Mechanics of the material point

Physics

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Chapter II: Kinematics of the material point

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The aim of point kinematics is to study the motion of a point over time independently of the causes that produce the motion. The objectives are to determine kinematic quantities such as acceleration vectors, velocity, position and the time equation of the trajectory of this point relative to a reference frame chosen by the observer

A. Characteristics of the motion

In kinematics the two fundamental concepts are space and time, because the motion takes place in space as a function of time. Mathematically solving kinematics problems in physics will involve understanding, calculating, and measuring several physical quantities:

Position vector $\vec{OM}$: determines the object's physical location in space relative to an origin in a defined coordinate system.

Velocity vector $\vec{V}$: which determines the variation in magnitude and position of the position vector.

Acceleration vector $\vec{a}$: which determines the variation in magnitude and position of the velocity vector.

B. Motions in various coordinate systems and bases

In mechanics, before studying the motion of a system, it is necessary to indicate the coordinate system in which the motion will be describe. We will explain the motion in different coordinate systems and bases, i.e. the set of three vectors on which we will give the expressions of the position vector, velocity vector and acceleration vector.

The elementary surface area and volume will also be given.

1. Cartesian coordinates

Consider an orthonormal basis denoted $\vec{u}_x, \vec{u}_y, \vec{u}_z$ .This base does not change over time.

Knowing the position vector $\vec{OM}$ also makes it possible to locate the point **M**, which is given by:

$$\vec{OM} = \vec{r} = x\,\vec{u}_x,\ y\,\vec{u}_y,\ z\,\vec{u}_z$$

Using the expression for the position vector, the velocity is:

$$\vec{V} = d\,\vec{OM}\,/\,dt = dx/dt\,\vec{u}_x + dy/dt\,\vec{u}_y + dz/dt\,\vec{u}_z$$

Its modulus is given by:

$$|\vec{V}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

The acceleration is:

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d^2x}{dt^2}\overrightarrow{u_x} + \frac{d^2y}{dt^2}\overrightarrow{u_y} + \frac{d^2z}{dt^2}\overrightarrow{u_z}$$

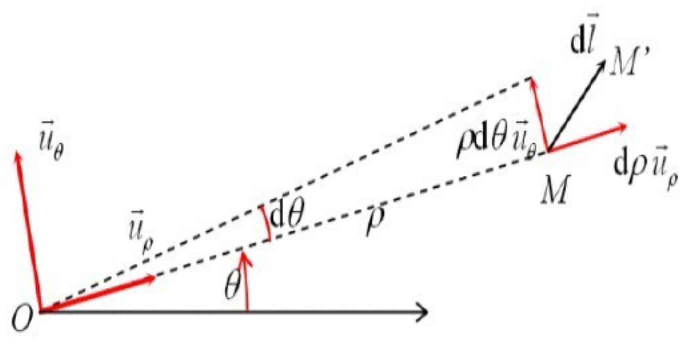
And its module:

$$|\vec{a}| = \sqrt{(a_x)^2 + \left(a_y\right)^2 + (a_z)^2}$$

2. Polar coordinates (in a plane)

The position vector in this frame of reference is written as:

$$\vec{\rho} = \vec{OM} = \|\vec{OM}\|\,\vec{u}_\rho = \rho\,\vec{u}_\rho$$



Relationship between polar and Cartesian coordinates

$$x = \rho\cos\theta$$

$$y = \rho \sin\theta$$

Then:

$$\vec{u}_x = (\cos\theta)\vec{u}_\rho + (\sin\theta)\vec{u}_\theta$$

$$\vec{u}_y = (\sin\theta)\vec{u}_\rho + (\cos\theta)\vec{u}_\theta$$

Switching from polar to Cartesian coordinates:

$$OM = \rho = \sqrt{(x^2 + y^2)}$$

$$cos\theta = x/\rho = x/\sqrt{(x^2 + y^2)}$$

$$sin\theta = y/\rho = y/\sqrt{(x^2 + y^2)}; \tan\theta = y/x$$

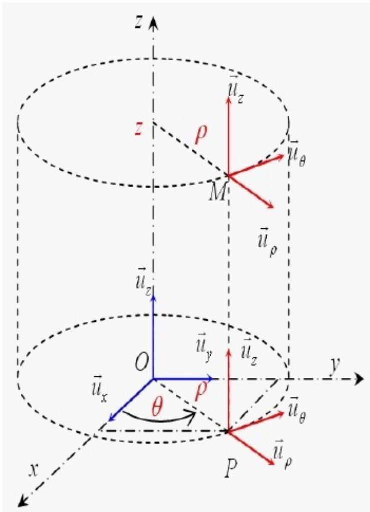
So:

$$\vec{u}_\rho = (\cos\theta)\vec{u}_x + (\sin\theta)\vec{u}_y$$

$$\vec{u}_\theta = (-\sin\theta)\vec{u}_x + (\cos\theta)\vec{u}_y$$

3. Cylindrical coordinates (in space)

To obtain the cylindrical coordinate system, all we need to do is add a third axis to the polar coordinate system (in the **xOy** plane): the **Oz** axis with its Cartesian **z** coordinate (called the coordinate).



The cylindrical coordinates are (ρ,θ,z), and the position vector is written as:

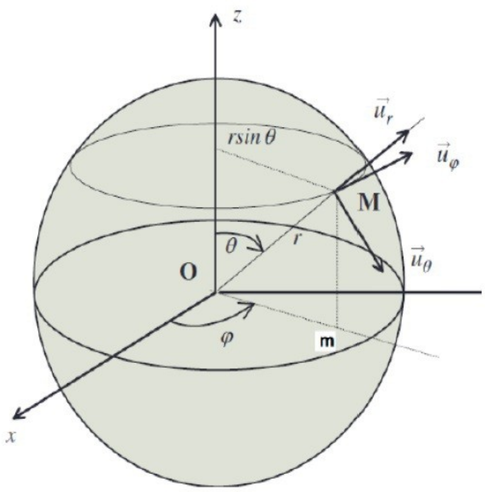
$$\vec{OM} = \rho\,\vec{u}_\rho + z\,\vec{k}$$

$$\vec{V} = d\,\vec{OM}/dt = d\rho/dt\,\vec{u}_\rho + \rho\,d\,\vec{u}_\rho/dt + dz/dt\,\vec{k}$$

And for acceleration:

4. Spherical coordinates (in space)

Spherical coordinates are used to locate a point on a sphere of radius OM= r.
This is typically the case for locating a point on the Earth, for which all you need to do is specify two angles: latitude and longitude. These unit vectors are: $\vec{u}_r, \vec{u}_\theta, \vec{u}_\varphi$
M is defined by the length.
$\vec{r} = \vec{OM} = r\,\vec{u}_r$ And the two angles φ and .



x=rcosθ sinφ
y=rsinθ sinφ
z=rsinθ
The position vector is written

$$\vec{OM} = r\,\vec{u}_r + r\,\theta\,\vec{u}_\theta + r\,\varphi\sin\theta\,\vec{u}_\varphi$$

In the spherical coordinate system, the velocity is given by the following relationship

$$\vec{V} = \dot{r}\,\vec{u}_r + r\,\dot{\theta}\,\vec{u}_\theta + r\,\dot{\varphi}\sin\theta\,\vec{u}_\varphi$$

C. Solved exercices

Exercise1

Rectangular coordinats (x, y, z) of a point are given. Find the cylindrical coordiantion (ρ,φ, z) of the
point:
P1: (1,√3,2) ; P2: (1,1,5) ; P3: (-2√2,2√2, 4)

Solution

$$\rho=\sqrt{(x^2+y^2)}=\sqrt{(1^2+(\sqrt{3})^2)}=\sqrt{4}=2$$

$$\varphi=tg^{-1}((\sqrt{3})/1)=\pi/3$$

$$Z=Z$$

$$P_1(2,\pi/3,2)$$

Exercise2

Spherical coordiantion (r, θ, φ) of a point are given. Find the rectangular coordinats (x, y, z)of the point:
P: (3,0,π) ; P2: (1,π/6,π/6) ; P3: (12,-π/4,π/4)

Solution

$$P_1\,(3,0,\pi)\quad x = r\sin\theta\cos\varphi = 3\sin(0)\cos\pi = 0$$

$$y = r\sin\theta\sin\varphi = 3\sin(0)\sin\pi = 0$$

$$z = r\cos\theta = 3\cos\theta = 3$$

$$P_1\,(0, 0, 3)$$

Exercise 3

Rectangular coordinats (x, y, z) of a point are given. Find the spherical coordiantion (r, θ, φ) of the point:
P_1: (4,0,0) ; P2: (-1,2,1) ; P3: (0,3,0)

Solution

$$P1(4,4,0): r=x^2+y^2+z^2=4^2+4^2+0^2=\sqrt{32}=5,65$$

$$\theta = tg^{-1}(4/4) = \pi/4$$

$$\varphi = tg^{-1}(y/x) = tg^{-1}(4/4) = tg^{-1}(1) = \pi/4$$