

Interrogation 01

Exercise 1 _____ 06 points

Solve the following differential equations :

1. $y' + 2y - 1 = x^2 e^{-2x} + 2e^{3x} + x$
2. $y' = |y - x|$.
3. $y' + 2y - (x+1)\sqrt{y} = 0, \quad y(0) = 1$

① $y' + 2y = x^2 e^{-2x} + 2e^{3x} + x + 1, \quad y_0 = y_H + y_P$
 $y_H = k e^{-2x} / k \in \mathbb{R}; \quad y_P = y_{P_1} + y_{P_2} + y_{P_3}$
 $y_{P_1} = \alpha(ax^2 + bx + c) e^{-2x} = \frac{1}{3} x^2 e^{-2x}$
 $y_{P_2} = A e^{3x} = \frac{2}{15} e^{3x}$
 $y_{P_3} = A x + b = \frac{1}{2} x + \frac{1}{4}$
 $y_0 = k e^{-2x} + \frac{x^2}{3} e^{-2x} + \frac{2}{15} e^{3x} + \frac{x}{2} + \frac{1}{4}$

② $y' = |y - x| = \begin{cases} y - x & x \geq 0 \\ -y + x & x < 0 \end{cases}$

$\begin{cases} y' - y = 0 & \Rightarrow y_H = C e^x / C \in \mathbb{R} \\ y' + y = 0 & \Rightarrow y_H = D e^{-x} / D \in \mathbb{R} \end{cases}$

$\begin{cases} y_{P_1}^? & y' - y = -x \Rightarrow y_{P_1} = C(x) e^x \Rightarrow y_{P_1} = x + 1 \\ y_{P_2}^? & y' + y = x \Rightarrow y_{P_2} = D(x) e^{-x} \Rightarrow y_{P_2} = x - 1 \end{cases}$

$y_0 = \begin{cases} C e^x + x + 1 & y \geq x \\ D e^{-x} + x - 1 & y < x \end{cases}$

③ $y' + 2y - (x+1)y^{1/2} = 0$

$y' + 2y = (x+1)y^{1/2} \Rightarrow \frac{y'}{y^{1/2}} + 2y^{-1/2} = x+1$

$\frac{y'}{y^{1/2}} + 2y^{-1/2} = x+1; \quad Z = \frac{1}{y^{-1/2}} = \sqrt{y}$

$2Z' + 2Z = x+1$

Ⓐ $2Z' + 2Z = 0 \Rightarrow Z' + Z = 0 \Rightarrow Z_H = k e^{-x}$

Ⓑ $Z_P = Ax + b \Rightarrow \begin{cases} 2A + 2Ax + 2b = x+1 \\ 2A = 1 \\ 2b + 2A = 1 \end{cases} \Rightarrow A = 1/2, b = 0$

$Z_P = \frac{x}{2}$

$Z = Z_H + Z_P = k e^{-x} + \frac{1}{2} x$

$Z = \frac{1}{y^{-1/2}} \Rightarrow y^{-1/2} = \frac{1}{Z} \Rightarrow y = Z^2$

$y = (k e^{-x} + \frac{1}{2} x)^2$

$y(0) = 1 = (k e^0 + \frac{1}{2} \cdot 0)^2 \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$

$y = (e^{-x} + \frac{1}{2} x)^2 \quad \text{et} \quad y = (\frac{x}{2} - e^{-x})^2$