q = 2.10^{-6}.32$

$Q = 64.10^{-6}\,C$

$\boxed{Q = 64\,\mu C}$

**5.** In the circuit shown in **Figure 9**,

**a)** After the switch ***S*** has been closed for a length of time sufficiently long, find the currents on each resistance.

**b)** Find the charges for each capacitors and the dissipated power on the resistance $R_2$.

**c)** If the switch ***S*** is opened, find the time constant of the discharging circuit.

**d)** After the switch ***S*** is opened, write the current on the resistance $R_1$ as a function of time.

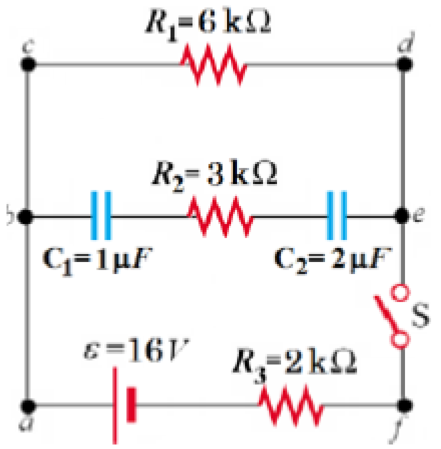



**Figure 9**

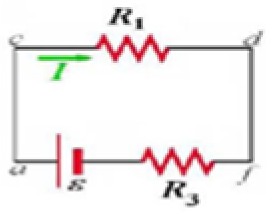
a) In the steady-state, there is no current through the capacitors (**be**) $\boxed{I_{R_2} = 0}$

for cdfac loop : $-IR_1 - IR_3 + \mathcal{E} = 0$

$$I = \frac{\mathcal{E}}{R_1 + R_3}$$

$$I = \frac{16}{(6+2).10^3} = 2.10^{-3}\ (A)$$

$$\boxed{I_{R_1} = I_{R_3} = 2(mA)}$$




b) for cdebc loop : $-IR_1 + \frac{Q}{C_1} + \frac{Q}{C_2} = 0$

$$-2.10^{-3} . 6.10^{3} + Q\left(\frac{1}{1.10^{-6}} + \frac{1}{2.10^{-6}}\right) = 0$$

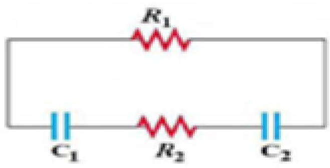
$$Q = 8.10^{-6}\ (C)$$

$$\boxed{Q = 8(\mu C)}$$

$$P_{R_2} = I_{R_2}^2 R_2$$

$$\boxed{P_{R_2} = 0} \qquad (I_{R_2} = 0)$$

c)




$$\tau = R_e . C_e$$

$$\tau = 9.10^3 . 6{,}66.10^{-7}$$

$$\tau = 6.10^{-3}\ (s)$$

$$\boxed{\tau = 6\ (ms)}$$

$$R_e = R_1 + R_2$$

$$R_e = 9\ (k\Omega)$$

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_e = 6{,}66.10^{-7}\ (F)$$

d) $I(t) = -\frac{Q}{\tau} e^{-t/\tau}$

$$I(t) = -\frac{8.10^{-6}}{6.10^{-3}} e^{-t/6.10^{-3}}$$

$$\boxed{I(t) = -\frac{4}{3} e^{-10^3 t/6}\ (mA)}$$

The (-) sign in this equation means, the current is the opposite direction compared to the current I in the charging.