Mohamed Khider University of Biskra

Faculty of FSESNV
Department of SM
University Year 2023/2024

Module: Series and Diff. Eq
Level: 2^{nd} Year LMD
Specialty: Physics

Dirigated Work N°1

(SINGLE, DOUBLE AND TRIPLE INTEGRALS)

Exercise 1 *Calculate the following integrals:*

$$\int \frac{dx}{1-x}, \qquad \int \frac{dx}{x^2-3x-4}, \qquad \int_0^{2\pi} \sin^2 x \cos x dx, \qquad \int \sin^2 x dx.$$

1. *Calculate the following integrals (Using a primitive):*

$$\int_{-1}^{2} x^2 dx, \qquad \int_0^2 x\left(x^3+1\right) dx, \qquad \int_0^1 \frac{e^x}{1+e^{2x}} dx.$$

2. *Calculate the following integrals (Using integration by parts):*

$$\int_1^e x^2 \ln x dx, \qquad \int_0^{\frac{\pi}{2}} x \cos x dx, \qquad \int_0^1 x e^{3x} dx.$$

3. *Calculate the following integrals (Using variable change):*

$$\int_e^{e^3} \frac{dx}{x \ln x} \quad (x>0), \qquad \int_0^1 x^2 \sqrt{a^2-x^2} dx \quad (a>0), \qquad \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(2x-\frac{\pi}{6}) dx.$$

Exercise 2 *Calculate the following integrals:*

$$\int_3^4 \int_1^2 \frac{dxdy}{(x+y)^2}, \qquad \int_1^2 \int_x^{x\sqrt{3}} xydxdy, \qquad \int_0^{2\pi} \int_{2\sin\theta}^{2} rdrd\theta, \qquad \int_0^1 \int_{y-1}^{2y} xydxdy.$$

Exercise 3 *Define the integration limits for* $\iint_D f(x,y)dxdy$, D *being delimited by:*

a) $x=2, x=3, y=-1, y=5$ *b)* $y=0, y=1-x^2$

c) $x^2+y^2=4$ *d)* $y=\frac{2}{1+x^2}, y=x^2$

Exercise 4 *Calculate the following integrals:*

a) $\iint\limits_D |x+y|\,dxdy$, *où* $D = \left\{(x,y) \in \mathbb{R}^2 / \, |x| < 1, |y| < 1\right\}$.

b) $\iint\limits_D \dfrac{1}{1+x^2+y^2} dxdy$, *où* $D = \left\{(x,y) \in \mathbb{R}^2 / x^2+y^2 < 1\right\}$.

c) $\iint\limits_D \dfrac{xy}{x^2+y^2} dxdy$, *où* $D = \left\{(x,y) \in \mathbb{R}^2 / x > 0, y > 0, x+y < 1\right\}$.

d) $\iint\limits_D \sqrt{x^2+y^2} dxdy$, *où* $D = \left\{(x,y) \in \mathbb{R}^2 / 0 < y < x < 1\right\}$.

Exercise 5 *Calculate the area of the figure bounded by the curves:*

a) $y^2 = 2x, y = x$; $\quad y^2 = 4x, x + y = 3, y = 0$

b) $y = \sin x, y = \cos x, x = 0$; $\quad y^2 = 4x + 4, y^2 = -4x + 4$

Exercise 6 *Calculate the volume bounded by the surfaces:*

a) $\dfrac{x^2}{9} + \dfrac{y^2}{16} + \dfrac{z^2}{25} = 1$ $\quad$ *b)* $x^2 + z^2 = R^2, y^2 + z^2 = R^2$ $\quad$ *c)* $x^2 + y^2 + z^2 = 1, x^2 + y^2 = z^2$

Exercise 7 *Calculate the volume bounded by the surfaces:*

$\iiint\limits_V z dxdydz$ $\quad$ *où* $V = \left\{(x,y,z) \in \mathbb{R}^3 / x \geq 0, y \geq 0, z \geq 0, z \leq 1 - y^2 \text{ et } x + y \leq 1\right\}$

$\iiint\limits_V xyz dxdydz$ $\quad$ *où* $V = \left\{(x,y,z) \in \mathbb{R}^3 / 0 < z < 1, x^2 + y^2 < z^2\right\}$

$\iiint\limits_V \left(\dfrac{1}{\sqrt{x^2+y^2}} + \dfrac{1}{z}\right) dxdydz$ $\quad$ *où* $V = \left\{(x,y,z) \in \mathbb{R}^3 / 0 < x^2 + y^2 + z^2 < 1, 0 < x^2 + y^2 < z^2, z > 0\right\}$

Charged of courses

Dr. OUAAR, F